SR Specialized Review

An In-Depth Review of Concrete Design Methods and Standards

Concrete Design

for the Civil PE and Structural SE Exams

Second Edition

C. Dale Buckner, PhD, PE, SECB

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for the Civil PE and Structural SE Exams

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Preface and Acknowledgments

I have written this book primarily for engineers who are studying to take the NCEES 16-hr structural engineering (SE) exam or the structural depth section of the NCEES civil Principles and Practice of Engineering (PE) exam. The civil PE and SE exams—even the breadth section of the civil PE exam—often contain structural questions that go beyond the basics. This book provides a more thorough review for those who want to be prepared for all questions in concrete design. It is also suitable as a reference for students taking introductory courses in reinforced or prestressed concrete.

For this second edition, nomenclature, equations, examples, and practice problems have been updated so that they are consistent with NCEES-adopted codes and specifications.

This is not a comprehensive textbook on the theory of reinforced concrete structures. I have included the basic theory you will need to solve the types of concrete design problems likely to appear on the exams, but I have not gone into detailed derivations and historical summaries of code criteria. Among the topics covered in this book are the effects of flexure, shear, torsion, and axial loads on members; serviceability; development of reinforcement; behavior of one-way and two-way floor systems; prestressed concrete members; and seismic design criteria.

I have included many examples to illustrate how ACI code criteria should be applied, and in the last chapter you will find 37 practice problems with complete solutions. Only U.S. customary units are used in these examples and practice problems, consistent with the exam format.

While studying this book, you'll need to have a copy of the building code ACI 318 at hand. Either the 2008 or the 2011 edition will do for this purpose, because there are only minor differences between them in regard to the problems covered in this book. (For the exam itself, however, you will want to have the same edition the current exam is based on. This is explained more fully in How to Use This Book.)

I appreciate the help provided by John Mercer, PE, who reviewed an early draft of the first edition. Thank you to PPI's product development and implementation staff, including Sarah Hubbard, director of product development and implementation; Cathy Schrott, production services manager; Jenny Lindeburg King, associate editor-in-chief; Magnolia Molcan, editorial project manager; Ellen Nordman, lead editor on this book; David Chu, Nicole Evans, Julia Lopez, Scott Marley, and Heather Turbeville, copy editors; Ralph Arcena, EIT, calculation checker; Tom Bergstrom, technical illustrator; and Kate Hayes, production associate.

Finally, if you find an error in this book, please let me know by using the error reporting form on the PPI website, found at **ppi2pass.com/errata**. Valid submitted errors will be posted to the errata page and incorporated into future printings of this book.

C. Dale Buckner, PhD, PE, SECB

How to Use This Book

What You'll Need

This book is designed to complement and be used with PPI's *Civil Engineering Reference Manual* (CERM), *Structural Depth Reference Manual* (CEST), or *Structural Engineering Reference Manual* (STRM). CERM, CEST, and STRM are the basic texts for anyone studying for the civil PE or structural engineering (SE) exams, and each book contains an introduction to the basic concepts and most common applications pertaining to concrete design.

It is essential that this book be used with the American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318) and Commentary (ACI 318R)*. The following chapters are meant to explain and clarify those aspects of the building code that are most likely to come up during the civil PE and SE exams, but it will be frequently assumed along the way that you can refer directly to the code itself when necessary.

Throughout the book, citations to code criteria refer to the 2011 edition of the ACI code. For example, the citation "ACI Sec. 7.12" refers to Sec. 7.12 of ACI 318-11. For the problems covered in this book, however, the differences between ACI 318-08 and ACI 318-11 are minor and amount to no more than the notation used for a few variables. That means you can study this book with either ACI 318-08 or ACI 318-11 at hand.

When it comes to the exam itself, of course, it's important to bring the editions of the design standards that the current exam is based on. Check the NCEES website at ncees.org for the current design standards for your exam. You can also check PPI's website at **ppi2pass.com/civil** or **ppi2pass.com/structural** for current information and answers to frequently asked questions (FAQs) about the civil PE or SE exams.

Appendix C in both ACI 318-08 and ACI 318-11 permits an alternative design approach using load and resistance factors from earlier code editions. Nevertheless, the examples and practice problems in this book employ only the unified approach consistent with the main body of ACI 318.

Studying with This Book

Each chapter in this book treats a different topic. If you only want to brush up on a few specific subjects, you may want to study only those particular chapters. However, later chapters frequently build on concepts and information that have been set out in earlier chapters, and the book is most easily studied by reading the chapters in order.

The civil PE and SE exams are open book, so it is a very good idea as you study to mark pages in both ACI 318 and this book that contain important information, such as tables, graphs, and commonly used equations, for quick reference during the exam. (Some states don't allow removable tabs in books brought into the exam. Check with your state board, or use permanent tabs.) Become as familiar as possible with this book and with ACI 318. Remember that preparation and organization are as important to passing the PE and SE exams as knowledge is.

Throughout the book, example problems illustrate how to use the standard design principles, methods, and formulas to tackle common situations you may encounter on the exam. Take your time with these and make sure you understand each example before moving ahead. Keep in mind, though, that in actual design situations there are often several correct solutions to the same problem.

Practice Problems for Each Exam

In the last chapter of the book you'll find 37 practice problems. Whether you're studying for the structural depth section of the civil PE exam, or the SE exam, you'll find practice problems that are similar in scope, subject matter, and difficulty to problems you'll encounter on the actual exam.

The NCEES PE exam in civil engineering consists of two 4-hour sections, separated by a one-hour lunch period. Both sections contain 40 multiple-choice problems, and you must answer all problems in each section to receive full credit. There are no optional questions.

The breadth section is taken in the morning by all examinees, and may include general concrete problems. In the afternoon, you are able to select from five depth sections: water resources and environmental, geotechnical, transportation, construction, and structural. The structural depth section covers a range of structural engineering topics including loads and load applications; forces and load effects; materials and material properties; component design and detailing; codes, standards, and guidance documents; and temporary structures and other topics. The first 25 practice problems in the last chapter of this book are appropriate for the topics covered on the structural depth section of the civil PE exam.

The structural engineering (SE) exam is a 16-hour exam offered in two parts. The first part, vertical forces (gravity/other) and incidental lateral, takes place on a Friday. The second part, lateral forces (wind/earthquake), takes place on a Saturday. Each part comprises a breadth section and a depth section. The breadth sections in the morning are each four hours and contain 40 multiple-choice problems that cover a range of structural engineering topics specific to vertical and lateral forces. The depth sections in the afternoon are also each four hours, but instead of multiple-choice problems, they contain essay (design) problems. You may choose either the bridges or the buildings depth section, but you must work the same depth section across both parts of the exam. That is, if you choose to work buildings for the lateral forces part, you must also work buildings for the vertical forces part.

According to NCEES, the vertical forces (gravity/other) and incidental lateral breadth section covers analysis of structures, including loads and methods; design and details of structures, including general structural considerations, structural systems integration, structural steel, light gage/cold-formed steel, concrete, wood, masonry, foundations, and retaining structures; and construction administration, including procedures for mitigating nonconforming work and inspection methods.

The lateral forces (wind/earthquake) breadth section covers analysis of structures, including lateral forces, lateral force distribution, and methods; design and detailing of structures, including general structural considerations, structural systems integration, structural steel, light gage/cold-formed steel, concrete, wood, masonry, foundations, and retaining structures; and construction administration, including structural observation.

The vertical forces (gravity/other) and incidental lateral depth section buildings module covers loads, lateral earth pressures, analysis methods, general structural considerations (e.g., element design), structural systems integration (e.g., connections), and foundations and retaining structures. The bridges module covers gravity loads, superstructures, substructures, and lateral loads other than wind and seismic. It may also require pedestrian bridge and/or vehicular bridge knowledge.

The lateral forces (wind/earthquake) depth section buildings module covers lateral forces, lateral force distribution, analysis methods, general structural considerations (e.g., element design), structural systems integration (e.g., connections), and foundations and retaining structures. The bridges module covers gravity loads, superstructures, substructures, and lateral forces. It may also require pedestrian bridge and/or vehicular bridge knowledge.

The first 35 practice problems in the last chapter of this book are patterned after questions on the breadth sections of the SE exam. These problems cover the full range of concrete design topics and show the range of effort needed to solve them. The last two problems in this book are scenario problems related to concrete building structures, and are intended to illustrate the type of problems likely to appear on the depth sections of the SE exam.

When you feel comfortable with the principles and methods taught by the example problems, work these practice problems under exam conditions. Try to solve them without referring to the solutions, and limit yourself to the tools and references you'll have with you during the actual exam—an NCEES-approved calculator, pencil and scratch paper, and the references you plan to bring. When you have finished with the practice problems that are tailored to the exam you're taking, try some of the others as well. Getting all the practice in problem-solving that you can is one of the best ways to improve how you do on exam day.

After studying this book, you should be able to solve most common problems in structural concrete, both on the exams and in real design applications. Good luck on the exam!

Codes and References Used to Prepare This Book

American Concrete Institute (ACI). *Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary (ACI 318R-11).*

American Society of Civil Engineers (ASCE). *Minimum Design Loads for Buildings and Other Structures* (ASCE 7-10).

Concrete Reinforcing Steel Institute (CRSI). *Manual of Standard Practice.*

Kamara, M. E., and B. G. Rabbat. *Notes on ACI 318-11 Building Code Requirements for Structural Concrete with Design Applications.* Portland Cement Association (PCA).

Kosmatka, S. H., B. Kerkhoff, and W. C. Panarese. *Design and Control of Concrete Mixtures.* Portland Cement Association (PCA).

Nilson, A. H., D. Darwin, and C. W. Dolan. *Design of Concrete Structures.* New York: McGraw-Hill.

Precast/Prestressed Concrete Institute. *PCI Design Handbook: Precast and Prestressed Concrete.*

Post-Tensioning Institute (PTI). *Post-Tensioning Manual.*

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Nomenclature

symbol	definition (units)
a	depth of equivalent rectangular stress block (in)
a	length of cantilevered beam (in)
a_2	depth of compression zone below bottom of trough (in)
a_v	shear span (in)
A	cross-sectional area (in^2)
A_1	loaded area (in^2)
A_2	area of lower base of largest frustum of pyramid, cone, or tapered wedge contained wholly within support and having loaded area as upper base with slide slopes of 1 vertical to 2 horizontal (in^2)
A_b	area of individual bar (in^2)
A_c	area of core of spirally reinforced compression member measured to outside diameter of spiral (in^2)
A_c	equivalent area of concrete compression zone (in^2)
A_{ch}	cross-sectional area of a structural member measured out-to-out of transverse reinforcement (in^2)
A_{cp}	area enclosed by outside perimeter of concrete cross section (in^2)
A_{cv}	gross area of concrete section bounded by web thickness and length of section in direction of shear force (in^2)
A_{cw}	area of concrete wall (in^2)
A_f	area of reinforcement in bracket or corbel resisting factored moment (in^2)
A_g	gross cross-sectional area of column (in^2)

symbol	definition (units)
A_h	area of shear reinforcement parallel to flexural tension reinforcement (in^2)
A_j	effective cross-sectional area within a joint (in^2)
A_l	total area of longitudinal reinforcement to resist torsion (in^2)
A_n	area of reinforcement in bracket or corbel resisting tensile force N_{uc} (in^2)
A_o	gross area enclosed by shear flow path (in^2)
A_{oh}	area enclosed by centerline of outermost closed transverse torsional reinforcement (in^2)
A_{ps}	area of prestressed reinforcement in tie or tension zone (in^2)
A_s	area of tension reinforcement (in^2)
A'_s	area of compression reinforcement (in^2)
A_{s1}, A_{s2}	area of steel in layers 1 and 2 (in^2)
A_{sc}	area of primary tension reinforcement in bracket or corbel (in^2)
A_{sh}	total cross-sectional area of transverse reinforcement (including crossties) within spacing s and perpendicular to dimension h_c (in^2)
A_{sl}	area of steel in longitudinal direction (in^2)
A_{st}	total area of longitudinal steel (in^2)
A_{sw}	area of steel in web area of wall (in^2)
A_t	area of one leg of closed stirrup resisting torsion within spacing s (in^2)
A_{tr}	transformed steel area (in^2)
A_v	area of shear reinforcement within spacing s (in^2)

symbol	definition (units)
A_{vf}	area of shear friction reinforcement (in^2)
A_{vo}	total area in outer legs of closed stirrups (in^2)
A_y	vertical reaction (kip)
b	width of compression face of member (in)
b_1	width of critical section defined in ACI 318 Sec. 13.5.3.2 measured in direction of span (in)
b_2	width of critical section defined in ACI 318 Sec. 13.5.3.2 measured in direction perpendicular to b_1 (in)
b_c	least dimension center to center of closed hoop (in)
b_e	effective width of T-beam (in)
b_o	perimeter of critical section for slabs and footings (in)
b_w	web width (in)
b_y	vertical reaction at support B (lbf)
B	width of support or member (in)
c	distance from extreme compression fiber to neutral axis (in)
c_1	size of rectangular or equivalent rectangular column, capital, or bracket measured in direction of span(in)
c_2	size of rectangular or equivalent rectangular column, capital, or bracket measured transversely to direction of span (in)
c_b	smaller of (a) distance from center of bar or wire to nearest concrete surface or (b) one-half distance between bars or wires being developed (in)
c_c	clear cover from nearest surface of flexural tension reinforcement (in)
C	component of force (lbf)
C	compressive force (lbf)
C_c	compressive force in concrete (lbf)
C_d	amplification factor for seismic deflection
C_m	factor relating actual moment diagram to equivalent uniform moment diagram
C_s	column strip width (in)

symbol	definition (units)
C'_s	compressive force in steel (in^2)
C_{s1}, C_{s2}	effective compressive force in left and right reinforcement, respectively (lbf)
C_T	coefficient of thermal expansion (in/in-°F)
d	distance from extreme compression fiber to centroid of longitudinal tension reinforcement (in)
d	equivalent diameter (in)
d'	distance from extreme compression fiber to centroid of compression reinforcement (in)
d_b	nominal diameter of bar, wire, or prestressing strand (in)
d_{bl}	diameter of longitudinal bar (in)
d_c	diameter of circle formed by spiral reinforcement (in)
d_h	diameter of transverse reinforcement (in)
d_p	distance from extreme compression fiber to centroid of prestressed reinforcement (in)
d_s	diameter of steel bar used in spiral reinforcement (in)
d_t	distance from extreme compression fiber to extreme tension steel (in)
D	dead loads (lbf)
e	eccentricity of force (in)
e_c	eccentricity of prestressing tendon at midspan of member (in)
e_e	eccentricity of prestressing tendon at end of member (in)
E	load effects of earthquakes (lbf)
E_c	modulus of elasticity of concrete (lbf/in^2)
E_{cb}	modulus of elasticity of beam concrete (lbf/in^2)
E_{cs}	modulus of elasticity of slab concrete (lbf/in^2)
E_{ps}	modulus of elasticity of prestressing steel (lbf/in^2)
E_s	modulus of elasticity of steel (lbf/in^2)
EI	flexural stiffness of compression member (in^2-lbf)
f	stress (lbf/in^2)

symbol	definition (units)
f_b	calculated stress at bottom fiber (lbf/in^2)
f_c	calculated compressive stress in concrete (lbf/in^2)
f'_c	specified compressive strength of concrete (lbf/in^2)
$\sqrt{f'_c}$	square root of specified compressive strength of concrete (lbf/in^2)
f'_{ci}	compressive strength of concrete at time of initial prestress (lbf/in^2)
$\sqrt{f'_{ci}}$	square root of compressive strength of concrete at time of initial prestress (lbf/in^2)
f_{ct}	average splitting tensile strength of lightweight aggregate concrete (lbf/in^2)
f_{ij}	flexibility influence coefficient: displacement at point i caused by a unit load applied at point j (lbf/in^2)
f_{pe}	compressive stress in concrete due to effective prestress forces only (lbf/in^2)
f_{pi}	stress in prestressed reinforcement immediately after anchorage (lbf/in^2)
f_{ps}	stress in prestressed reinforcement at nominal strength (lbf/in^2)
f_{pu}	specified tensile strength of prestressing steel (lbf/in^2)
f_{py}	specified yield strength of prestressing steel (lbf/in^2)
f_r	modulus of rupture of concrete (lbf/in^2)
f_s	calculated stress in reinforcement at service loads (lbf/in^2)
f'_s	stress in compression reinforcement (lbf/in^2)
f^*_s	calculated stress in transformed steel area (lbf/in^2)
f_{se}	effective stress in prestressed reinforcement after allowance for all prestress losses (lbf/in^2)
f_t	extreme fiber stress in tension in precompressed tensile zone (lbf/in^2)
f_y	specified yield strength of reinforcement (lbf/in^2)
f_{yt}	specified yield strength of transverse reinforcement (lbf/in^2)

symbol	definition (units)
F_v	vertical component of prestress force (lbf)
h	height (in)
h	overall thickness of member (in)
h_1	depth of trough (in)
h_c	cross-sectional dimension of column core measured center to center of confining reinforcement (in)
h_s	overall thickness of slab (in)
h_w	height of wall from base to top (in)
h_x	maximum horizontal spacing of hoop or crosstie legs on all faces of column (in)
I	moment of inertia (in^4)
I_{cr}	moment of inertia of cracked section transformed to concrete (in^4)
I_e	effective moment of inertia for computation of deflection (in^4)
I_g	moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement (in^4)
I_{se}	moment of inertia of reinforcement about centroidal axis of member cross section (in^4)
J_c	polar moment of inertia of critical area around column (in^4)
k	effective length factor for compression members
k	ratio of distance between extreme compression edge and elastic neutral axis to effective beam depth
K	relative rigidity (lbf/in)
K_n	parameter for column interaction diagram (lbf/in^2)
K_{tr}	transverse reinforcing index for calculating development length
l_1	length of span in direction that moments are being determined, measured center to center of supports (in)
l_2	length of span transverse to l_1, measured center to center of supports (in)
l_a	additional embedment length at support or point of inflection (in)

symbol	definition (units)
l_c	length of compression member in frame (in)
l_c	length of concrete (in)
l_d	development length (in)
l_{dc}	development length in compression (in)
l_{dcc}	development length in confined concrete (in)
l_{dh}	development length of standard hook in tension, measured from critical section to outside end of hook (in)
l_{dm}	required development length of bar outside confined core (in)
l_e	length of embedment (in)
l_n	length of clear span from face to face of beams or other supports (in)
l_o	minimum length, measured from joint face along axis of structural member, over which transverse reinforcement must be provided (in)
l_o	overall length of continuous beam (in)
l_s	length of tendon (in)
l_t	transfer length (in)
l_u	unbraced length of column (in)
l_w	overall length of wall (in)
L	length (in)
L	live loads (lbf)
m_{cs}	factored moment in column strip per foot width (in-lbf)
M	maximum unfactored moment due to service load, including $P\Delta$ effects (in-lbf)
M_1, M_2	smaller and larger factored end moment on compression member, respectively (in-lbf)
M_a	maximum moment in member at stage deflection is computed (in-lbf)
M_c	moment at the face of the joint, corresponding to the nominal flexural strength of the column framing into that joint (in-lbf)
M_{cr}	moment causing flexural cracking at section due to externally applied loads (in-lbf)

symbol	definition (units)
M_g	moment at face of joint, corresponding to nominal flexural strength of girder including slab where in tension (in-lbf)
M_n	nominal moment strength at section (in-lbf)
M_{nb}	nominal flexural strength of beam (in-lbf)
M_{nc}	nominal flexural strength of concrete (in-lbf)
M_o	total factored static moment (in-lbf)
M_{pr}	probable flexural strength of members, with or without axial load, determined using properties of member at joint faces (in-lbf)
M_{ps}	moment caused by prestress force (in-lbf)
M_s	moment due to loads causing appreciable sway (in-lbf)
M_u	factored moment at section (in-lbf)
M_{ul}, M_{ur}	factored moment at left and right section, respectively (in-lbf)
n	modular ratio of elasticity
n	number of bars or wires
N	normal force in depressed prestress strand (lbf)
N_{uc}	factored tensile force applied at top of bracket or corbel acting simultaneously with V_u, taken as positive for tension (lbf)
p_{cp}	outside perimeter of the concrete cross section (in)
p_h	perimeter of centerline of outermost closed transverse torsional reinforcement (in)
P	unfactored axial load (lbf)
P_0	nominal axis load strength at zero eccentricity (lbf)
P_c	critical load (lbf)
P_i	initial prestress force (lbf)
P_n	nominal strength of cross section subject to compression (lbf)
$P_{n,\text{max}}$	nominal axial load strength adjusted for accidental eccentricity (lbf)
P_u	factored axial load at given eccentricity (lbf)

symbol	definition (units)
q_u	factored distributed load on two-way slab (lbf/ft^2)
r	radius of gyration of cross section of compression member (in)
R	beam reaction (lbf)
R_n	nominal strength (lbf or in-lbf)
R_n	parameter for column interaction diagram (lbf/in^2)
R_u	structural action caused by factored loads (lbf or in-lbf)
s	pitch
s	sag of prestressing tendon (in)
s	spacing of shear or torsion reinforcement (in)
s	spacing of transverse reinforcement measured along longitudinal axis of structural member (in)
s_1, s_2	spacing of adjacent beams to left and right, respectively (in)
s_o	maximum spacing of transverse reinforcement (in)
S	elastic section modulus of section (in^3)
t	thickness
t_w	thickness of wall (lbf)
T	temperature
T	tension force (lbf)
T_n	nominal torsional moment strength (in-lbf)
T_u	factored torsional moment at section (in-lbf)
T_{ut}	threshold torsional moment (in-lbf)
v_c	shear stress caused by punching shear and unbalanced moment (lbf/in^2)
v_u	nominal factored shear stress (lbf/in^2)
V_c	nominal shear strength provided by concrete (lbf)
V_d	shear force at section due to unfactored dead load (lbf)
V_e	effective shear strength (lbf)
V_n	nominal shear strength (lbf)

symbol	definition (units)
V_s	nominal shear strength provided by shear reinforcement (lbf)
V_u	factored shear force at section (lbf)
w	unfactored load (lbf/ft)
w_c	unit weight of concrete (lbf/ft^3)
w_d	dead load (lbf/ft or lbf/ft^2)
w_e	equivalent uniform load from prestress (lbf/ft or lbf/ft^2)
w_g	unfactored weight of girder (lbf)
w_l	live load (lbf/ft^2)
w_u	factored load per unit length of beam or unit area of slab (lbf/ft or lbf/ft^2)
W	wind load
x	distance along member axis (in)
x	shorter overall dimension of rectangular part of cross section (in)
$\overline{x}$	distance from column edge to plastic centroid (in)
y	longer overall dimension of rectangular part of cross section (in)
$\overline{y}$	vertical position of centroid (in)
y_t	distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension (in)
α	constant used to compute minimum stirrup requirement for beams
α	reinforcement location factor
α_c	constant relating wall height to length for computing shear strength of wall
α_f	ratio of flexural stiffness of beam section to flexural stiffness of width of slab
α_{fm}	average value of α_f for all beams on edges of a panel
α_s	constant used to compute V_u in slabs and footings
β	ratio of clear spans in long direction to short direction of two-way slabs
β_1	factor defined in ACI 318 Sec. 10.2.7.3
β_c	ratio of long side to short side of concentrated load of reaction area

symbol	definition (units)
β_{dns}	ratio of factored dead load to total load
γ	reinforcement size factor
γ_f	fraction of unbalanced moment transferred by flexure at slab-column connections
γ_p	factor for type of prestressing steel
γ_v	fraction of unbalanced moment transferred by eccentricity of shear at slab-column connections
δ	moment magnification factor for frames braced against sidesway
δ_s	moment magnification factor for frames not braced against sidesway
δ_{xe}	elastic deflection (in)
Δ	deflection (in)
Δ_i	initial deflection of a prestressed member (in)
Δ_{lt}	maximum long-term deflection (in)
Δ_o	total elongation of tendon (in)
ε	strain (in/in)
ε_c	ultimate strain in concrete (in/in)
ε_{ce}	strain in concrete under effective prestress (in/in)
ε_{ps}	strain in prestressed steel at nominal flexural strength (in/in)
ε_s	strain in tension steel (in/in)
ε_s'	strain in compression steel (in/in)
ε_{se}	strain in prestressed steel under effective prestress (in/in)
ε_{su}	ultimate steel strain (in/in)
ε_t	net tensile strain in extreme tension steel at nominal strength (in/in)
ε_y	strain in reinforcement at first yield (in/in)
λ	lightweight aggregate concrete factor
λ	multiplier for additional long-term deflection as defined in ACI 318 Sec. 9.5.2.5
λ_1	factor to adjust member overall thickness to account for steel strength
λ_2	factor to adjust member overall thickness to account for concrete unit weight
μ	coefficient of static friction

symbol	definition (units)
ξ	time-dependent factor for sustained load
ρ	ratio of non-prestressed tension reinforcement $= A_s/bd$
ρ'	ratio of compression reinforcement $= A_s'/bd$
ρ'	ratio of nonprestressed compression reinforcement
ρ_g	ratio of total reinforcement area to cross-sectional area of column
ρ_l	ratio of longitudinal wall steel to gross area of wall
$\rho_{\max}$	maximum steel ratio for which extreme tension strain equals or exceeds 0.005
$\rho_{\min}$	minimum steel ratio $= A_{s,\min}/bd$
ρ_p	ratio of prestressed reinforcement $= A_{ps}/bd_p$
ρ_s	ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced compression member
ρ_t	ratio of transverse wall steel to gross area of wall
ϕ	strength reduction factor
ψ	ratio of summation of column stiffness to summation of beam stiffness for joint in frame
ψ_e	factor used to modify development length for bar coating
ψ_s	factor used to modify development length for bar size
ψ_t	factor used to modify development length for bar position
ω	tension steel index $(\rho f_y/f_c')$
ω'	compression steel index $(\rho' f_y/f_c')$
Ω	bearing strength coefficient

Subscripts

a	above
ave	average
b	balanced or below
bm	beam
c	concrete
col	column

cs	column strip
d	dead load
e	effective or equivalent
ext	exterior
f	final
i	initial
int	interior
j	joint or joist
l	left or live load
lw	lightweight
max	maximum
min	minimum
ped	pedestal
r	right
rt	right
s	steel
sec	secondary
sl	slab
sup	superimposed
t	torsional
w	wall or wind

1

Materials

1. Properties of Fresh and Hardened Concrete

Concrete is a material composed of aggregates (which may be gravel, sand, and so forth) cemented together.

Cement is mixed with water to form a paste. This mixture coats and surrounds the aggregates. A chemical reaction between the cement and water, called *hydration*, produces heat and causes the mixture to solidify and harden, binding the aggregates into a rigid mass.

The cement used for most structural concrete is portland cement. A portion of the portland cement is sometimes replaced by fly ash, silica fume, or other supplemental cementitious material.

The properties of the hardened concrete can be affected by a number of factors, but the most important is the ratio of water to cementitious materials. More water is always added to the mix than is necessary for the chemical reaction with the concrete, so that the fresh concrete has a workable consistency. The excess water eventually evaporates, causing shrinkage and making the concrete more porous. As the water content of the cement paste is increased, then, the workability of the fresh concrete is also increased, but the strength and durability of the hardened concrete is reduced. Several chemical admixtures, called *plasticizers*, are available that can improve fresh concrete's workability without increasing its water-cement ratio. An alternative use is to reduce the water needed in a mix while maintaining workability, and plasticizers are thus often called *water reducers* or *water-reducing admixtures.*

Several other chemical admixtures are used to alter the properties of either the fresh or the hardened concrete. Calcium chloride, for example, may be added to accelerate hydration. But calcium chloride is a source of free chloride ions, and these can cause the steel reinforcement in the concrete to deteriorate. For this reason, design specifications either forbid or limit the amount of calcium chloride that can be used.

As the cement paste is mixed, small bubbles of air are trapped in it. Sometimes a chemical admixture called an *air entrainer* or *air-entraining agent* is added to the paste to increase the creation of these air bubbles. This entrained air makes the resulting concrete more resistant to freeze-thaw deterioration. A secondary benefit is that it also improves the workability of the fresh concrete. Typical air content, by volume, ranges from about 1% to 2% in non-air-entrained concrete to as much as 6% in air-entrained concrete.

Cement hardens more rapidly at higher temperatures. *Retarders* are sometimes added in hot weather and other cases where it is desirable to slow the rate of hydration. This can help prevent partial hardening of the concrete before pouring is complete.

A variety of materials are used for aggregate. (*Aggregate* can refer to all the material, such as gravel, or to one piece of the material, such as a single stone.) The general requirement is that aggregates must be sound, durable, and nonreactive with other constituents in the concrete. Since aggregates are relatively inexpensive, it is desirable that they occupy as much of the concrete's volume as possible. This is accomplished by controlling the *gradation* of the aggregates in such a way that the voids between the larger aggregates are filled by progressively smaller particles.

Aggregates are classified as fine or coarse. Fine aggregates will pass through a sieve of 1/4 in mesh openings, and coarse aggregates will not. It is desirable to use the largest aggregates that can be placed without causing *segregation*—the uneven distribution of coarse aggregates in the mixture—as the fresh concrete flows around reinforcing steel, inserts, or other items embedded in the concrete element. For most buildings, bridges, and comparable structures, the maximum size of the aggregates is within a range of about 3/4 in to 1 1/2 in.

Every aspect of batching, mixing, placing, consolidating, and curing concrete can significantly influence its behavior. It is particularly important to ensure that the temperature of the concrete during curing is within a tolerable range and that the mixture remains moist. Techniques for producing quality concrete are described in detail in *Design and Control of Concrete Mixtures*, by the Portland Cement Association.

2. Specifying Concrete

Structural concrete is specified in terms of two basic parameters: unit weight, w_c, and compressive strength, f'_c.

A. Unit Weight

The unit weight of concrete is defined as the weight of a cubic foot of hardened concrete. It is denoted by the symbol w_c. The type of aggregates used in the concrete controls its unit weight. Unit weights range from 90 lbf/ft^3 for structural lightweight concrete up to about 160 lbf/ft^3 for normal weight concrete.

Special applications, such as insulation, require extremely lightweight concrete, but concretes lighter than 90 lbf/ft^3 are not permitted for structural applications. Heavyweight concrete uses iron ore or steel slugs for aggregate and yields concrete with a unit weight in excess of 200 lbf/ft^3. But heavyweight concrete is rarely encountered in routine design.

In the ACI code, distinction is made between *all-lightweight* and *sand-lightweight* concrete. All-lightweight concrete contains only lightweight aggregates, whereas sand-lightweight concrete contains lightweight coarse aggregate along with natural sand for the fine aggregate. All-lightweight concrete results in unit weights near the lower bound of 90 lbf/ft^3, and sand-lightweight concrete has a unit weight approaching 115 lbf/ft^3.

The unit weight of plain normal weight concrete is approximately 145 lbf/ft^3. The unit weight of a reinforced concrete member is estimated as 150 lbf/ft^3, a slightly heavier unit weight that allows for the fact that the steel in the member is heavier than the concrete it displaces. These values are used throughout this book whenever normal weight concrete is specified.

B. Specified Compressive Strength

The specified compressive strength of concrete is the expected compressive stress at failure of a cylinder of a standard size that is cast, cured, and tested in accordance with ASTM specifications. The symbol for specified compressive strength is f'_c. The concrete cylinder typically has a diameter of either 4 in or 6 in and a height equal to twice its diameter. Test cylinders are cast using properly selected samples of fresh concrete and cured under controlled temperature and humidity until tested at a specified age, which is usually 28 days. When information on strength gain is needed at earlier or later ages, additional cylinders are made from the same batch sample, and these are tested at intervals (such as at seven days, 14 days, and so on) to determine the rate of strength gain.

Most structural concrete produced today has a compressive strength from 3000 psi to 6000 psi. Job-cast elements such as footings, slabs, beams, walls, and so forth typically use 3000 psi to 4000 psi concrete, while plant-produced precast concrete elements typically use higher strength 5000 psi to 6000 psi concrete. Strengths significantly above 6000 psi can be achieved, and such concrete is sometimes used in columns and walls of high-rise buildings and other applications when higher performance is needed.

Many equations in ACI 318 refer to the quantity $\sqrt{f'_c}$. By convention, this means the square root of only the numerical value of f'_c as expressed in pounds per square inch (psi). The units themselves are not changed by the operation, so that the result is also in psi. If f'_c is given in kips per square inch (ksi), convert to psi before taking the square root. For example, if f'_c equals 4 ksi, $\sqrt{f'_c}$ equals $\sqrt{4000 \text{ psi}}$ or 63.2 psi.

3. Mechanical Properties of Concrete

The design of concrete structures requires an understanding of the behavior of concrete under various states of stress and strain. Of particular importance are the uniaxial compressive stress-strain relationship, tensile strength, and the volume changes that occur in hardened concrete.

A. Compressive Stress-Strain Relationship

The compressive stress-strain relationship is determined from a uniaxial compression test performed on a cylinder of hardened concrete. This requires a stiff test machine—that is, one that will not itself be deflected by the test—that is capable of measuring strain beyond the peak compressive stress, f'_c. Figure 1.1 shows representative stress-strain curves for normal weight concretes having compressive strengths of 3000 psi, 4000 psi, and 5000 psi. The following characteristics are evident.

- Behavior is essentially linearly elastic up to a stress of about $0.65f'_c$, and becomes distinctly nonlinear beyond that stress.
- The slope of the linear portion (that is, the modulus of elasticity) increases as f'_c increases.
- The compressive strength is reached at a strain of approximately 0.002.
- There is a descending branch of the curve beyond f'_c, reaching an ultimate strain of at least 0.003.

Based on similar tests involving a wide range of compressive strengths and unit weights, the ACI code adopts the following criteria for the design of structural concrete.

- The modulus of elasticity (in psi) is defined in ACI Sec. 8.5 by the equation

$$E_c = 33w_c^{1.5}\sqrt{f'_c} \tag{1.1}$$

In this equation, w_c is unit weight in lbf/ft^3 and f'_c is compressive strength in psi. The equation was derived empirically, however, and the canceling of units should be disregarded.

- The ultimate strain in concrete in compression is $\varepsilon_c = 0.003$.

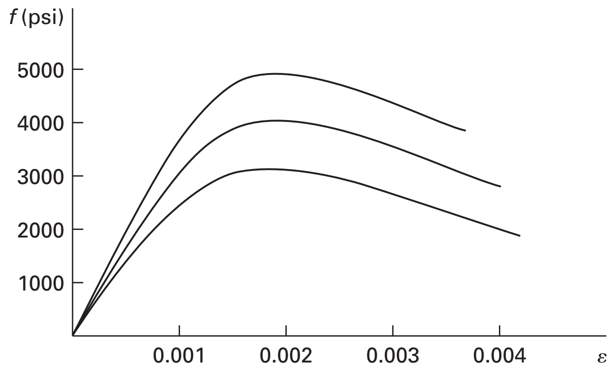


Figure 1.1 Representative Stress-Strain Curves for Concrete in Uniaxial Compression

B. Tensile Strength

Concrete is a brittle material, and its tensile strength is small compared to its compressive strength—only about a tenth. There are several ways to measure tensile strength, but the most important in design is the *modulus of rupture*. This is the flexural tensile stress at failure in a prism of plain concrete when subjected to pure bending. For convenience, ACI uses an empirical equation, ACI Eq. 9-10, that relates the modulus of rupture, f_r, to the specified compressive strength, f'_c (both in psi).

$$f_r = 7.5\lambda\sqrt{f'_c} \qquad 1.2$$

The parameter λ is 1.0 for normal weight concrete, 0.85 for sand-lightweight concrete, and 0.75 for all-lightweight concrete.

The *splitting tensile strength*, f_{ct}, is an alternative measure of tensile strength that may be determined by laboratory test. The splitting tensile strength is defined in terms of the principal tensile stress at failure of a test cylinder of a standard size that is loaded in compression along a main diameter as shown in Fig. 1.2.

C. Volume Changes

Changes in volume will occur for various reasons in a completed structure. The design of concrete structures, then, must account for the effects of these changes. Depending on circumstances, these effects can be either beneficial or detrimental.

For example, if the deflection of a reinforced concrete beam increases significantly over time, attached finish items may bend and crack. On the other hand, similar

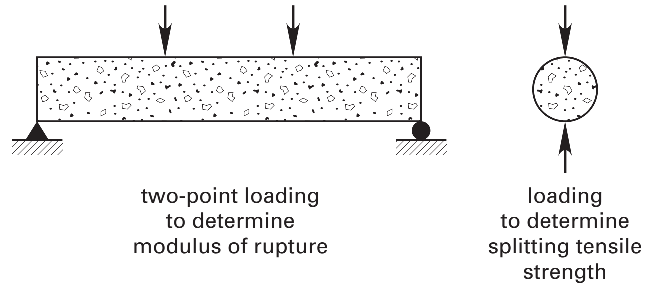


Figure 1.2 Test Arrangements to Determine Concrete Tensile Strength

changes in volume may "soften" the effects of support settlements, reducing the internal stresses that generally occur.

There are three primary sources of volume changes to consider: temperature change, creep, and shrinkage.

Temperature Change

The axial deformation of concrete caused by a temperature change ΔT is

$$\Delta L = C_T(\Delta T)L \qquad 1.3$$

ΔT is the change in temperature, L is the original unstrained length in the direction under consideration, and C_T is the coefficient of thermal expansion. The coefficient of thermal expansion varies with the aggregate type and the other ingredients in the concrete, but for practical design purposes it is assumed to be the same as for steel, about 0.000006 in/in-°F.

Creep

In the discussion of the compressive stress-strain relationship earlier in this chapter, the duration of loading was assumed to be short. In reality, concrete that must sustain stress for long periods will undergo additional strain which is termed *creep*. This strain occurs more quickly at first, and the rate of creep gradually diminishes over time.

Several equations have been proposed to predict the amount of creep at a specified time after loading. In the case of plain concrete, the ACI code uses a multiplier, ξ, which is applied to the immediate deformation to predict the cumulative additional deformation at a later time. In the ACI approach, the multiplier is as follows.

$\xi = 1.0$ for stress sustained 3 mo

$\xi = 1.2$ for stress sustained 6 mo

$\xi = 1.4$ for stress sustained 12 mo

$\xi = 2.0$ for stress sustained 60 mo or longer

Shrinkage

Shrinkage is strain associated with the evaporation of the excess water in the concrete mixture. The ultimate shrinkage strain depends on the mixture's properties. Typical values range from 0.0004 to 0.0008 in/in. The rate at which the shrinkage occurs depends on many factors, the two most important being the average ambient relative humidity and the volume-to-surface ratio of the structural member.

For structural members of usual proportions, the ultimate shrinkage strain requires several years to develop. Because the expected life of a structure is considerably longer, the expected ultimate strain is usually the controlling value used in design.

4. Properties of Reinforcing Steel

Concrete is brittle, prone to creep, and relatively weak in tension. Most structural applications, then, require ways of overcoming these deficiencies.

There are two common approaches. The conventional method is to embed steel reinforcement bars, or rebars. The rebars bond with the hardened concrete and reinforce it. An alternative method is to prestress the concrete. This is accomplished by inducing compressive stresses in those regions that will experience tensile stresses when loads are applied.

The current ACI code attempts to unify many of the concepts for these two approaches. Nevertheless, there are many differences in means, methods, and materials of construction, and the two approaches are considered separately in this book. This section describes the essential material properties of steel used for reinforcement. The properties of materials used in prestressing are discussed in a later chapter devoted to prestressed concrete.

A. Reinforcing Bars

Reinforcing bars, or rebars, are round steel bars produced by hot rolling. Raised ribs on the surface of the bars, called *deformations*, create a mechanical interlock between the steel and the hardened concrete, helping to maintain the bond between the two. An ASTM specification controls the percentage of the cross section that must comprise the deformations. Figure 1.3 shows schematically a typical reinforcing bar.

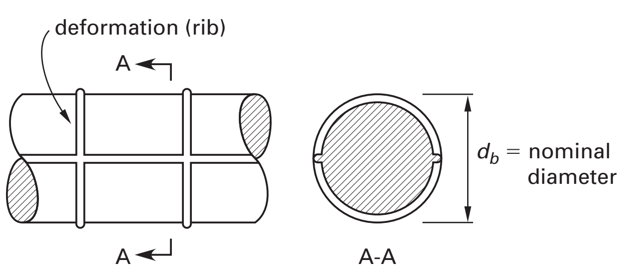


Figure 1.3 Elevation and Cross Section of Typical Reinforcing Bar

Reinforcing bars are designated by a number that gives the number of eighths of an inch in the nominal diameter. For example, a no. 5 bar has a nominal diameter of 5/8 in (0.625 in). Because of the irregularities caused by the deformations, the actual diameter differs from the nominal diameter. For the same reason, the cross-sectional area of the bar, which is an important design property, does not match precisely the area of a circle of the same nominal diameter.

In current practice, deformed bars ranging from no. 3 to no. 11 reinforce structures of usual proportions. Bars of special size, no. 14 and no. 18, are used in exceptionally large or heavily loaded members. Table 1.1 gives the properties of the standard sizes of deformed bars.

Table 1.1 Properties of Standard Reinforcing Bars (no. 14 and no. 18 omitted)

bar no.	nominal diameter d_b (in)	nominal area A_b (in^2)	weight (lbf/ft)
3	0.375	0.11	0.38
4	0.500	0.20	0.67
5	0.625	0.31	1.04
6	0.750	0.44	1.50
7	0.875	0.60	2.04
8	1.000	0.79	2.67
9	1.128	1.00	3.40
10	1.270	1.27	4.30
11	1.410	1.56	5.31

B. Smooth Bars and Wire Fabric

Prior to 1971, the ACI Building Code permitted the use of smooth bars without deformations as reinforcing bars. These were used primarily in situations where low bond stresses between steel and concrete could be tolerated. The current codes permit smooth bars only in the form of continuous spirals so that bond is not a consideration.

Another type of reinforcement is welded wire fabric. This consists of longitudinal and transverse wires that are machine welded to produce a rectangular grid. The wires may be either smooth or deformed. They are designated by a letter (either W for smooth wire or D for deformed) followed by a number indicating the cross-sectional area of the wires in hundreds of a square inch. For example, the designation W5 indicates a smooth wire with a cross-sectional area of 0.05 in^2; D30 indicates a deformed wire with a cross-sectional area of 0.30 in^2.

The designation of wire fabric gives the wire spacing first in the longitudinal direction and then in the transverse direction, followed by the wire sizes for the longitudinal and transverse directions. For example, the designation 12×6 W1.4 $\times$ W2.5 indicates a fabric with wires spaced 12 in on centers longitudinally and

6 in on centers transversely, and providing 0.014 in^2 per foot of reinforcement in the longitudinal direction and 0.025 in^2 per 6 in (or 0.050 in^2 per foot) in the transverse direction. Appendix E of ACI 318 includes a table of commonly used patterns of wire fabric.

C. Mechanical Properties

Reinforcement is specified by a designation that refers first to an appropriate ASTM specification and then to a grade which corresponds to the yield stress of the steel in kips per square inch. A commonly specified reinforcement, for example, is ASTM A615 grade 60, which has a minimum yield stress of 60 ksi. The distinction between specifications has to do primarily with whether the steel will be welded or not. From a design standpoint, the most important item specified is the yield stress.

Unlike concrete, steel does not creep under sustained stress at normal temperatures. Fortunately, the coefficients of thermal expansion for steel and concrete are nearly the same (about 0.000006 in/in-°F), which means that embedded reinforcement can expand and contract with temperature changes without breaking its bond with the surrounding concrete.

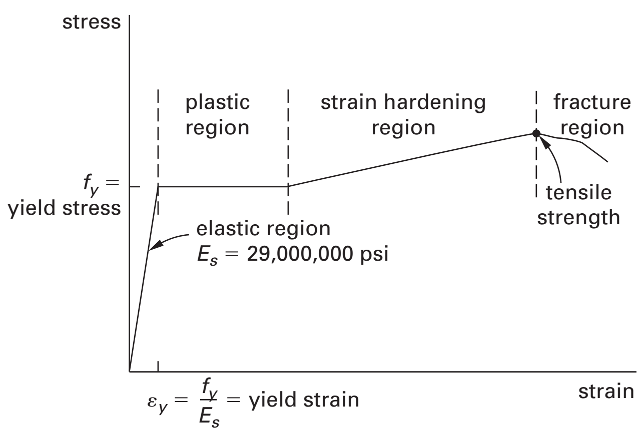


Figure 1.4 Idealized Uniaxial Stress-Strain Curve for Grade 60 Reinforcing Steel

The most important properties for reinforcing steel are associated with the uniaxial stress-strain relationship. Figure 1.4 shows the idealized stress-strain relationship for grade 60 reinforcement, the most commonly used grade in modern construction. Curves like this show some important characteristics.

- Steel is linearly elastic practically up to its yield stress.
- The slope of the stress-strain curve, which is the modulus of elasticity, is constant for all steel grades and is equal to 29,000,000 psi or 29,000 ksi.
- There is a well-defined yield plateau at the yield stress, indicating large plastic deformation.
- Beyond the yield plateau is a region over which stress increases with strain. This increase is called *strain hardening*, and it produces a tensile strength that is significantly larger than the yield stress.
- Reinforcing steel is very ductile, and can stretch to about 25 times its original length before fracturing.

2

Design Specifications

Concrete has been used in construction for many centuries. A well-known application of concrete made with hydraulic cement is the Pantheon, built by the Romans in about A.D. 100. This structure is a testament to the durability of concrete when used in a properly constructed building and loaded in compression.

Reinforcing concrete with ductile steel bars is a relatively recent development, introduced in the early 1900s. Most early applications involved empirically designed structures that were load tested to ensure satisfactory behavior, and then replicated in many similar structures.

Among the earliest U.S. standards of practice for reinforced concrete was a joint bulletin published in 1940 by the American Concrete Institute (ACI), the American Society for Testing and Materials (ASTM), the American Society of Civil Engineers (ASCE), and others. The first edition of the ACI 318 Building Code followed in 1951, and this document became the accepted design specification for structural concrete in the United States. The first editions of the ACI code assumed linear elastic behavior of the steel and concrete. This method, known as *working stress design* (WSD), was used to set allowable stresses for concrete, steel, and the bond between them.

In 1963, ACI introduced a new, easier, and more economical design approach as an alternative to WSD. This method, called *strength design*, gradually replaced WSD for structural concrete, although WSD is still widely used for design of reinforced masonry.

The primary design criteria for structural concrete in ACI 318 ensure

- adequate strength
- adequate ductility
- serviceability
- practical and economical constructability

ACI 318 does not consider aesthetics, even though this may be an important consideration for exposed elements. ACI 318 design criteria are discussed in general in the following sections, and specific criteria related to typical members and systems are summarized in the appropriate chapters.

1. Strength

The basic strength requirement for structural concrete is

$$R_u \leq \phi R_n \qquad 2.1$$

R_u represents a particular structural action (for example, shear, bending moment, or axial force) caused by an appropriate combination of factored loads. R_n is the corresponding nominal strength, and ϕ is a capacity reduction factor that reduces the nominal strength to account for variations in materials, workmanship, and type and consequence of failure.

Factored loads are the result of multiplying the actual expected loads on a structure, called *service loads*, by appropriate *load factors*. The service loads and load factors are set by ASCE 7 (see Codes and References Used to Prepare This Book). For example, in the case of gravity dead and live loads (denoted by D and L, respectively), ASCE 7 requires that R_u be taken as the more severe action caused by the combinations $(1.2D + 1.6L)$ and $1.4D$. ASCE 7 gives other load combinations and factors for cases involving dead and live loads in combination with wind, earthquake, lateral earth pressure, and so forth. Load and resistance factors are defined in ACI Secs. 9.2 and 9.3.

As the ACI strength design method has evolved, the load and resistance factors have been revised from one code to another. This has sometimes caused confusion. The current factors (adopted by ACI 318 for the 2002 and later editions) align the ACI code with ASCE 7, which is the most widely used loading standard in building codes and design specifications. It is important to be aware how the factors have changed because many existing textbooks and design documents employ earlier codes and reflect older factors. Table 2.1 summarizes the changes in some of the more commonly used load and resistance factors in ACI 318.

Table 2.1 Variation in Some Load and Resistance Factors in ACI 318

	load factors		resistance factors		
year	dead	live	flexure	shear	bearing on concrete
1963–1971	1.5	1.8	0.90	0.85	0.70
1971–2002	1.4	1.7	0.90	0.85	0.70
2002–	1.2	1.6	0.90	0.75	0.65

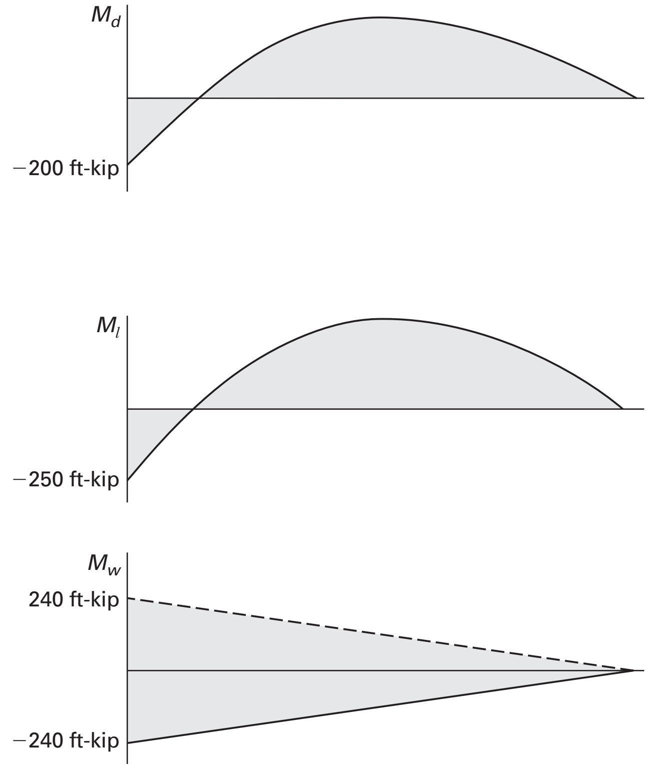
The current load factors are given in ACI Sec. 9.2 and are directly from ASCE 7 Sec. 2.3.2. For example, for a common case involving dead load D, live load L, and wind load W, the appropriate factored load combinations from ACI Sec. 9.2 are

- $1.4D$
- $1.2D + 1.6L$
- $1.2D + 1.0W + L$
- $0.9D + 1.0W$

Because the effects of wind are reversible, the intent is that wind's effect is additive to dead and live load effects in the third combination and is opposite to the dead load effect in the fourth combination.

Example 2.1
Factored Load Combinations for Gravity and Wind

A girder in a reinforced concrete frame is analyzed for service-level (that is, unfactored) dead and live load, and design-level wind load bending moments at the face of support. The results are shown. Determine the appropriate combination of factored moments for the moment at the left face of support.



Solution:

ASCE 7 requires the following four combinations involving dead load moment, M_d, live load moment, M_l, and wind moment, M_w.

$$\begin{aligned} M_u &= 1.4M_d \\ &= (1.4)(-200 \text{ ft-kip}) \\ &= -280 \text{ ft-kip} \end{aligned}$$

$$\begin{aligned} M_u &= 1.2M_d + 1.6M_l \\ &= (1.2)(-200 \text{ ft-kip}) + (1.6)(-250 \text{ ft-kip}) \\ &= -640 \text{ ft-kip} \end{aligned}$$

$$\begin{aligned} M_u &= 1.2M_d + 1.0M_w + M_l \\ &= (1.2)(-200 \text{ ft-kip}) + (1.0)(-240 \text{ ft-kip}) \\ &\quad + (-250 \text{ ft-kip}) \\ &= -730 \text{ ft-kip} \end{aligned}$$

$$\begin{aligned} M_u &= 0.9M_d + 1.0M_w \\ &= (0.9)(-200 \text{ ft-kip}) + (1.0)(240 \text{ ft-kip}) \\ &= 60 \text{ ft-kip} \end{aligned}$$

The sign of the bending moment follows the usual convention that flexure causing tension on the top surface is negative and tension on the bottom is positive. In the last combination, wind can act in either direction, so that the wind load moment is reversible. It is the positive sense of this moment that combines with $0.9M_d$. Thus, the design factored moment, M_u, ranges from −730 ft-kip to 60 ft-kip.

2. Ductility

Ductility is the ability of a material or member to deform visibly without fracture. Plain concrete is a brittle material, but if reinforcement is properly placed inside, concrete members can behave in a ductile manner.

A ductile member can generally redistribute loads to less highly stressed regions. This can protect the member in the event of an accidental overload, and in the case of an extraordinary overload can warn of impending collapse. In the ACI code, adequate ductility is assured by placing minimum limits on the amount of steel that must be provided in particular members and by imposing upper limits on the amount of reinforcement that can be considered effective in a member.

3. Serviceability

Serviceability is the characteristic of a structure to serve its intended function under the service loads (that is, unfactored loads). Important serviceability issues for structural concrete include defections, crack widths, and durability.

4. Constructability Issues

Many of the design rules in ACI 318 exist to alleviate the difficulties of placing and consolidating fresh concrete. These take the form of minimum bar spacing, maximum steel percentages, or minimum member size for various types of members.

3

Flexural Design of Reinforced Concrete Beams

Flexural members are slender members that deform primarily by bending moments caused by concentrated couples or transverse forces. In modern construction these members may be joists, beams, girders, spandrels, lintels, and other specially named elements. But their behavior in every case is essentially the same. Unless otherwise specified in a problem, flexural members will be referred to as beams throughout this book.

In the following sections, the ACI 318 provisions for the strength, ductility, serviceability, and constructability of beams are summarized and illustrated.

1. Strength

The basic strength requirement for flexural design is

$$M_u \leq \phi M_n \tag{3.1}$$

M_n is the nominal moment strength of the member, M_u is the bending moment caused by the factored loads, and ϕ is the capacity reduction factor. For most practical designs, ACI specifies the value of ϕ as 0.9; however, special cases exist for which lower values apply, as explained in Sec. 2 of this chapter.

A. M_n for a Singly Reinforced Concrete Beam

The simplest case is that of a rectangular beam containing steel in the tension zone only. A beam of this sort is referred to as *singly reinforced*. Figure 3.1 shows a typical cross section of a singly reinforced beam and the notation used.

a = equivalent depth of compression zone
A_s = total area of steel in tension zone
b = width of compression edge
c = distance from compression edge to neutral axis
d = effective depth, distance from compression edge to centroid of tension steel
f'_c = specified compressive strength of concrete
f_y = yield stress of tension steel
h = overall depth of beam

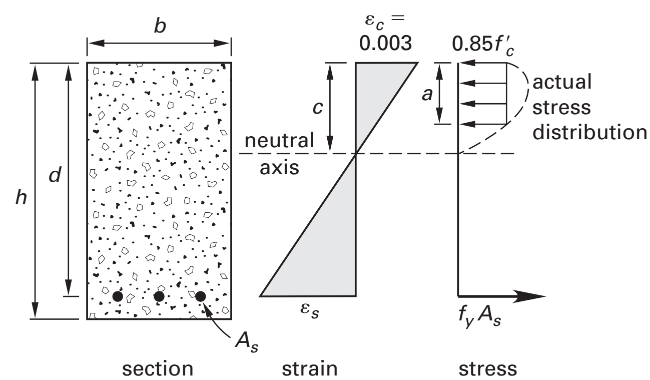


Figure 3.1 Notation for Moment Strength of a Singly Reinforced Rectangular Beam

ACI Secs. 10.2 and 10.3 give the principles governing the flexural strength.

- Strain varies linearly through the depth of the member.
- A complete bond exists between the steel and the concrete; that is, the strain in the steel is the same as in the adjacent concrete.
- Tension stress in the concrete is negligible (that is, all tension is resisted by steel).
- The ultimate strain in concrete is 0.003.
- In a properly designed beam, the tension steel yields; thus, $T = A_s f_y$.
- The concrete stress distribution may be replaced by an equivalent rectangular distribution with uniform stress $0.85f'_c$ acting over an area ba and creating a compression resultant, $C = 0.85f'_c ba$, that acts at distance $a/2$ from the compression edge.

For bending without axial force applied, equilibrium requires

$$\sum F_{\text{horizontal}} = C - T = 0 \text{ lbf} \tag{3.2}$$

$$0.85f'_c ba = A_s f_y \tag{3.3}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{3.4}$$

The resultant compression force in the concrete, C, forms a couple with the resultant tension force, T.

$$M_n = T\left(d - \frac{a}{2}\right) \qquad 3.5$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \qquad 3.6$$

Example 3.1
Singly Reinforced Beam Analysis

A cantilevered singly reinforced beam is subjected to a service dead load of 1.5 kip/ft, which includes the self-weight of the beam. The beam is reinforced with three no. 9 bars, and the yield stress of the tension steel is 60,000 psi. The beam's span is 9.5 ft, and its capacity reduction factor is 0.9. The width of the beam's compression edge is 16 in, and the beam's effective depth is 22 in. The concrete's specified compressive strength is 3000 psi. Determine the maximum uniformly distributed service live load that the beam can support based on its flexural strength.

Solution:
From Table 1.1, the cross-sectional area of one no. 9 bar is

$$A_b = 1.00 \text{ in}^2$$

The total cross-sectional area of the steel is

$$\begin{aligned} A_s &= n_{\text{bars}}A_b = (3)(1.00 \text{ in}^2) \\ &= 3.00 \text{ in}^2 \end{aligned}$$

From Eq. 3.4, the equivalent depth of the compression zone is

$$\begin{aligned} a &= \frac{A_s f_y}{0.85 f'_c b} \\ &= \frac{(3.00 \text{ in}^2)\left(60{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(3000 \ \frac{\text{lbf}}{\text{in}^2}\right)(16 \text{ in})} \\ &= 4.41 \text{ in} \end{aligned}$$

From Eq. 3.6,

$$\begin{aligned} \phi M_n &= \phi A_s f_y\left(d - \frac{a}{2}\right) \\ &= (0.9)(3.00 \text{ in}^2)\left(60{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right) \\ &\quad \times \left(22 \text{ in} - \frac{4.41 \text{ in}}{2}\right) \\ &= 3{,}207{,}000 \text{ in-lbf} \end{aligned}$$

Converting to foot-kips,

$$\phi M_n = \frac{3{,}207{,}000 \text{ in-lbf}}{\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)} = 267 \text{ ft-kip}$$

For a uniformly loaded cantilevered beam,

$$M_u = \frac{w_u L^2}{2}$$

L is the span length in feet and w_u is the factored uniformly distributed load in kips per foot. Using the strength requirement (Eq. 3.1) and solving for w_u gives

$$\begin{aligned} \frac{w_u L^2}{2} &= \phi M_n \\ w_u &= \frac{2\phi M_n}{L^2} \\ &= \frac{(2)(267 \text{ ft-kip})}{(9.5 \text{ ft})^2} \\ &= 5.92 \text{ kip/ft} \end{aligned}$$

In terms of the service loads,

$$\begin{aligned} w_u &= 1.2w_d + 1.6w_l \\ &= (1.2)\left(1.5 \ \frac{\text{kip}}{\text{ft}}\right) + 1.6w_l \\ &= 5.92 \text{ kip/ft} \\ w_l &= 2.58 \text{ kip/ft} \end{aligned}$$

B. Beams with Irregular Cross Sections

Many reinforced concrete beams have cross sections that are not rectangular. Figure 3.2 shows three typical cross sections with irregularly shaped compression regions.

Fortunately, the same principles that govern the behavior of rectangular beams apply more generally to these cases as well. In the absence of axial forces, in a properly designed beam (that is, a beam for which tension steel yields) the compression region is determined using the condition of equilibrium.

$$C = T$$

$$0.85 f'_c A_c = A_s f_y \qquad 3.7$$

$$A_c = \frac{A_s f_y}{0.85 f'_c} \qquad 3.8$$

Geometric relationships determine the depth of compression region and a summation of moments gives the nominal moment strength of the section.

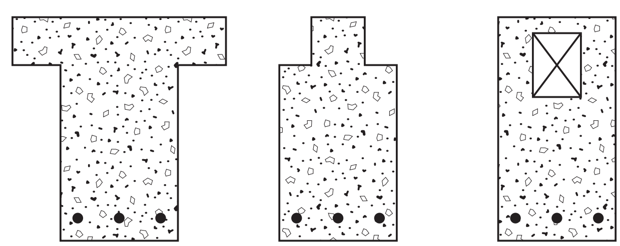


Figure 3.2 Representative Cross Sections of Irregular Reinforced Concrete Beams

For most cast-in-place floor systems, the slab and beams are cast monolithically and the slab functions as the flange of a T- or L-shaped beam, as shown in Fig. 3.3. ACI Sec. 8.12 limits the effective flange width, b_e, of such members by the following criteria.

Slab Extending Both Sides (T-Beam)

$$b_{e,\text{int}} \le \begin{cases} L/4 \\ b_w + 16h_s \\ b_w + \dfrac{s_1 + s_2}{2} \end{cases} \quad 3.9$$

Slab Extending One Side Only (L-Beam)

$$b_{e,\text{ext}} \le \begin{cases} b_w + \dfrac{L}{12} \\ b_w + 6h_s \\ b_w + \dfrac{s_1}{2} \end{cases} \quad 3.10$$

L is the span. Other symbols are as defined in Fig. 3.3.

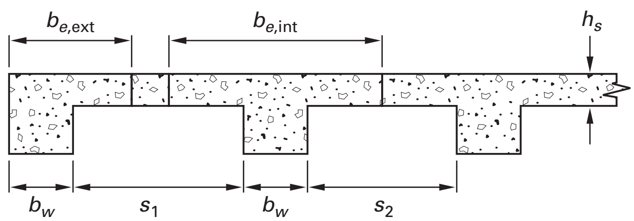
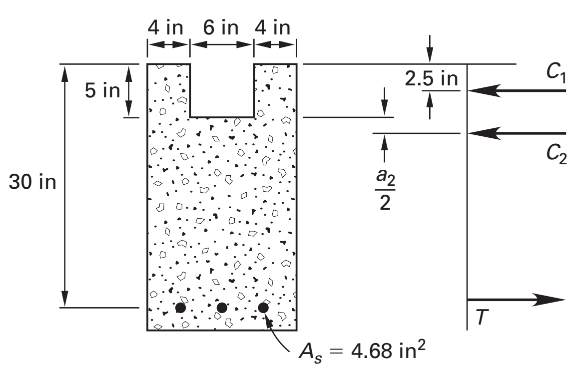


Figure 3.3 Effective Widths of T-Beams and L-Beams

Example 3.2
Analysis of an Irregularly Shaped Beam

Calculate the design moment strength of the section shown. The compressive strength of the concrete is 4000 psi, and the yield stress of the reinforcement is 60,000 psi.



Solution:

The equivalent area of the compression zone can be found from Eq. 3.8.

$$\begin{aligned} A_c &= \frac{A_s f_y}{0.85 f'_c} \\ &= \frac{(4.68\ \text{in}^2)\left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(4\ \dfrac{\text{kip}}{\text{in}^2}\right)} \\ &= 82.6\ \text{in}^2 \end{aligned}$$

Because the equivalent area of the compression zone exceeds the areas in the rectangular regions to the left and right of the trough, the compression zone extends to some depth below the bottom of the trough. This depth is

$$\begin{aligned} a_2 &= \frac{A_c - 2b_1h_1}{b_w} \\ &= \frac{82.6\ \text{in}^2 - (2)(4\ \text{in})(5\ \text{in})}{14\ \text{in}} \\ &= 3.04\ \text{in} \end{aligned}$$

The equivalent compression force can be expressed in terms of a component acting in the rectangular regions adjacent to the trough, C_1, and a component acting over the region below the trough, C_2.

$$\begin{aligned} C_1 &= 2(0.85 f'_c b_1 h_1) \\ &= (2)(0.85)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(4\ \text{in})(5\ \text{in}) \\ &= 136\ \text{kip} \\ C_2 &= 0.85 f'_c b_w a_2 \\ &= (0.85)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(14\ \text{in})(3.04\ \text{in}) \\ &= 145\ \text{kip} \end{aligned}$$

Taking moments of the two forces about the line of action of the tension force gives the design moment strength of the section.

$$\begin{aligned} \phi M_n &= \phi\left(C_1\left(d - \frac{h_1}{2}\right) + C_2\left(d - h_1 - \frac{a_2}{2}\right)\right) \\ &= (0.9)\left(\begin{array}{l} (136\ \text{kip})\left(30\ \text{in} - \dfrac{5\ \text{in}}{2}\right) \\ \quad + (145\ \text{kip})\left(30\ \text{in} - 5\ \text{in} - \dfrac{3.04\ \text{in}}{2}\right) \end{array}\right) \\ &= (6430\ \text{in-kip})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &= 536\ \text{ft-kip} \end{aligned}$$

2. Ductility Criteria

ACI Secs. 10.3.5 and 10.5 limit both the minimum and maximum amount of tension steel that is acceptable in a beam. The minimum limit ensures that the flexural strength of the reinforced beam is appropriately larger than that of the gross section when it cracks. This requires

$$A_{s,\min} \geq \begin{cases} \left(\dfrac{200}{f_y}\right) b_w d \\ \left(\dfrac{3\sqrt{f'_c}}{f_y}\right) b_w d \end{cases} \qquad 3.11$$

The code makes an exception to this requirement for slabs and footings, which require minimum temperature and shrinkage steel, and for special cases in which the amount of steel provided in a flexural member is at least one-third greater at every point than required by analysis. For cantilevered T-beams with the flange in tension, the value of b_w used in the expressions is the smaller of either the flange width or twice the actual web width.

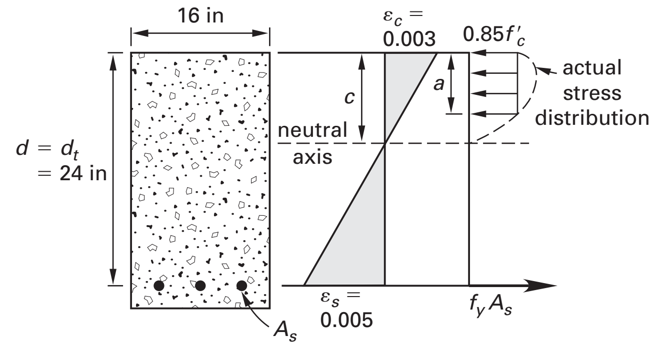
The maximum limit on the amount of tension steel ensures that the steel yields well before the concrete crushes, so that the beam fails in a gradual, ductile manner and not a sudden, brittle manner. This provides warning in the event of failure. ACI Sec. 10.3.5 limits the strain in the extreme tension reinforcement at the nominal strength. For sections subject to bending with negligible axial force (axial force less than $0.1f'_cA_g$), the strain in the extreme tension steel must exceed 0.004 (that is, approximately twice its yield strain) when the extreme compression edge of the member reaches the ultimate concrete strain of 0.003.

However, the code imposes a capacity reduction factor of 0.65 when the strain in the tension steel equals 0.002. The capacity reduction factor increases linearly to a maximum value of 0.9 as the tension strain increases from 0.002 to 0.005. There is rarely a practical advantage to designing beams for which the tension strain is less than 0.005, so this limit, which permits a capacity reduction factor of 0.9, will be used throughout this book.

The following example illustrates the method of determining the minimum and maximum limits.

Example 3.3
Maximum and Minimum Flexural Steel in a Rectangular Beam

For the section shown, calculate the minimum area of tension steel and the maximum steel for which a capacity reduction factor of 0.9 is applicable. The compresson strength of concrete is 4000 psi, and the yield stress of the tension steel is 60,000 psi.



Solution:
From Eq. 3.11, the minimum steel is

$$A_{s,\min} \geq \begin{cases} \left(\dfrac{200}{f_y}\right) b_w d \\ \quad = \left(\dfrac{200\ \dfrac{\text{lbf}}{\text{in}^2}}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}}\right)(16\text{ in})(24\text{ in}) \\ \quad = 1.28\text{ in}^2 \quad [\text{controls}] \\ \left(\dfrac{3\sqrt{f'_c}}{f_y}\right) b_w d \\ \quad = \left(\dfrac{3\sqrt{4000\ \dfrac{\text{lbf}}{\text{in}^2}}}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}}\right)(16\text{ in})(24\text{ in}) \\ \quad = 1.21\text{ in}^2 \end{cases}$$

The maximum steel for which a ϕ-factor of 0.9 is applicable corresponds to the quantity of steel that makes the depth to the neutral axis small enough that the strain at the extreme steel is exactly 0.005 when concrete strain reaches 0.003. As shown in the illustration, using similar triangles gives

$$\frac{c}{0.003} = \frac{d_t}{0.003 + 0.005}$$

$$c = \frac{0.003d_t}{0.008} = 0.375d_t$$

For the most common case, in which steel is in a single layer, d_t is equal to d. When the steel is in two or more layers, d_t is taken as the distance from the compression edge to the center of the steel in the extreme layer.

The maximum steel area is found by equating the tension force to the compression force when c equals $0.375d_t$.

$$\begin{aligned} A_{s,\max} f_y &= 0.85 f'_c b a \\ &= 0.85 f'_c b(\beta_1 c) \end{aligned}$$

$$A_{s,\max} = \frac{0.85 f'_c b \beta_1 (0.375 d_t)}{f_y}$$

$$= \frac{\left(\begin{array}{c} (0.85)\left(4000\ \dfrac{\text{lbf}}{\text{in}^2}\right)(16\ \text{in}) \\ \times (0.85)(0.375)(24\ \text{in}) \end{array} \right)}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}}$$

$$= 6.94\ \text{in}^2$$

---

3. Design of Singly Reinforced Rectangular Beams

The design of a singly reinforced rectangular beam to resist a factored bending moment requires solving for appropriate dimensions and tension reinforcement. In practical problems, the specified compressive strength and yield strength of the reinforcement are known. When the dimensions b and d are known, the solution for A_s is a straightforward solution of the equation

$$\begin{aligned} M_u &= \phi M_n \\ &= \phi A_s f_y \left(d - \frac{a}{2}\right) \\ &= \phi A_s f_y \left(d - \frac{A_s f_y}{1.7 f'_c b}\right) \end{aligned} \qquad 3.12$$

The calculated A_s must satisfy the ductility limits imposed by $A_{s,\min}$ and $A_{s,\max}$.

Example 3.4
Flexural Steel in a Rectangular Beam

For a rectangular beam, the width of the compression edge is 16 in, the effective depth is 24 in, the compressive strength of the concrete is 4000 psi, and the yield stress of the tension steel is 60,000 psi. Calculate the area of flexural steel needed to resist a bending moment of 200 ft-kip.

Solution:
Equation 3.12 gives

$$\phi A_s f_y \left(d - \frac{A_s f_y}{1.7 f'_c b}\right) = M_u$$

$$0.9 A_s \left(60\ \frac{\text{kip}}{\text{in}^2}\right)\left(24\ \text{in} - \frac{A_s \left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)}{(1.7)\left(4\ \dfrac{\text{kip}}{\text{in}^2}\right)(16\ \text{in})}\right) = (200\ \text{ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right)$$

$$(29.78\ \text{in}^{-1})A_s^2 + (-1296\ \text{in})A_s + 2400\ \text{in}^3 = 0$$

Solving this quadratic equation gives

$$A_s = \frac{-(-1296\ \text{in}) - \sqrt{\begin{array}{r} (-1296\ \text{in})^2 \\ - (4)(29.78\ \text{in}^{-1}) \\ \times (2400\ \text{in}^3) \end{array}}}{(2)(29.78\ \text{in}^{-1})} = \frac{115.44\ \text{in}}{59.56\ \text{in}^{-1}} = 1.94\ \text{in}^2$$

As computed in Ex. 3.3, the minimum and maximum steel areas for this beam are 1.28 in^2 and 6.94 in^2, respectively; thus, the calculated 1.94 in^2 is acceptable.

A. Design Equation in Terms of the Steel Ratio

In many cases, it is more convenient to express the moment strength of a singly reinforced section in terms of the nondimensional steel ratio, ρ, defined as

$$\rho = \frac{A_s}{bd} \qquad 3.13$$

In terms of the steel ratio, the equations for moment strength, minimum steel, and maximum steel are

$$\phi M_n = \phi \rho b d^2 f_y \left(1 - 0.59\ \rho \left(\frac{f_y}{f'_c}\right)\right) \qquad 3.14$$

$$\rho_{\min} = \frac{A_{s,\min}}{bd} \leq \begin{cases} \dfrac{200}{f_y} \\ \dfrac{3\sqrt{f'_c}}{f_y} \end{cases} \qquad 3.15$$

$$\rho_{\max} = \frac{A_{s,\max}}{bd} = \frac{(0.85)(0.375)\beta_1 f'_c}{f_y} \qquad 3.16$$

Table 3.1 gives values of the maximum and minimum steel ratios for representative material strengths, where the maximum ratio assumes a limiting steel strain of 0.005.

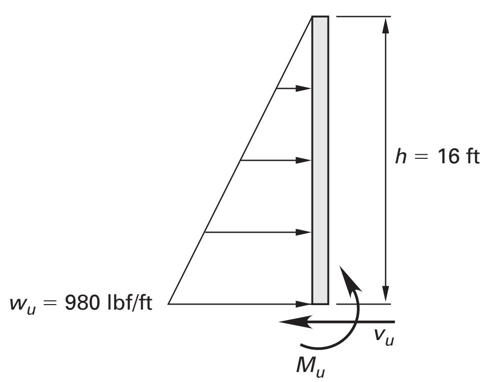
Table 3.1 Limiting Steel Ratios for Commonly Used Materials

f'_c (psi)	β_1	$f_y = 40{,}000$ psi		$f_y = 60{,}000$ psi	
		$\rho_{\min}$	$\rho_{\max}$	$\rho_{\min}$	$\rho_{\max}$
3000	0.85	0.0050	0.0203	0.0033	0.0135
3500	0.85	0.0050	0.0240	0.0033	0.0160
4000	0.85	0.0050	0.0271	0.0033	0.0181
4500	0.825	0.0050	0.0296	0.0033	0.0197
5000	0.80	0.0053	0.0319	0.0033	0.0212
6000	0.75	0.0058	0.0359	0.0039	0.0239

The steel area is determined uniquely when the dimensions of the member are known. However, if one of the dimensions b or d is unknown, an infinite number of combinations of steel area and beam dimensions will satisfy the strength requirement. In these cases, it is necessary to select a feasible steel ratio and solve the problem. The best choice for the steel ratio is that which satisfies construction and economic constraints. In the absence of specific directions, a reasonable approach is to select a steel ratio midway between the minimum and maximum permitted.

Example 3.5
Flexural Steel Calculated Using the Steel Ratio

A retaining wall is loaded by factored load as shown. Design flexural steel for the lowest level of the wall. Base the design on a unit length (that is, b = 1 ft = 12 in) and use a steel ratio midway between the minimum and maximum steel ratios. Use a compressive strength for the concrete of 3000 psi and a yield stress for the steel of 60,000 psi.



Solution:

The formula for the moment at the base of the retaining wall gives

$$M_u = \left(\frac{w_u h}{2}\right)\left(\frac{h}{3}\right) = \left(\frac{\left(980\ \frac{\text{lbf}}{\text{ft}}\right)(16\ \text{ft})}{2}\right)\left(\frac{16\ \text{ft}}{3}\right)\left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) = 41.8\ \text{ft-kip}$$

From Table 3.1, the steel ratio to use is

$$\rho = \frac{\rho_{\text{max}} + \rho_{\text{min}}}{2} = \frac{0.0135 + 0.0033}{2} = 0.0084$$

$$\begin{aligned}\phi M_n &= M_u \\ &= \phi \rho b d^2 f_y \left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right) \\ &= (0.9)(0.0084)(12\ \text{in})\,d^2\left(60\ \frac{\text{kip}}{\text{in}^2}\right) \\ &\quad \times \left(1 - (0.59)(0.0084)\left(\frac{60\ \frac{\text{kip}}{\text{in}^2}}{3\ \frac{\text{kip}}{\text{in}^2}}\right)\right) \\ &= (41.8\ \text{ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right) \\ d &= 10.1\ \text{in} \\ A_s &= \rho b d = (0.0084)\left(12\ \frac{\text{in}}{\text{ft}}\right)(10.1\ \text{in}) \\ &= 1.02\ \text{in}^2/\text{ft}\end{aligned}$$

4. Doubly Reinforced Beams

When longitudinal reinforcement exists near the compression edge of a beam as well as in the tension region, a beam is *doubly reinforced*. Reinforcement near the compression edge is most often due either to construction requirements (such as when bars are placed to support shear reinforcement) or to a situation where the surface may be in tension and in compression at different times and from different loads. In these cases, the steel near the compression edge is usually ignored, as it contributes very little to the flexural strength of the beam.

There are cases, however, when the compression steel is added in order to add one or more of the following.

- compression resistance when beams are compression controlled as singly reinforced members
- stiffness to improve immediate and long-term deflection behavior
- ductility

In these cases, the section is usually analyzed to assess the effect of the additional reinforcement. To find a moment strength that satisfies strain compatibility, stress-strain relationships, and equilibrium, a trial-and-error process is used. This leads to a solution in a few iterations, as shown in the following example.

The principles are the same as in Sec. 1 of this chapter, with one additional assumption given in ACI Sec. 10.2.4: Stress in steel is E_s times the steel strain when the strain is below yield, and is equal to the yield stress for all strains greater than yield. The additional notation involved is shown in Fig. 3.4, where A'_s is the area of steel near the compression edge and d' denotes the distance from compression edge to centroid of this steel.

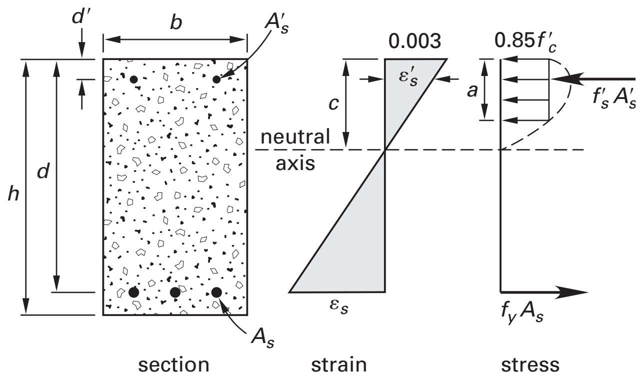


Figure 3.4 Notation for Moment Strength of a Doubly Reinforced Rectangular Beam

Example 3.6
Doubly Reinforced Beam Analysis

A doubly reinforced rectangular beam has three no. 9 bars in the tension zone and two no. 4 bars near the top compression edge. The beam's width, b, is 16 in. Its effective depth, d, is 22 in, and the distance from the top of the beam to the compression reinforcement, d', is 2.5 in. The compressive strength of the concrete, f'_c, is 3000 psi, and the yield stress of the steel, f_y, is 60,000 psi. Compute the nominal moment capacity and verify the failure mode.

Solution:

For the initial trial, ignore the two no. 4 bars. Calculate the corresponding depth of the neutral axis and use strain compatibility to determine a value for the strain. Using this value, determine the stress in A'_s and then repeat the analysis while including A'_s. Iterate until the trial value gives a computed value within about 5%.

From Eq. 3.4,

$$\begin{aligned} a &= \frac{A_s f_y}{0.85 f'_c b} \\ &= \frac{(3.00\ \text{in}^2)\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)(16\ \text{in})} \\ &= 4.41\ \text{in} \end{aligned}$$

The distance to the neutral axis is

$$\begin{aligned} c &= \frac{a}{\beta_1} \\ &= \frac{4.41\ \text{in}}{0.85} \\ &= 5.19\ \text{in} \end{aligned}$$

Using similar triangles, the strain in A'_s for this trial is

$$\begin{aligned} \frac{\varepsilon'_s}{c-d'} &= \frac{0.003}{c} \\ \varepsilon'_s &= \frac{0.003(c-d')}{c} \\ &= \frac{(0.003)(5.19\ \text{in} - 2.5\ \text{in})}{5.19\ \text{in}} \\ &= 0.00155 \end{aligned}$$

The yield strain is

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60\ \frac{\text{kip}}{\text{in}^2}}{29{,}000\ \frac{\text{kip}}{\text{in}^2}} = 0.00207$$

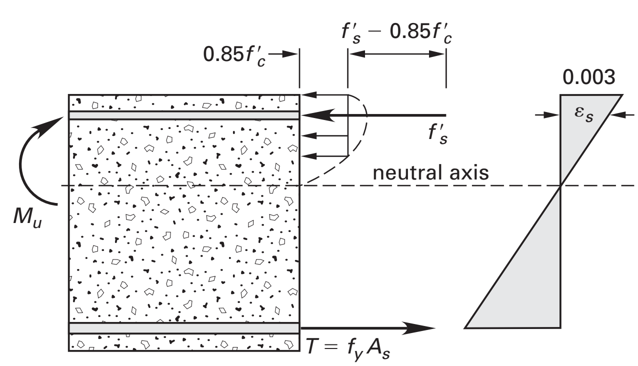
The calculated strain is below the yield strain, so the stress is within the linearly elastic range. Note, however, that this initial trial ignores A'_s, which is equivalent to assuming that the strain ε'_s is zero. Including a nonzero compressive stress in A'_s causes the neutral axis to shift upward, which results in a smaller value of ε'_s. Thus it is reasonable to choose a smaller value than was calculated for this first trial.

For the second trial, assume

$$\varepsilon'_s = 0.0015$$

Then the stress in the compression steel is

$$\begin{aligned} f'_s &= E_s \varepsilon'_s \\ &= \left(29{,}000\ \frac{\text{kip}}{\text{in}^2}\right)(0.0015) \\ &= 43.5\ \text{ksi} \end{aligned}$$



The distance to the neutral axis is found from

$$A_s f_y = 0.85 f'_c b\beta_1 c + (f'_s - 0.85 f'_c)A'_s$$

$$\begin{aligned} c &= \frac{A_s f_y - (f'_s - 0.85 f'_c)A'_s}{0.85 f'_c b \beta_1} \\ &= \frac{\left(\begin{array}{l}(3.00\ \text{in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right) \\ \quad - \left(43.5\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(3\ \frac{\text{kip}}{\text{in}^2}\right)\right) \\ \quad \times (0.4\ \text{in}^2)\end{array}\right)}{(0.85)\left(3\ \frac{\text{kip}}{\text{in}^2}\right)(0.85)(16\ \text{in})} \\ &= 4.72\ \text{in} \end{aligned}$$

The strain corresponding to this trial is

$$\begin{aligned} \varepsilon'_s &= \frac{0.003(c-d')}{c} \\ &= \frac{(0.003)(4.72\ \text{in} - 2.5\ \text{in})}{4.72\ \text{in}} \\ &= 0.00141 \end{aligned}$$

This is still below the yield strain of 0.00207. Thus, assuming a strain of 0.0015 leads to a calculated value of 0.00141. The true strain must be between these two values, and is probably closer to the lower, 0.00141.

Make a third attempt using an assumed strain of 0.00142. Then the stress in the compression steel is

$$\begin{aligned} f'_s &= E_s \varepsilon'_s \\ &= \left(29{,}000\ \frac{\text{kip}}{\text{in}^2}\right)(0.00142) \\ &= 41.2\ \text{ksi} \end{aligned}$$

The distance to the neutral axis is

$$\begin{aligned} c &= \frac{A_s f_y - (f'_s - 0.85 f'_c)A'_s}{0.85 f'_c b \beta_1} \\ &= \frac{\left(\begin{array}{l}(3.00\ \text{in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right) \\ \quad - \left(41.2\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(3\ \frac{\text{kip}}{\text{in}^2}\right)\right) \\ \quad \times (0.4\ \text{in}^2)\end{array}\right)}{(0.85)\left(3\ \frac{\text{kip}}{\text{in}^2}\right)(0.85)(16\ \text{in})} \\ &= 4.74\ \text{in} \end{aligned}$$

The strain corresponding to this trial is

$$\begin{aligned} \varepsilon'_s &= \frac{0.003(c-d')}{c} \\ &= \frac{(0.003)(4.74\ \text{in} - 2.5\ \text{in})}{4.74\ \text{in}} \\ &= 0.00142 \end{aligned}$$

This is close enough. The process converges in three iterations to give the stress in the compression steel as 41.2 ksi.

Now use similar triangles again to calculate the strain in the tension steel.

$$\begin{aligned} \frac{\varepsilon_s}{d-c} &= \frac{0.003}{c} \\ \varepsilon_s &= \frac{0.003(d-c)}{c} \\ &= \frac{(0.003)(22\ \text{in} - 4.74\ \text{in})}{4.74\ \text{in}} \\ &= 0.011 \end{aligned}$$

This is greater than the lower limit of 0.005, as explained in the earlier section on ductility criteria, so the beam is tension controlled. The values of C_c and C'_s are

$$\begin{aligned} C_c &= 0.85 f'_c b \beta_1 c \\ &= (0.85)\left(3\ \frac{\text{kip}}{\text{in}^2}\right)(16\ \text{in})(0.85)(4.74\ \text{in}) \\ &= 164.4\ \text{kip} \\ C'_s &= (f'_s - 0.85 f'_c)A'_s \\ &= \left(41.2\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(3\ \frac{\text{kip}}{\text{in}^2}\right)\right)(0.4\ \text{in}^2) \\ &= 15.5\ \text{kip} \end{aligned}$$

The depth of the compression zone is

$$\begin{aligned} a &= \beta_1 c \\ &= (0.85)(4.74\ \text{in}) \\ &= 4.029\ \text{in} \end{aligned}$$

The design moment capacity is

$$\begin{aligned} \phi M_n &= \phi\left(C_c\left(d - \frac{a}{2}\right) + C'_s(d-d')\right) \\ &= (0.9)\left(\begin{array}{l}(164.4\ \text{kip})\left(22\ \text{in} - \frac{4.029\ \text{in}}{2}\right) \\ \quad + (15.5\ \text{kip})(22\ \text{in} - 2.5\ \text{in})\end{array}\right) \\ &= (3229\ \text{in-kip})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &= 269\ \text{ft-kip} \end{aligned}$$

This iterative procedure can be applied to any cross section containing multiple layers of longitudinal steel to find the neutral axis position that satisfies strain compatibility, equilibrium, and the stress-strain relationships.

5. Construction Considerations

There is generally more than one way to select reinforcement to furnish the required steel area. Certain criteria related to crack control and development of reinforcement—discussed in later sections—may influence the choice. However, the constraint that usually controls the choice is that the spacing of the reinforcement bars must provide for reasonable consolidation of the concrete.

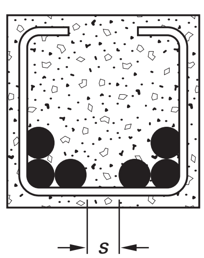
The limits on bar spacing for beams and girders are contained in ACI Secs. 3.3 and 7.6. The clear spacing between adjacent bars must be

$$s \geq \begin{cases} d_b \\ \frac{4}{3} \times \text{maximum aggregate size} \\ 1 \text{ in} \end{cases} \quad 3.17$$

Here, s is the clear spacing and d_b is the nominal bar diameter. In some cases, restrictions on beam width make it impractical to use separate bars and the code permits bars to be bundled in groups of two, three, or four bars in contact. For bundled bars, the nominal diameter used in the spacing limit is that of a fictitious round bar with the same area as the total areas of bars in the bundle.

Example 3.7
Spacing Limits for Bundled Bars

An interior beam contains no. 4 U-stirrups and main reinforcement consisting of six no. 10 bars arranged in bundles of three as shown. The maximum aggregate size is 1.5 in. Compute the clear distance required between bars.



Solution:

From Table 1.1, the cross-sectional area of one no. 10 bar is

$$A_b = 1.27 \text{ in}^2$$

The cross-sectional area of each bundle of three bars is

$$A_{\text{bundle}} = n_{\text{bars}} A_b = (3)(1.27 \text{ in}^2) = 3.81 \text{ in}^2$$

Find the equivalent diameter of a bundle.

$$\begin{aligned} A &= \pi r^2 \\ d_{\text{bundle}} &= 2r = 2\sqrt{\frac{A_{\text{bundle}}}{\pi}} \\ &= 2\sqrt{\frac{3.81 \text{ in}^2}{\pi}} \\ &= 2.20 \text{ in} \end{aligned}$$

From Eq. 3.17,

$$s \geq \begin{cases} d_b = 2.20 \text{ in} \quad \text{[controls]} \\ \frac{4}{3} \times \text{maximum aggregate size} = \left(\frac{4}{3}\right)(1.5 \text{ in}) \\ \qquad = 2.0 \text{ in} \\ 1 \text{ in} \end{cases}$$

The minimum spacing between the bars is 2.20 in.

4

Serviceability of Reinforced Concrete Beams

In addition to meeting requirements of flexural strength and ductility, reinforced concrete beams must meet serviceability requirements related to rigidity (such as deflection limits) and durability (such as crack width limits).

Serviceability issues are treated differently from the strength and ductility issues described in the previous chapter in two important ways. First, serviceability limits employ unfactored loadings, which are known as the service loads. Second, behavior is assumed to be within the linear elastic stress range. The following sections summarize and illustrate the ACI 318 serviceability provisions for beams.

1. Linear Elastic Behavior

In a properly designed reinforced concrete beam, the steel yields well before the concrete crushes. If the concrete were to crush before the steel yielded, failure would occur suddenly and without warning.

A properly designed beam, then, achieves its moment strength, M_n, by the yielding of its extreme tension steel. When the concrete in the beam crushes (that is, reaches its assumed ultimate strain of 0.003), steel strains are usually in excess of 0.005.

Figure 4.1 shows the relationship between moment and midspan deflection for a typical beam loaded to flexural failure. Initially, the beam is uncracked and the response is essentially linearly elastic, with stresses resisted by the gross section. Cracking is predicted to occur when the maximum tension stress reaches the modulus of rupture, f_r. For purposes of serviceability checks, the value of the modulus of rupture used by ACI Sec. 9.5 is

$$f_r = 7.5\sqrt{f'_c} = \frac{M_{\text{cr}} y_t}{I_g} \quad 4.1$$

$$M_{\text{cr}} = \frac{7.5\sqrt{f'_c} I_g}{y_t} \quad 4.2$$

I_g here is the moment of inertia of the gross concrete section about its neutral axis, y_t is the distance from the neutral axis to extreme tension fiber prior to cracking, and M_{cr} is the cracking moment.

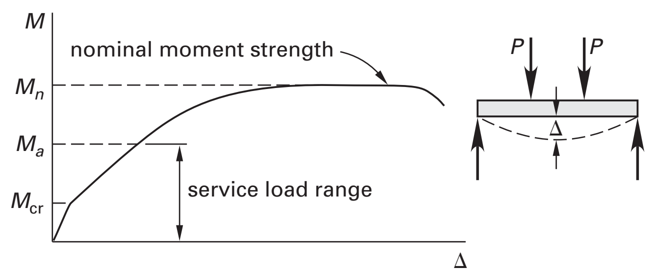


Figure 4.1 Typical Relationship Between Bending Moment and Deflection in a Reinforced Concrete Beam

After the section cracks, tension is resisted only by the steel, and the neutral axis shifts to a new position. Within the service load range, the member continues to behave linearly under short-term loading, but the moment of inertia is markedly lower than it was for the section before it cracked. To calculate deflection under short-term loading, ACI 318 employs an effective moment of inertia, I_e, that weights the gross and cracked moments of inertia.

$$I_e = \left(\frac{M_{\text{cr}}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{\text{cr}}}{M_a}\right)^3\right) I_{\text{cr}} \le I_g \quad 4.3$$

M_a is the maximum service load moment ever applied to the beam, and I_{cr} is the moment of inertia of the cracked section.

The section consists of two materials, steel and concrete, that have different properties. To simplify the calculation of elastic stresses and deformations, one material in the section can be replaced with an *equivalent area* of the other material—that is, the area that would give the same properties—resulting in a fictitious section composed of a single homogeneous material that has the same properties as the actual section. By assuming that a complete bond exists between steel and concrete and that strains must vary linearly through the depth of a section, the steel area can be replaced by an equivalent area, A_{tr}, as shown in Fig. 4.1.

$$f_s A_s = f_s^* A_{tr} \quad (4.4)$$

$$A_{tr} = \left(\frac{f_s}{f_s^*}\right) A_s = \left(\frac{E_s\varepsilon}{E_c\varepsilon}\right) A_s = \left(\frac{E_s}{E_c}\right) A_s = nA_s \quad (4.5)$$

n is the modular ratio, which is the modulus of elasticity of steel, E_s, divided by the modulus of elasticity of concrete, E_c.

$$n = \frac{E_s}{E_c} \quad (4.6)$$

In practice, the modular ratio is usually rounded to the nearest integral value; if the calculated value of n is, say, 9.34, it is usually taken as $n = 9$. Multiplying the actual steel area, A_s, by the modular ratio replaces the steel with an equivalent strip of concrete smeared across the section at depth d. After this replacement, the usual equations of solid mechanics apply to stresses and deflections. Note, however, that the computed stress, f_s^*, is a fictitious stress acting over the strip A_{tr}. The actual steel stress is obtained by multiplying the computed stress by n.

$$f_s = nf_s^* \quad (4.7)$$

Example 4.1
Elastic Stresses in a Singly Reinforced Rectangular Beam

A simply supported beam supports total uniformly distributed service load of 2.6 kip/ft. The beam has a span of 22 ft and is reinforced with three no. 9 tension bars. The width of the compression edge, b, is 16 in and the effective depth, d, is 22 in. The concrete is normal weight with a compressive strength, f_c', of 3000 psi, and the yield stress of the tension steel, f_y, is 60,000 psi. Calculate the maximum service load stresses in steel and concrete.

Solution:

From Eq. 1.1, the modulus of elasticity of the concrete is

$$E_c = 33w_c^{1.5}\sqrt{f_c'} = (33)\left(145\ \frac{\text{lbf}}{\text{ft}^3}\right)^{1.5}\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}} = 3{,}160{,}000\ \text{psi}$$

From Eq. 4.6, the modular ratio, rounded to the nearest integer, is

$$n = \frac{E_s}{E_c} = \frac{29{,}000{,}000\ \frac{\text{lbf}}{\text{in}^2}}{3{,}160{,}000\ \frac{\text{lbf}}{\text{in}^2}} = 9.18 \quad \text{[use 9]}$$

From Table 1.1, the area of three no. 9 bars is

$$A_s = n_{\text{bars}}A_b = (3)(1.00\ \text{in}^2) = 3.00\ \text{in}^2$$

The transformed steel area is

$$A_{tr} = nA_s = (9)(3)(1.00\ \text{in}^2) = 27.00\ \text{in}^2$$

The elastic neutral axis is a centroidal axis, so the moment of area above the neutral axis is equal to the moment of area below it.

$$\frac{b(kd)^2}{2} = A_{tr}(d - kd)$$

$$\frac{(16\ \text{in})(kd)^2}{2} = (27.00\ \text{in}^2)(22\ \text{in} - kd)$$

$$(kd)^2 + (3.375\ \text{in})(kd) - 74.25\ \text{in}^2 = 0$$

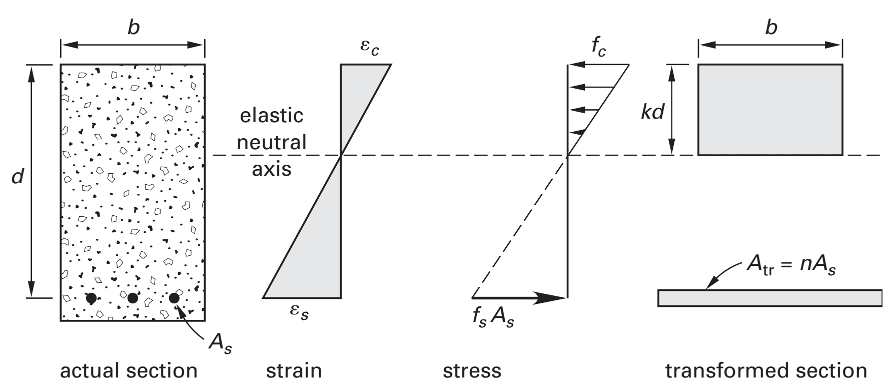


Figure 4.2 Notation for Transformed Area of a Singly Reinforced Rectangular Beam

Using the quadratic formula to solve for kd gives

$$\begin{aligned} kd &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(3.375\ \text{in}^2) \pm \sqrt{\begin{array}{l}(3.375\ \text{in})^2 \\ \quad -(4)(1)(-74.25\ \text{in}^2)\end{array}}}{(2)(1)} \\ &= 7.09\ \text{in} \end{aligned}$$

The formula for the moment of inertia of a cracked section is

$$\begin{aligned} I_{\text{cr}} &= \frac{b\,(kd)^3}{3} + A_{\text{tr}}\,(d - kd)^2 \\ &= \frac{(16\ \text{in})\,(7.09\ \text{in})^3}{3} + (27.00\ \text{in}^2)\,(22\ \text{in} - 7.09\ \text{in})^2 \\ &= 7900\ \text{in}^4 \end{aligned}$$

Maximum stresses occur at midspan. The maximum moment is

$$\begin{aligned} M_{\max} &= \frac{wL^2}{8} \\ &= \frac{\left(2.6\ \frac{\text{kip}}{\text{ft}}\right)(22\ \text{ft})^2}{8} \\ &= 157\ \text{ft-kip} \end{aligned}$$

The compressive stress in the concrete is

$$\begin{aligned} f_c &= \frac{M_{\max} kd}{I_{\text{cr}}} \\ &= \frac{(157\ \text{ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right)(7.09\ \text{in})}{7900\ \text{in}^4} \\ &= 1.69\ \text{ksi} \end{aligned}$$

The maximum service load steel stress is

$$\begin{aligned} f_s &= \frac{nM_{\max}(d - kd)}{I_{\text{cr}}} \\ &= \frac{(9)(157\ \text{ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right)(22\ \text{in} - 7.09\ \text{in})}{7900\ \text{in}^4} \\ &= 32.0\ \text{ksi} \end{aligned}$$

Example 4.2
Elastic Deflection of a Singly Reinforced Rectangular Beam

For the beam of Ex. 4.1, compute the immediate deflection at midspan when the uniform load of 2.6 kip/ft is applied. The overall depth of beam is 25 in.

Solution:
The gross moment of inertia of the section is

$$\begin{aligned} I_g &= \frac{bh^3}{12} = \frac{(16\ \text{in})\,(25\ \text{in})^3}{12} \\ &= 20{,}833\ \text{in}^4 \end{aligned}$$

The cracking moment is the moment that causes an extreme fiber tension stress equal to the modulus of rupture. From Eq. 4.2,

$$\begin{aligned} M_{\text{cr}} &= \frac{7.5\sqrt{f_c'}I_g}{y_t} \\ &= \frac{\left(\begin{array}{c}(7.5)\left(\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\right)\left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) \\ \times\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)(20{,}833\ \text{in}^4)\end{array}\right)}{12.5\ \text{in}} \\ &= 57\ \text{ft-kip} \end{aligned}$$

The effective moment of inertia is

$$\begin{aligned} I_e &= \left(\frac{M_{\text{cr}}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{\text{cr}}}{M_a}\right)^3\right) I_{\text{cr}} \le I_g \\ &= \left(\frac{57\ \text{ft-kip}}{157\ \text{ft-kip}}\right)^3 (20{,}833\ \text{in}^4) \\ &\quad + \left(1 - \left(\frac{57\ \text{ft-kip}}{157\ \text{ft-kip}}\right)^3\right)(7900\ \text{in}^4) \\ &= 8520\ \text{in}^4 \end{aligned}$$

The midspan deflection in a uniformly loaded simply supported beam is

$$\begin{aligned} \Delta &= \frac{5}{384}\frac{(wL)L^3}{E_c I_e} \\ &= \left(\frac{5}{384}\right)\frac{\left(2.6\ \frac{\text{kip}}{\text{ft}}\right)(22\ \text{ft})\left((22\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^3}{\left(3160\ \frac{\text{kip}}{\text{in}^2}\right)(8520\ \text{in}^4)} \\ &= 0.51\ \text{in} \quad [\text{downward}] \end{aligned}$$

2. Long-Term Behavior

Beams with non-rectangular cross sections and those that contain steel in the compression region are analyzed using basic principles in a manner similar to that illustrated in the previous section. For beams containing steel in the compression region, long-standing practice is to use a modular ratio of $2n$ to account for the effect of creep deformation in the concrete (in effect, taking the concrete modulus of elasticity as half the instantaneous modulus). The current ACI 318 approach, however, applies a multiplier, λ, to the immediate deflection to account for the long-term deformation.

$$\lambda = \frac{\xi}{1+50\rho'} \quad 4.8$$

In this formula,

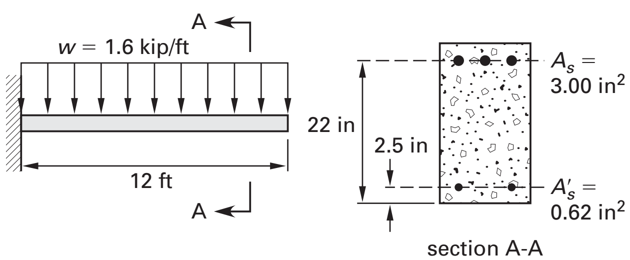
$$\rho' = \frac{A'_s}{bd} \quad 4.9$$

ξ is an empirical factor to account for the rate of additional deflection.

$\xi = 1.0$ for loads sustained 3 mo
$\xi = 1.2$ for loads sustained 6 mo
$\xi = 1.4$ for loads sustained 12 mo
$\xi = 2.0$ for loads sustained 5 yr or longer

Example 4.3
Elastic and Long-Term Deflection of a Doubly Reinforced Rectangular Beam

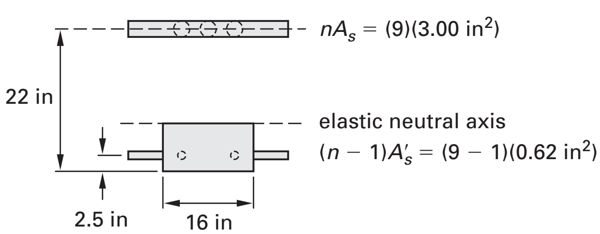
The concrete used in the cantilevered beam shown is of normal weight with a compressive strength of 3000 psi, a modulus of elasticity of 3160 ksi, and a modular ratio of 9. The width of the beam's compression edge is 16 in, its effective depth is 22 in, and its overall depth is 25 in. As calculated in the previous example, the gross moment of inertia of the section is 20,833 in^4, and the cracking moment is 57 ft-kip. Compute the immediate deflection and the total deflection if the load is sustained for 10 yr.



(not to scale)

Solution:

In this case, the tension steel is on top and the steel in the lower region is in compression. The transformed area for calculating immediate deflection is shown.



(not to scale)

The neutral axis is at the centroid of the transformed section.

$$\frac{b(kd)^2}{2} + (n-1)A'_s(kd-d') = nA_s(d-kd)$$

$$\frac{(16\text{ in})(kd)^2}{2} + (8)(0.62\text{ in}^2)(kd-2.5\text{ in}) = (9)(3.00\text{ in}^2)(22\text{ in}-kd)$$

$$kd = 6.94\text{ in}$$

The moment of inertia of the cracked section is

$$\begin{aligned} I_{cr} &= \frac{b(kd)^3}{3} + (n-1)A'_s\left(kd-d'\right)^2 + nA_s\left(d-kd\right)^2 \\ &= \frac{(16\text{ in})\left(6.94\text{ in}\right)^3}{3} \\ &\quad + (8)(0.62\text{ in}^2)\left(6.94\text{ in}-2.5\text{ in}\right)^2 \\ &\quad + (9)(3.00\text{ in}^2)\left(22\text{ in}-6.94\text{ in}\right)^2 \\ &= 8000\text{ in}^4 \end{aligned}$$

The maximum service load moment is at the left support.

$$M_a = \frac{wL^2}{2} = \frac{\left(1.6\ \frac{\text{kip}}{\text{ft}}\right)(12\text{ ft})^2}{2} = 115\text{ ft-kip}$$

The effective moment of inertia is

$$\begin{aligned} I_e &= \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1-\left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} \le I_g \\ &= \left(\frac{57\text{ ft-kip}}{115\text{ ft-kip}}\right)^3 (20{,}833\text{ in}^4) \\ &\quad + \left(1-\left(\frac{57\text{ ft-kip}}{115\text{ ft-kip}}\right)^3\right)(8000\text{ in}^4) \\ &= 9560\text{ in}^4 \end{aligned}$$

The immediate elastic deflection is

$$\begin{aligned} \Delta &= \frac{1}{8}\frac{(wL)L^3}{E_cI_e} \\ &= \left(\frac{1}{8}\right)\left(\frac{\left(1.6\ \frac{\text{kip}}{\text{ft}}\right)(12\text{ ft})\left((12\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^3}{\left(3160\ \frac{\text{kip}}{\text{in}^2}\right)(9560\text{ in}^4)}\right) \\ &= 0.24\text{ in}\quad\text{[downward]} \end{aligned}$$

From Eq. 4.8, the multiplier λ is

$$\lambda = \frac{\xi}{1+50\rho'}$$

From Eq. 4.9, the ratio ρ' is

$$\rho' = \frac{A'_s}{bd} = \frac{0.62\ \text{in}^2}{(16\ \text{in})(22\ \text{in})} = 0.00176$$

The factor ξ is 2.0 for a load sustained longer than 5 yr, so

$$\lambda = \frac{2.0}{1 + (50)(0.00176)} = 1.84$$

The long-term deflection after 10 yr is

$$\begin{aligned}\Delta_{\text{lt}} &= \Delta + \lambda\Delta \\ &= 0.24\ \text{in} + (1.84)(0.24\ \text{in}) \\ &= 0.68\ \text{in} \quad [\text{downward}]\end{aligned}$$

---

3. Durability Issues

In many respects, steel and concrete are ideal materials to use in combination. For one, the coefficient of thermal expansion is approximately the same for both, so temperature changes do not induce significant stresses. For another, properly proportioned concrete is practically inert in harsh environments and with adequate cover can protect the steel against corrosion and high temperatures. To ensure this protection, ACI 318 sets limits on certain concrete ingredients and admixtures—especially those containing chloride ions—and in ACI Sec. 7.7 appropriate minimum clear covers are prescribed. These minimums depend on the type of member, the service environment, and the diameter of the reinforcing bars.

To enhance the durability and in some cases the appearance of members, ACI 318 prescribes rules on crack control and distribution of flexural steel. ACI Sec. 10.6 gives the following rules.

- According to ACI Sec. 10.6.4, the spacing of reinforcement closest to the tension face of a flexural member must not exceed

$$s \leq \begin{cases} 15\left(\dfrac{40{,}000}{f_s}\right) - 2.5c_c \\ 12\left(\dfrac{40{,}000}{f_s}\right) \end{cases} \qquad 4.10$$

  s is the center-to-center spacing, f_s is the calculated service load stress in the steel, and c_c is the least distance from surface of reinforcement to the extreme tension face. In lieu of calculating f_s using linear elastic theory, ACI 318 permits the stress to be approximated as $2f_y/3$.
- Where the flanges of T-beams are in tension, some of the tension reinforcement must extend over a width defined as the lesser of the effective flange width and one-tenth of the span.
- Where the height of a flexural member, h, exceeds 36 in, longitudinal reinforcement is required on both faces, and must be spread over a distance $d/2$ (half the effective depth) from the tension face with a maximum spacing as given above, except that c_c in this case is the distance from surface of reinforcing to nearest face of member (see Fig. R10.6.7 in ACI 318).

5

Shear Design of Reinforced Concrete

Design shear stresses generally act along planes perpendicular to the longitudinal axis of a member. These stresses, either alone or in combination with normal stresses, create principal tensile stresses that can lead to sudden failure. ACI design criteria alleviate the risk of such failures by applying more conservative theories and resistance factors than are used for more ductile modes of failure.

In the following sections, the ACI 318 shear provisions are summarized and illustrated for some of the more commonly encountered design cases.

1. Shear Strength of Slender Reinforced Concrete Beams

The basic strength requirement for shear design is

$$\begin{aligned} V_u &\le \phi V_n \\ &\le \phi(V_c + V_s) \end{aligned} \qquad 5.1$$

V_u is the shear caused by the factored loads, V_n is the nominal shear strength of the member, V_c is the contribution of concrete to shear resistance, V_s is the contribution of shear reinforcement to shear resistance, and ϕ is the capacity reduction factor, which is 0.75.

The flowchart in Fig. 5.1 summarizes the ACI 318 shear design provisions that apply to the most commonly encountered case, in which the slender reinforced concrete beam is subject to the following restrictions.

- The span-to-depth ratio is greater than or equal to four.
- The beam is supported and loaded such that the reaction induces compression in the region near the support.
- The beam is transversely loaded by static forces so that the torsion and axial forces caused are negligible.
- Only vertical leg stirrups are used, and their total cross-sectional area is A_v.

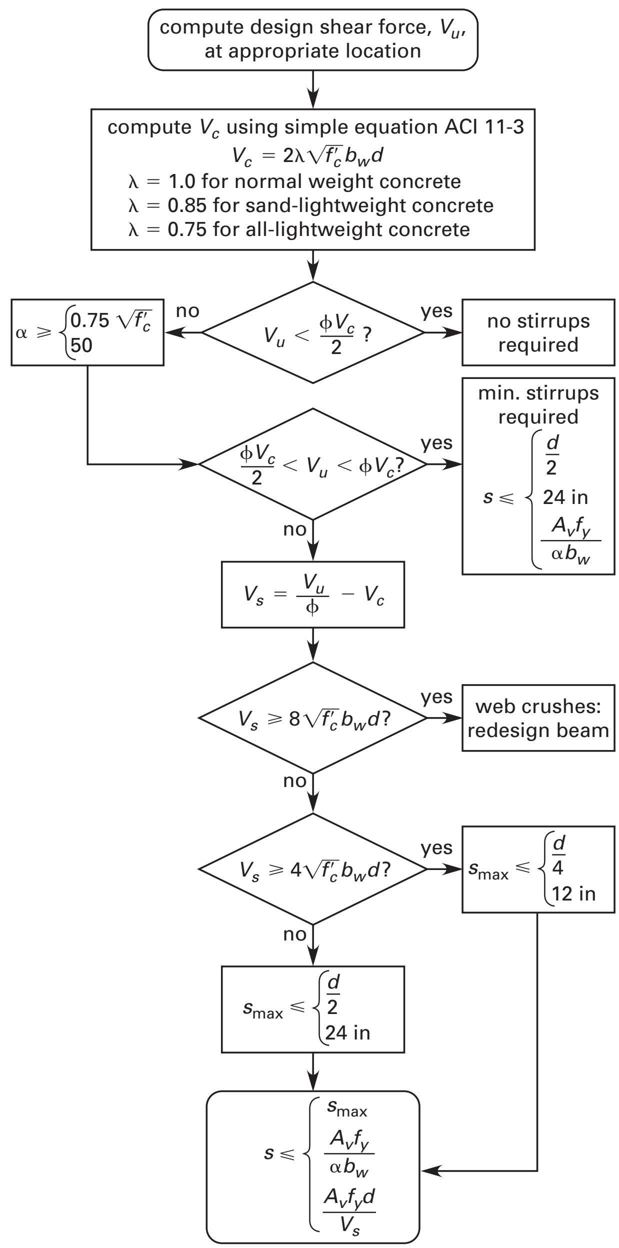


Figure 5.1 Shear Design Provisions for a Slender Reinforced Concrete Beam

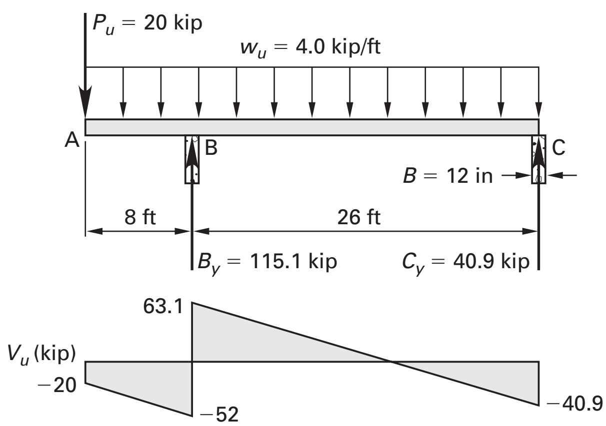
- The simpler expression for the contribution of the concrete to shear resistance is used (that is, $V_c = 2\lambda\sqrt{f'_c}b_w d$).

Comprehensive textbooks on reinforced concrete treat more general cases involving deep beams, beams with axial forces or torsion, or inclined shear reinforcement. Other special cases such as prestressed concrete members, two-way slabs, and members subjected to seismic loading are discussed in later chapters of this book.

Example 5.1

Stirrup Design for a Reinforced Concrete Beam

The loads shown in the following diagram are the factored loads, which include the beam weight. The effective depth is 20 in and the web width is 18 in. The distance from the support centerline to the face of support is 6 in. The concrete is normal weight with a compressive strength of 3000 psi, and the rebars are of grade 60 steel. The shear diagram given assumes simple "knife-edge" supports at the centers of supports. Calculate the spacing of no. 3 U-stirrups that is required by ACI 318.



Solution:

The solution to this problem has two parts. First, find the stirrup spacing required by ACI 318. Then, calculate the interval between points B and C over which ACI 318 does not require stirrups.

Start by determining the design shear at the most critical location of the beam. This occurs at a distance equal to the effective depth to the right of the support face at B.

$$\begin{aligned} V_u &= V_{\mathrm{B,rt}} - w_u\left(\frac{B}{2} + d\right) \\ &= 63.1\ \text{kip} - \left(4\ \frac{\text{kip}}{\text{ft}}\right)\left(\frac{12\ \text{in}}{2} + 20\ \text{in}\right) \\ &\quad \times \left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &= 54.4\ \text{kip} \end{aligned}$$

The shear strength of the concrete is

$$\begin{aligned} V_c &= 2\lambda\sqrt{f'_c}b_w d \\ &= (2)(1.0)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\ (18\ \text{in})(20\ \text{in}) \\ &\quad \times \left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) \\ &= 39.4\ \text{kip} \end{aligned}$$

Because the factored shear exceeds the resistance of the concrete, stirrups are required. From Eq. 5.1, the resistance that the stirrups must provide is

$$\begin{aligned} V_s &= \frac{V_u}{\phi} - V_c \\ &= \frac{54.4\ \text{kip}}{0.75} - 39.4\ \text{kip} \\ &= 33.1\ \text{kip} \end{aligned}$$

For 3000 psi concrete, using criteria from ACI Sec. 11.4.6.3, the constant α is

$$\begin{aligned} \alpha &\geq \begin{cases} 0.75\sqrt{f'_c} = 0.75\sqrt{3000\ \dfrac{\text{lbf}}{\text{in}^2}} = 41.1\ \dfrac{\text{lbf}}{\text{in}^2} \\ 50\ \dfrac{\text{lbf}}{\text{in}^2} \end{cases} \\ &= 50\ \text{psi} \end{aligned}$$

The yield stress of the grade 60 steel is 60,000 psi, so, from ACI Secs. 11.4.5 to 11.4.7, the required stirrup spacing is

$$s \leq \begin{cases} \dfrac{d}{2} = \dfrac{20\ \text{in}}{2} = 10\ \text{in} \\ \dfrac{A_v f_y}{\alpha b_w} = \dfrac{(0.22\ \text{in}^2)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{\left(50\ \dfrac{\text{lbf}}{\text{in}^2}\right)(18\ \text{in})} = 14.6\ \text{in} \\ \dfrac{A_v f_y d}{V_s} = \dfrac{(0.22\ \text{in}^2)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)(20\ \text{in})}{(33.1\ \text{kip})\left(1000\ \dfrac{\text{lbf}}{\text{kip}}\right)} \\ \quad = 8.0\ \text{in}\quad [\text{controls}] \end{cases}$$

Use no. 3 U-stirrups spaced at 8 in centers.

Next, determine the interval between B and C over which ACI 318 does not require stirrups. For this condition, the magnitude of the factored shear must be less than or equal to $\phi V_c/2$. Therefore,

$$|V_u| \le \frac{\phi V_c}{2}$$

$$|V_{\mathrm{B,rt}} - w_u x| \le \frac{\phi V_c}{2}$$

$$\left| 63.1 \text{ kip} - \left(4 \ \frac{\text{kip}}{\text{ft}}\right) x \right| \le \frac{(0.75)(39.4 \text{ kip})}{2}$$

$$\le 14.8 \text{ kip}$$

$$12.1 \text{ ft} \ge x \ge 19.5 \text{ ft}$$

Stirrups are not required over the region from 12.1 ft to 19.5 ft from the left support.

---

2. Shear Friction

There are many situations in which shear force is transferred from one concrete element to another or between a concrete element and another material. A model has been developed that has been shown to correlate with the nominal strength of this force transfer. This model is analogous to static friction with a normal force provided by properly developed reinforcement that crosses the shear plane, modified by an appropriate coefficient of static friction, μ.

ACI Sec. 11.6.4 defines the coefficient of friction as follows.

- For concrete placed monolithically, $\mu = 1.4\lambda$.
- For concrete placed against hardened and deliberately roughened concrete, with surface clean and roughened to a minimum amplitude of 1/4 in, $\mu = 1.0\lambda$.
- For concrete placed against hardened concrete cleaned of laitance, but not roughened, $\mu = 0.6\lambda$.
- For concrete placed against as-rolled, unpainted structural steel, $\mu = 0.7\lambda$.

The constant λ is 1.0 for normal weight concrete, 0.85 for sand-lightweight concrete, and 0.75 for all-lightweight concrete.

When shear friction reinforcement with an area of A_{vf} crosses the shear plane at right angles, which is the usual case, the nominal shear strength is

$$V_n \le \begin{cases} \mu A_{vf} f_y \\ 0.2 f'_c A_c \\ \left(800 \ \frac{\text{lbf}}{\text{in}^2}\right) A_c \end{cases} \qquad 5.2$$

f_y, the yield strength of reinforcement, is limited to a maximum of 60,000 psi. A_c is the area of the concrete section resisting shear. ACI 318 also gives a more general expression for shear transfer when tensile shear friction reinforcement crosses the shear plane at an acute angle.

Example 5.2
Shear Friction Reinforcement

A concrete floor slab 6 in thick is cast against the hardened surface of a shear wall that has been cleaned but not deliberately roughened. The slab acts as a rigid diaphragm and must transfer a factored lateral force of 200 kip into the wall over a length of 11 ft. Concrete in both wall and slab is normal weight having a compressive strength of 4000 psi. Compute the area of grade 60 reinforcement that must cross the shear plane.

Solution:

The interface area for shear transfer is

$$A_c = Bh = (11 \text{ ft})(6 \text{ in})\left(12 \ \frac{\text{in}}{\text{ft}}\right) = 792 \text{ in}^2$$

From Eq. 5.2, the maximum shear transfer across this area is

$$V_u \le \begin{cases} \phi 0.2 f'_c A_c \\ \quad = (0.75)(0.2)\left(4000 \ \frac{\text{lbf}}{\text{in}^2}\right) \times (792 \text{ in}^2)\left(\frac{1 \text{ kip}}{1000 \text{ lbf}}\right) \\ \quad = 475 \text{ kip} \\ \phi\left(800 \ \frac{\text{lbf}}{\text{in}^2}\right) A_c \\ \quad = (0.75)\left(800 \ \frac{\text{lbf}}{\text{in}^2}\right)(792 \text{ in}^2)\left(\frac{1 \text{ kip}}{1000 \text{ lbf}}\right) \\ \quad = 475 \text{ kip} \end{cases}$$

The maximum shear transfer is greater than the required shear transfer of 200 kip. Thus, with properly developed shear friction reinforcement, the force transfer can be done. For these conditions, ACI Sec. 11.6.4 defines the coefficient of friction as

$$\mu = 0.6\lambda = (0.6)(1.0) = 0.6$$

The basic strength requirement is found from Eqs. 5.1 and 5.2.

$$V_u = \phi V_n \le \phi \mu A_{vf} f_y$$

$$A_{vf} \ge \frac{V_u}{\phi \mu f_y} \ge \frac{200 \text{ kip}}{(0.75)(0.6)\left(60 \ \frac{\text{kip}}{\text{in}^2}\right)} \ge 7.40 \text{ in}^2$$

---

3. Brackets and Corbels

A bracket or corbel is a reinforced concrete element that projects from a wall or column and supports a reaction from another element. In most cases, the reaction is primarily a vertical force whose resultant acts at a short distance a_v from the face of the supporting wall or column, as shown in Fig. 5.2. Unless special provisions are made to alleviate horizontal forces (for example, by using very low friction-bearing pads or roller supports), there will also be a horizontal tensile force, N_{uc}, transmitted into the support. ACI Sec. 11.8.3 requires that the horizontal force be treated as a live load with a magnitude of at least 20% of the vertical force, V_u. For most practical cases, when the ratio a_v/d is less than 1, the prescriptive procedure in Sec. 11.8 of ACI 318 is used to design the bracket or corbel.

The following restrictions apply.

- The yield strength of reinforcement, f_y, must not exceed 60,000 psi.
- The horizontal tensile force, N_{uc}, must not be less than $0.2V_u$ or greater than V_u.
- The effective depth, d, is determined at face of support.
- The main reinforcement, with an area of A_{sc}, must fully develop at face of support. This usually requires mechanical anchorage of the main steel, either by welding a cross bar of the same diameter or by welding to steel plates.
- The depth at the outside edge of bearing must be not less than $0.5d$.
- The capacity reduction factor, ϕ, is 0.75 in all calculations.
- Shear transfer occurs through shear friction.
- Supplementary shear friction steel in the form of closed ties is distributed over the upper $2d/3$ of the member.

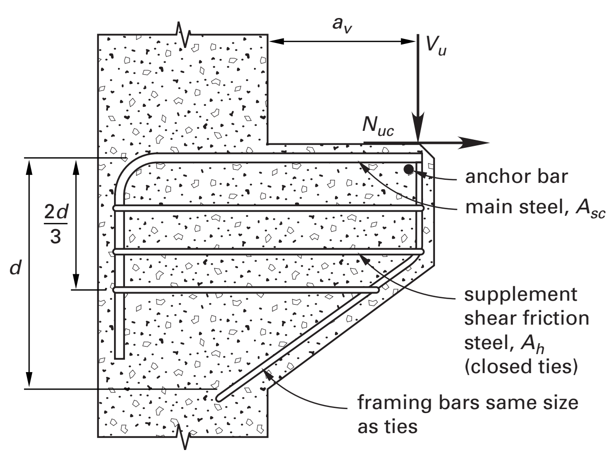


Figure 5.2 Elevation of Bracket or Corbel Supported by Column (column steel omitted for clarity)

The flowchart in Fig. 5.3 summarizes the prescriptive design provisions in ACI 318 for a bracket or corbel. Example 5.3 illustrates their application.

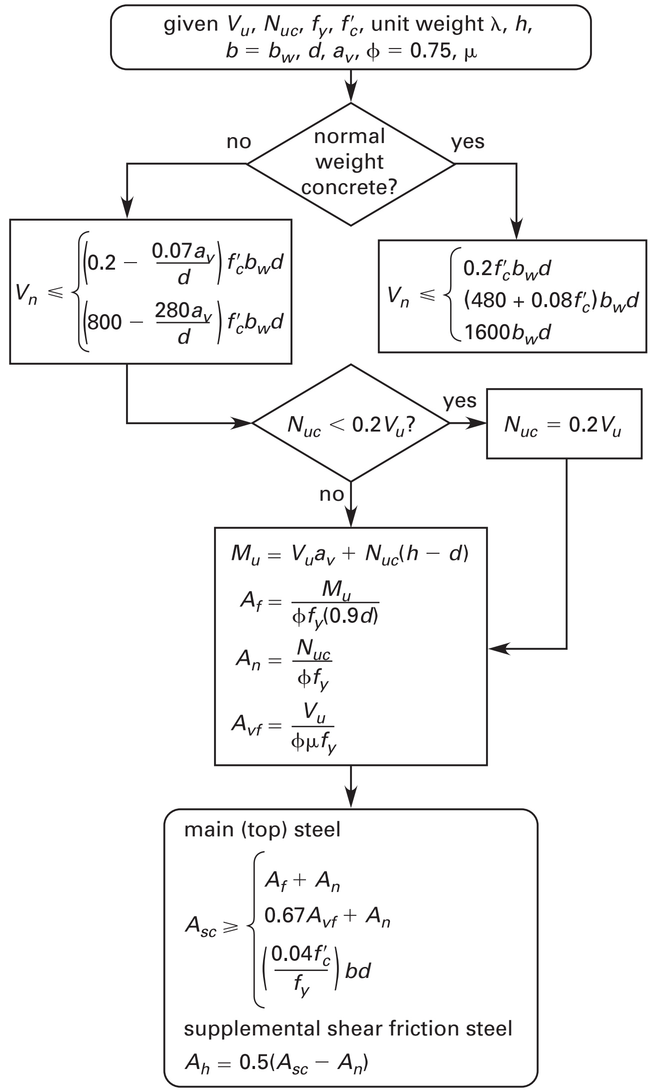


Figure 5.3 ACI Design Provisions for a Reinforced Concrete Bracket or Corbel

Example 5.3
Design of a Reinforced Concrete Corbel

A concrete corbel supports a vertical factored force of 75 kip that acts 5 in from face of support, and an unfactored normal force computed as 12 kip, based on anticipated volume change effects. The corbel is monolithic with the supporting member. The concrete is normal weight with a compressive strength of 5000 psi, and the steel is grade 60. Preliminary design gives a corbel projecting 10 in from face of support with a width of 12 in and an overall depth of 16 in. The distance from the top surface of the corbel to the centroid of the main steel is 1.25 in. Calculate the areas of main steel and supplementary shear friction steel.

Solution:
The effective depth at the face of support is

$$\begin{aligned} d &= h - 1.25 \text{ in} = 16 \text{ in} - 1.25 \text{ in} \\ &= 14.75 \text{ in} \end{aligned}$$

Concrete is normal weight, so from Fig. 5.3 the maximum shear strength is

$$V_n \le \begin{cases} 0.2f'_c b_w d \\ \quad = (0.2)\left(5000\ \frac{\text{lbf}}{\text{in}^2}\right)(12 \text{ in})(14.75 \text{ in}) \\ \quad = 177{,}000 \text{ lbf} \\ (480 + 0.08f'_c)b_w d \\ \quad = \left(480\ \frac{\text{lbf}}{\text{in}^2} + (0.08)\left(5000\ \frac{\text{lbf}}{\text{in}^2}\right)\right) \\ \qquad \times (12 \text{ in})(14.75 \text{ in}) \\ \quad = 156{,}000 \text{ lbf} \quad \text{[controls]} \\ 1600 b_w d \\ \quad = \left(1600\ \frac{\text{lbf}}{\text{in}^2}\right)(12 \text{ in})(14.75 \text{ in}) \\ \quad = 283{,}000 \text{ lbf} \end{cases}$$

No special provisions exist to alleviate the restraint to volume changes, so ACI Sec. 11.8.3.4 requires

$$N_{uc} \ge \begin{cases} 0.2V_u = (0.2)(75 \text{ kip}) \\ \qquad = 15 \text{ kip} \\ 1.6N = (1.6)(12 \text{ kip}) \\ \qquad = 19.2 \text{ kip} \quad \text{[controls]} \end{cases}$$

The factored moment at support face is

$$\begin{aligned} M_u &= V_u a + N_{uc}(h - d) \\ &= (75 \text{ kip})(5 \text{ in}) + (19.2 \text{ kip})(16 \text{ in} - 14.75 \text{ in}) \\ &= 399 \text{ in-kip} \end{aligned}$$

The usual approximation is that the lever arm of the internal couple is $0.9d$, so the area of steel to resist bending is

$$\begin{aligned} A_f &= \frac{M_u}{\phi f_y (0.9d)} \\ &= \frac{399 \text{ in-kip}}{(0.75)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)(0.9)(14.75 \text{ in})} \\ &= 0.67 \text{ in}^2 \end{aligned}$$

The steel needed to resist the direct tension is

$$\begin{aligned} A_n &= \frac{N_{uc}}{\phi f_y} \\ &= \frac{19.2 \text{ kip}}{(0.75)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)} \\ &= 0.43 \text{ in}^2 \end{aligned}$$

The total shear friction steel needed is

$$\begin{aligned} A_{vf} &= \frac{V_u}{\phi \mu f_y} \\ &= \frac{75 \text{ kip}}{(0.75)(1.4)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)} \\ &= 1.19 \text{ in}^2 \end{aligned}$$

The main steel required is

$$A_{sc} \ge \begin{cases} A_f + A_n \\ \quad = 0.67 \text{ in}^2 + 0.43 \text{ in}^2 \\ \quad = 1.10 \text{ in}^2 \\ \frac{2A_{vf}}{3} + A_n \\ \quad = \frac{(2)\left(1.19 \text{ in}^2\right)}{3} + 0.43 \text{ in}^2 \\ \quad = 1.23 \text{ in}^2 \quad \text{[controls]} \\ \left(\frac{0.04f'_c}{f_y}\right) bd \\ \quad = \left(\frac{(0.04)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)}{60\ \frac{\text{kip}}{\text{in}^2}}\right)(12 \text{ in})(14.75 \text{ in}) \\ \quad = 0.59 \text{ in}^2 \end{cases}$$

The area of supplemental shear friction steel required is

$$\begin{aligned} A_h &= 0.5(A_{sc} - A_n) \\ &= (0.5)(1.23 \text{ in}^2 - 0.43 \text{ in}^2) \\ &= 0.40 \text{ in}^2 \end{aligned}$$

One way to satisfy these requirements would be to use two no. 8 bars for the main steel and two no. 3 closed ties in the upper two-thirds of the corbel.

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4. Torsion

Most reinforced concrete members are loaded either axially or transversely in such a manner that twisting of the cross section is negligible. ACI Sec. 11.5 sets threshold limits for torsional moments beyond which their effects must be included in designs. From ACI Sec. 11.5.1, for nonprestressed members, the threshold limit is

$$T_u < \phi\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) \tag{5.3}$$

T_u is the factored torsional moment, A_{cp} is the area of the outside perimeter of section resisting torsion, p_{cp} is the outside perimeter of the section resisting torsion.

If the applied factored torsional moment exceeds the threshold, the member must be reinforced with additional transverse ties and longitudinal steel to resist the torsional moment. ACI 318 considers two cases: equilibrium torsion and compatibility torsion.

Equilibrium torsion applies to situations where redistribution of loads cannot occur and the torsional resistance is necessary to maintain equilibrium. A common example is a precast concrete spandrel beam. *Compatibility torsion* describes a situation in which loads can redistribute after torsional cracking. In such a case, a reduction is permitted in the design value of torsional moment. In the case of compatibility torsion, ACI Sec. 11.5.2 allows the maximum torsional moment that the member must be designed to carry to be limited to four times the threshold value.

Figure 5.4 shows two spandrel beams: one in a precast floor system and the other in a cast-in-place system. For the precast beam, full torsional resistance is needed to maintain equilibrium and this member must be designed for equilibrium torsion. In contrast, for the cast-in-place spandrel, a reduction in torsional resistance reduces the negative bending moment transferred from the slab with a corresponding redistribution of moment to the positive region of the slab. This is a case of compatibility torsion.

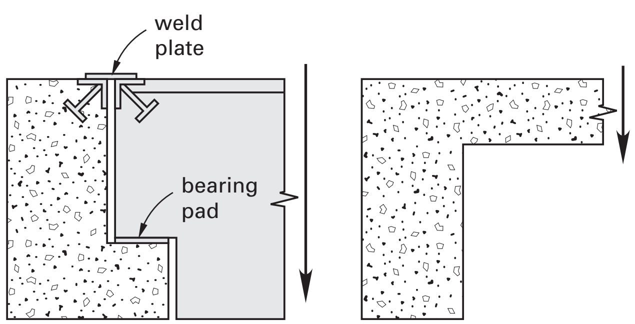


Figure 5.4 Sections Through Precast and Cast-in-Place Spandrel Beams

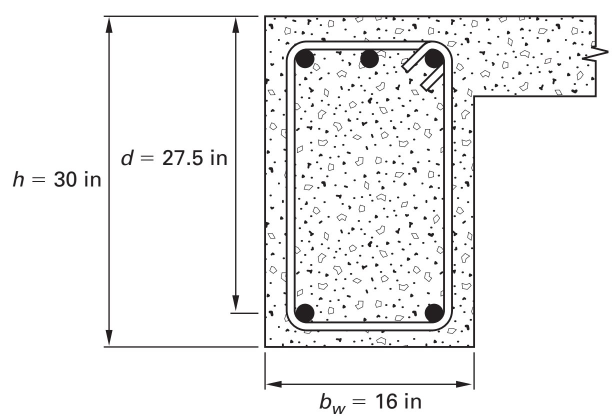
Design for torsion is performed using a space truss model and assuming an equivalent hollow tube, compression diagonals developing in the concrete at 45° to the longitudinal axis, and longitudinal and transverse steel resisting all tension. The ties must be closed, with details defined in ACI Sec. 11.5.4. The transverse steel needed to resist torsion is additive to the stirrups resisting shear. Because only the outer legs of the stirrups provide torsional resistance, the transverse steel required is

$$\text{stirrup reinforcement} = \frac{A_v}{s} + 2\left(\frac{A_t}{s}\right) \qquad 5.4$$

s is the spacing, A_v is the area of outer stirrup legs resisting shear, and A_t is the required area of one outer leg to resist torsion. The space truss analogy also requires longitudinal steel, A_l, that is generally additive to any other longitudinal steel required to resist axial forces or bending. ACI Sec. 11.5.6.2 requires that the longitudinal torsional steel must be distributed around the perimeter resisting torsion at a spacing of no more than 12 in. The design procedure for both transverse and longitudinal reinforcement for members with torsional moment, T_u, exceeding the threshold is summarized in the flowchart of Fig. 5.5.

Example 5.4
Torsional Design Moment

A spandrel beam supports floor loads from a one-way slab. The slab creates a factored shear force of 54 kip and a factored torsional moment of 50 ft-kip at a critical section located 27.5 in from the face of support. The concrete is normal weight with a compressive strength of 3000 psi. Stirrups are no. 3 with 1.5 in cover. The overhanging portion of the slab is not detailed to resist torsion. Determine whether torsional stresses are significant and, if so, the maximum torsional moment that ACI specifies.



Solution:
The outside perimeter of the cross section is

$$\begin{aligned} p_{cp} &= 2(b_w + h) \\ &= (2)(16 \text{ in} + 30 \text{ in}) \\ &= 92 \text{ in} \end{aligned}$$

The area of the cross section is

$$\begin{aligned} A_{cp} &= b_w h \\ &= (16 \text{ in})(30 \text{ in}) \\ &= 480 \text{ in}^2 \end{aligned}$$

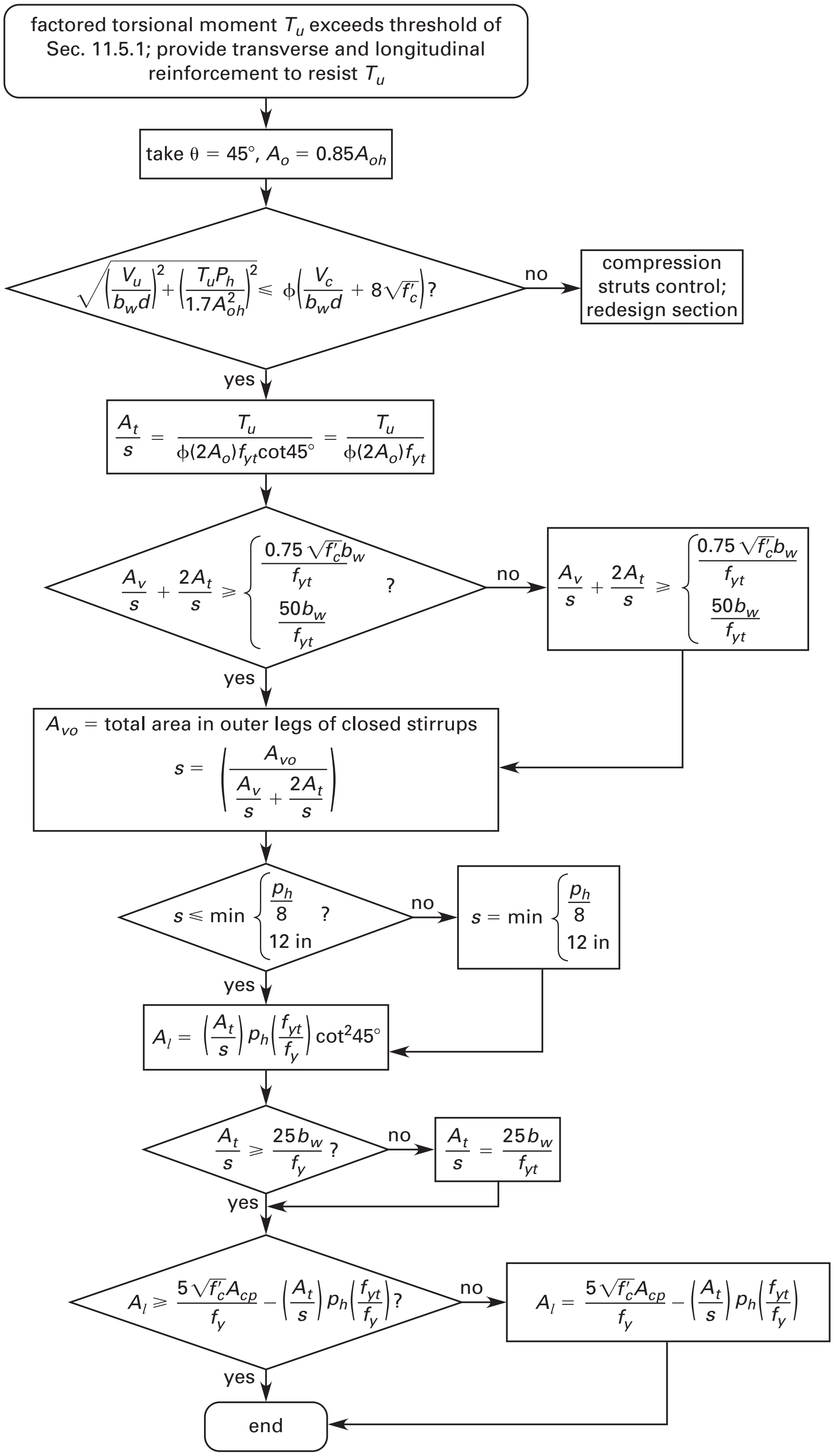


Figure 5.5 Torsion Design Provisions for a Slender Reinforced Concrete Beam

From Eq. 5.3, the threshold torsion for non-prestressed members is

$$\begin{aligned} T_{ut} &= \phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) \\ &= 0.75\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\left(\frac{\left(480\ \text{in}^2\right)^2}{92\ \text{in}}\right) \\ &\quad \times\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)\left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) \\ &= 8.6\ \text{ft-kip} \end{aligned}$$

Because the factored torsional moment exceeds the threshold torsional moment, torsional shear stresses are significant. The spandrel beam is an example of compatibility torsion, for which the code permits a redistribution of forces after torsional cracking. The maximum torsional moment that applies in this case is

$$\begin{aligned} T_u &= 4\phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) \\ &= 4T_{ut} \\ &= (4)(8.6\ \text{ft-kip}) \\ &= 34.4\ \text{ft-kip} \end{aligned}$$

The critical section should be designed to resist a factored shear force of 54 kip and a factored torsional moment of 34.4 ft-kip.

Example 5.5
Torsional Reinforcement

Design the torsional reinforcement for the beam of Ex. 5.4, at a section for which the factored shear is 54 kip and the reduced torsional moment is 34.4 ft-kip. Assume that no. 3 stirrups at 12 in centers acting alone are adequate to resist the factored shear. Use grade 60 reinforcement for both transverse and longitudinal steel.

Solution:

The outside perimeter of the cross section is

$$\begin{aligned} p_{cp} &= 2(b_w + h) \\ &= (2)(16\ \text{in} + 30\ \text{in}) \\ &= 92\ \text{in} \end{aligned}$$

The area of the cross section is

$$\begin{aligned} A_{cp} &= b_w h \\ &= (16\ \text{in})(30\ \text{in}) \\ &= 480\ \text{in}^2 \end{aligned}$$

The perimeter of the centerline of the outermost closed transverse torsional reinforcement is

$$\begin{aligned} p_h &= 2\big((b_w - 2(\text{cover}) - d_b) + (h - 2(\text{cover}) - d_b)\big) \\ &= (2)\left(\begin{array}{l} (16\ \text{in} - (2)(1.5\ \text{in}) - 0.375\ \text{in}) \\ \quad + (30\ \text{in} - (2)(1.5\ \text{in}) - 0.375\ \text{in}) \end{array}\right) \\ &= 78.5\ \text{in} \end{aligned}$$

The area enclosed by this centerline is

$$\begin{aligned} A_{oh} &= (b_w - 2(\text{cover}) - d_d)(h - 2(\text{cover}) - d_b) \\ &= (16\ \text{in} - (2)(1.5\ \text{in}) - 0.375\ \text{in}) \\ &\qquad \times (30\ \text{in} - (2)(1.5\ \text{in}) - 0.375\ \text{in}) \\ &= 336\ \text{in}^2 \end{aligned}$$

Check that the section can be reinforced to resist the loading. ACI Eq. 11-18 requires

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right)$$

The nominal shear strength of the concrete is given by ACI Eq. 11-3.

$$\begin{aligned} V_c &= 2\lambda\sqrt{f'_c}b_w d \\ &= (2)(1.0)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}(16\ \text{in})(27.5\ \text{in}) \\ &= 48{,}200\ \text{lbf} \end{aligned}$$

Calculating the left side of ACI Eq. 11-18,

$$\begin{aligned} &\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \\ &= \sqrt{\begin{array}{l} \left(\dfrac{54\ \text{kip}}{(16\ \text{in})(27.5\ \text{in})}\right)^2 \\ \quad + \left(\dfrac{(34.4\ \text{ft-kip})\left(12\ \dfrac{\text{in}}{\text{ft}}\right)(78.5\ \text{in})}{(1.7)\left(336\ \text{in}^2\right)^2}\right)^2 \end{array}} \\ &= 0.208\ \text{ksi} \end{aligned}$$

Calculating the right side,

$$\begin{aligned} &\phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \\ &\quad = (0.75)\left(\frac{48{,}200\ \text{lbf}}{(16\ \text{in})(27.5\ \text{in})} + 8\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\right) \\ &\qquad \times\left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) \\ &\quad = 0.410\ \text{ksi} \end{aligned}$$

The left side of ACI Eq. 11-18 is less than the right side, so the cross section is sufficient.

The required additional leg area of stirrups to resist torsion can be calculated from ACI Eqs. 11-20 and 11-21.

$$\begin{aligned}
T_n\phi &\geq T_u \\
T_n &= \frac{2A_oA_tf_{yt}}{s}\cot\theta \\
\frac{T_u}{\phi} &= \frac{2(0.85A_{oh})A_tf_{yt}}{s}\cot\theta \\
\frac{A_t}{s} &= \frac{T_u}{1.7\phi A_{oh}f_{yt}\cot 45^\circ} \\
&= \frac{(34.4\text{ ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{(1.7)(0.75)(336\text{ in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)(1)} \\
&= 0.016\text{ in}
\end{aligned}$$

The given shear reinforcement, $A_v = 0.22\text{ in}^2$ spaced at 12 in centers, is needed to resist the factored shear force of 54 kip. ACI Eq. 11-23 gives the minimum stirrups requirement.

$$\frac{A_v}{s} + \frac{2A_t}{s} \geq \begin{cases} 0.75\sqrt{f'_c}\left(\dfrac{b_w}{f_{yt}}\right) \\ \dfrac{50b_w}{f_{yt}} \end{cases}$$

$$\frac{0.22\text{ in}^2}{12\text{ in}} + (2)(0.016\text{ in}) \geq \begin{cases} 0.75\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}} \\ \qquad \times \left(\dfrac{16\text{ in}}{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right) \\ \dfrac{(50)(16\text{ in})}{\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)} \end{cases}$$

$$0.050\text{ in} \geq \begin{cases} 0.011\text{ in} \\ 0.013\text{ in} \end{cases}$$

Thus, 0.050 in controls. For no. 3 closed stirrups, the required spacing is found from

$$\frac{A_v}{s} + \frac{2A_t}{s} = 0.050\text{ in} = \frac{0.22\text{ in}^2}{s}$$

$$s = \frac{0.22\text{ in}^2}{0.050\text{ in}} = 4.4\text{ in}$$

ACI Sec. 11.5.6.1 sets the maximum spacing limit for the stirrups.

$$s \leq \begin{cases} \dfrac{p_h}{8} = \dfrac{78.5\text{ in}}{8} = 9.8\text{ in} & \text{[does not control]} \\ 12\text{ in} \end{cases}$$

Thus, no. 3 closed ties at 4 in centers are adequate for the transverse reinforcement at the critical section. ACI Eq. 11-22 gives the additional longitudinal steel required.

$$\begin{aligned}
A_l &= \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yt}}{f_y}\right)\cot^2\theta \\
&= (0.016\text{ in})(78.5\text{ in})\left(\frac{60\ \frac{\text{kip}}{\text{in}^2}}{60\ \frac{\text{kip}}{\text{in}^2}}\right)\cot^2 45^\circ \\
&= 1.26\text{ in}^2
\end{aligned}$$

The minimum amount of additional longitudinal steel is set by ACI Sec. 11.5.5.3.

$$\begin{aligned}
\frac{A_t}{s} &\geq \frac{25b_w}{f_{yt}} \\
&\geq \frac{\left(25\ \frac{\text{lbf}}{\text{in}^2}\right)(16\text{ in})}{60{,}000\ \frac{\text{lbf}}{\text{in}^2}} \\
&\geq 0.0067\text{ in} \qquad \left[\begin{array}{l}\text{the previously computed} \\ \text{value of 0.050 in controls}\end{array}\right] \\
A_{l,\min} &= \frac{5\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yt}}{f_y}\right) \\
&= \frac{5\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}(480\text{ in}^2)}{60{,}000\ \frac{\text{lbf}}{\text{in}^2}} \\
&\qquad - (0.016\text{ in})(78.5\text{ in})\left(\frac{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right) \\
&= 0.93\text{ in}^2 \quad \text{[does not control]}
\end{aligned}$$

Provide 1.26 in^2 of additional longitudinal steel distributed around the perimeter of the closed stirrups at a spacing not to exceed 12 in.

6

Columns and Compression Members

A compression member is defined as a column in ACI 318 if it has a ratio of unsupported length to least cross-sectional dimension greater than three. When stability effects are important, a column is *slender*; otherwise it is *stocky*.

ACI 318 requires that columns have at least some minimum percentage of longitudinal and transverse reinforcement. Section 10.10 of ACI 318 contains design provisions for slender columns. Columns are further classified as either *tied* or *spiral* according to their transverse reinforcement. Details of reinforcement and requirements for size, spacing, and pitch of the transverse reinforcement are in ACI Secs. 7.10 and 10.9.3.

The transverse reinforcement restrains the longitudinal reinforcement to prevent its buckling. Spirals also serve to confine the concrete within, significantly increasing the confined concrete's strength and ductility. The code rewards this greater ductility and toughness by applying more liberal criteria to a spiral column than it does for a tied column with the same nominal strength.

The more commonly encountered design criteria for columns are summarized and illustrated in the following sections.

1. Stocky Column Behavior

Figure 6.1 shows the notation used to define the cross-sectional properties of a rectangular reinforced concrete column.

For convenience, this book uses a longitudinal reinforcement ratio, ρ_g, relating the total area of longitudinal steel, A_{st}, to the gross cross-sectional area of concrete, A_g.

$$\rho_g = \frac{A_{\text{st}}}{A_g} \qquad 6.1$$

To alleviate congestion and improve the consolidation of fresh concrete, ACI Sec. 10.9 imposes an upper limit of 0.08 on ρ_g. To provide ductility, the code also imposes a lower limit of 0.01.

ACI Sec. 10.9 requires a minimum of three longitudinal bars if triangular ties are used, four bars for rectangular or circular ties, and six for spirals.

ACI Secs. 10.2 and 10.3 give the basic assumptions that govern the strength of stocky columns.

- Strain varies linearly across the section.
- A complete bond exists between the steel and the concrete (that is, the strain in the steel is the same as in the adjacent concrete).
- The steel stress is linearly elastic for strains less than f_y/E_s. For strains greater than f_y/E_s, the steel stress is perfectly plastic and equal to f_y.
- The tension stress in the concrete is negligible (that is, all tension stress is resisted by steel).
- The ultimate strain in the concrete is 0.003.
- The concrete stress distribution may be replaced by an equivalent rectangular distribution with uniform stress $0.85f'_c$ acting over an area ba and creating a compression resultant that acts at distance $a/2$ from the compression edge.

The plastic centroid of the cross section is the location at which an axial compressive force, P, can be applied and create uniform compressive strains throughout. For the usual case of a symmetrically reinforced section, the plastic centroid coincides with the geometric centroid. Most columns are loaded with both an axial force and bending moment about a principal axis. For convenience, a statically equivalent system with the axial force acting at an eccentricity e from the plastic centroid replaces the bending moment, as shown in Fig. 6.1. For a rectangular cross section, h denotes the dimension parallel to e, and b is the dimension perpendicular to e.

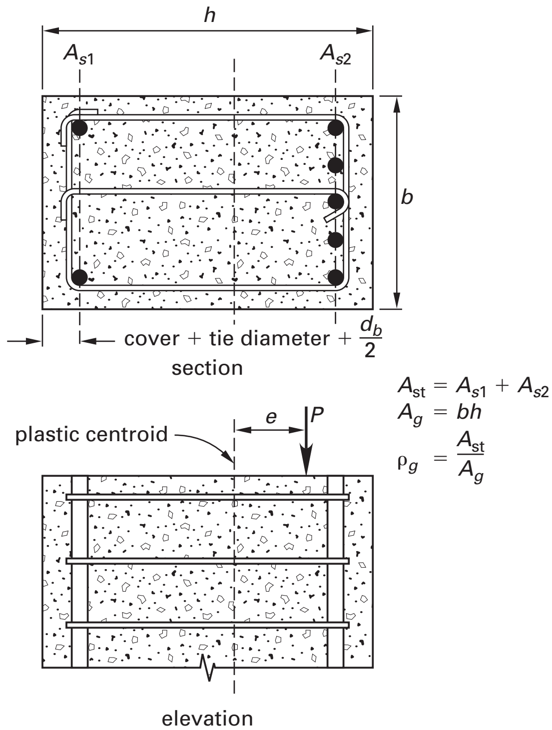
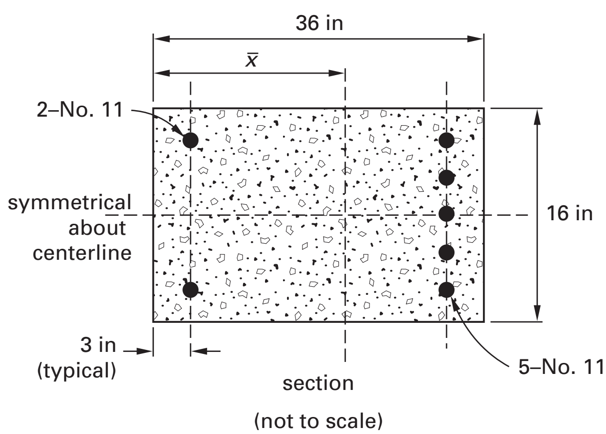


Figure 6.1 Notation for a Rectangular Reinforced Concrete Column

Example 6.1
Plastic Centroid for an Asymmetrical Column Section

An asymmetrical reinforced cross section is shown. The concrete is normal weight with a compressive strength of 5 ksi, and the steel has a yield strength of 60 ksi. Cover and stirrup size are such that the distance from the edge of the concrete to the centers of the bars is 3 in. Compute the location of the plastic centroid.



Solution:

Because the reinforcement is symmetric about a horizontal axis, the plastic centroid is 8 in (half the height of the section) from the top and bottom edges.

To determine the distance from the left edge to the plastic centroid, use the fact that the cross section is uniformly strained to 0.003, which means that the steel stress is at yield, 60 ksi. Let P_0 denote the concentric load strength of the section, C_{s1} the effective compressive force in the left side reinforcement at nominal strength, and C_{s2} the force in the right side reinforcement at nominal strength. From Table 1.1, the cross-sectional area of two no. 11 bars is $2 \times 1.56 \text{ in}^2 = 3.12 \text{ in}^2$ and that of five no. 11 bars is $5 \times 1.56 \text{ in}^2 = 7.80 \text{ in}^2$. Then

$$\begin{aligned} C_c &= 0.85 f'_c bh \\ &= (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)(36 \text{ in})(16 \text{ in}) \\ &= 2448 \text{ kip} \end{aligned}$$

$$\begin{aligned} C_{s1} &= (f_y - 0.85 f'_c)A_{s1} \\ &= \left(60\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)\right)(3.12 \text{ in}^2) \\ &= 174 \text{ kip} \end{aligned}$$

$$\begin{aligned} C_{s2} &= (f_y - 0.85 f'_c)A_{s2} \\ &= \left(60\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)\right)(7.80 \text{ in}^2) \\ &= 435 \text{ kip} \end{aligned}$$

Equilibrium requires that the sum of forces longitudinally must equal zero, which gives the magnitude of the axial force.

$$\begin{aligned} P_0 &= C_c + C_{s1} + C_{s2} \\ &= 2448 \text{ kip} + 174 \text{ kip} + 435 \text{ kip} \\ &= 3057 \text{ kip} \end{aligned}$$

By definition, the plastic centroid produces uniform axial compression. Therefore, the moments computed at the edge of the section must be zero. This gives the required distance.

$$\bar{x}P_0 = C_c\left(\frac{h}{2}\right) + C_{s1}d_1 + C_{s2}d_2$$

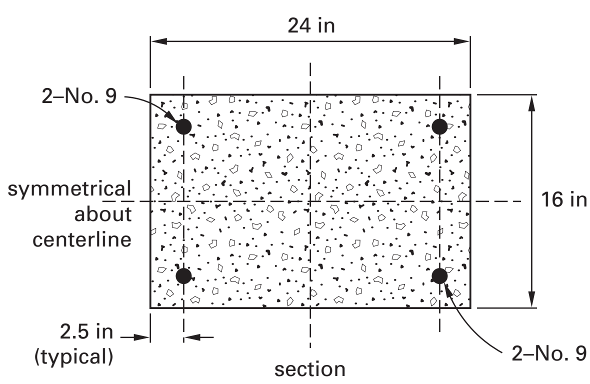
$$\begin{aligned} \bar{x} &= \frac{C_c\left(\frac{h}{2}\right) + C_{s1}d_1 + C_{s2}d_2}{P_0} \\ &= \left(\frac{(2448 \text{ kip})\left(\frac{36 \text{ in}}{2}\right) + (174 \text{ kip})(3 \text{ in}) + (435 \text{ kip})(33 \text{ in})}{3057 \text{ kip}}\right) \\ &= 19.3 \text{ in} \end{aligned}$$

The distribution of strain varies linearly across a section, and failure occurs when the strain on the extreme compression fiber reaches 0.003. A strain compatibility analysis and the stress-strain relationship for the steel can be used to determine the stress in the longitudinal reinforcement for every possible strain variation. Of particular significance is the *balanced strain distribution*, which corresponds to initial yielding of the longitudinal steel on the extreme tension side when the concrete strain reaches the ultimate of 0.003. Equilibrium equations determine the corresponding axial compression force and bending moment for any feasible strain distribution. In this way, an interaction diagram can be constructed for any given cross section, as illustrated in the following example.

Example 6.2
Interaction Diagram for a Reinforced Concrete Column

In the column shown, the concrete is of normal weight with a compressive strength of 5 ksi, and the steel has a yield strength of 60 ksi. The cover and tie size are such that the distance from the edge of the concrete to the centers of the bars is 2.5 in. For the section oriented as shown, bending is about a vertical axis. Compute the concentric load strength of the section, P, and maximum unfactored moment, M (equal to Pe), corresponding to

A. concentric loading
B. balanced strain conditions
C. compression failure with strain in the leftmost longitudinal steel equal to zero
D. tension failure with the steel strain equal to 0.01



Solution:

Because the reinforcement is symmetric, the plastic centroid is at the geometric center of the cross section.

A. In the case of concentric loading, the strain is uniformly distributed and the eccentricity is zero.

$$\begin{aligned} C_c &= 0.85 f'_c bh \\ &= (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)(24\ \text{in})(16\ \text{in}) \\ &= 1632\ \text{kip} \end{aligned}$$

From Table 1.1, the area of two no. 9 bars is 2.00 in^2, so

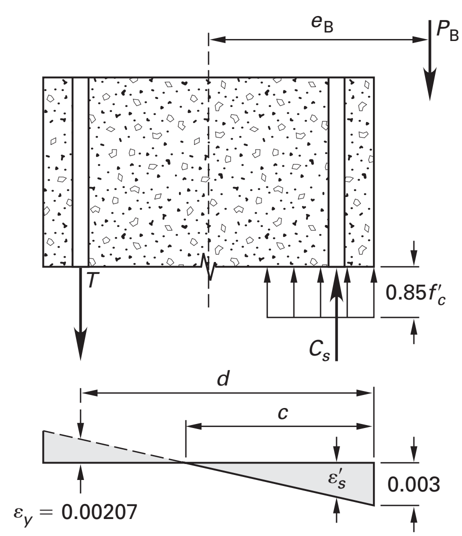
$$\begin{aligned} C_{s1} = C_{s2} &= (f_y - 0.85 f'_c)A_{s1} \\ &= \left(60\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)\right)(2.00\ \text{in}^2) \\ &= 112\ \text{kip} \end{aligned}$$

$$\begin{aligned} P_{0,\text{A}} &= C_c + C_{s1} + C_{s2} \\ &= 1632\ \text{kip} + 112\ \text{kip} + 112\ \text{kip} \\ &= 1856\ \text{kip} \end{aligned}$$

$$M_\text{A} = P_{0,\text{A}} e_\text{A} = (1856\ \text{kip})(0\ \text{in}) = 0\ \text{in-kip}$$

B. In balanced strain conditions, the tensile reinforcement reaches its yield strain (0.00207 for grade 60 reinforcement) exactly when the concrete reaches the crushing strain, 0.003, as shown in the following figure. The position of the neutral axis is found by using similar triangles.

$$\begin{aligned} \frac{c}{0.003} &= \frac{d}{0.003 + 0.00207} \\ c &= \frac{0.003d}{0.003 + 0.00207} \\ &= \frac{(0.003)(24\ \text{in} - 2.5\ \text{in})}{0.00507} \\ &= 12.7\ \text{in} \end{aligned}$$



The strain in the compression side reinforcement is also found by similar triangles.

$$\begin{aligned} \frac{\varepsilon'_s}{c - 2.5\ \text{in}} &= \frac{0.003}{c} \\ \varepsilon'_s &= \frac{0.003(c - 2.5\ \text{in})}{c} \\ &= \frac{(0.003)(12.7\ \text{in} - 2.5\ \text{in})}{12.7\ \text{in}} = 0.0024 \end{aligned}$$

The strain ε_s' exceeds the yield strain ε_y, so the stress in the compression side steel is the yield stress, 60 ksi. The force components are

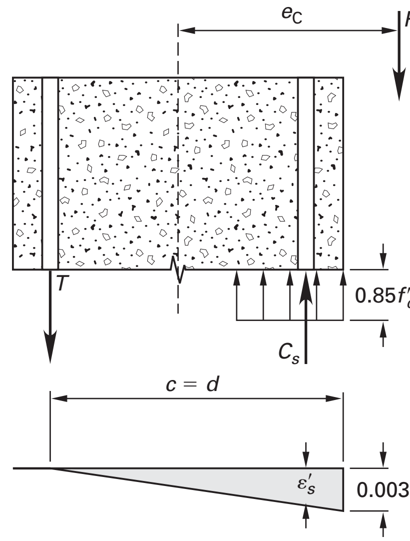
$$\begin{aligned} C_c &= 0.85 f_c' b(\beta_1 c) \\ &= (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)(16\ \text{in})(0.8)(12.7\ \text{in}) \\ &= 691\ \text{kip} \\ C_s' &= (f_y - 0.85 f_c')A_s' \\ &= \left(60\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)\right)(2.00\ \text{in}^2) \\ &= 112\ \text{kip} \\ T &= f_y A_s \\ &= \left(60\ \frac{\text{kip}}{\text{in}^2}\right)(2.00\ \text{in}^2) \\ &= 120\ \text{kip} \end{aligned}$$

Equilibrium gives

$$\begin{aligned} P_\text{B} &= C_c + C_s' - T \\ &= 691\ \text{kip} + 112\ \text{kip} - 120\ \text{kip} \\ &= 683\ \text{kip} \\ M_\text{B} = P_\text{B} e_\text{B} &= C_c\left(\frac{h}{2} - \frac{\beta_1 c}{2}\right) + C_s'\left(\frac{h}{2} - 2.5\ \text{in}\right) \\ &\quad + T\left(d - \frac{h}{2}\right) \\ &= (691\ \text{kip})\left(\frac{24\ \text{in}}{2} - \frac{(0.8)(12.7\ \text{in})}{2}\right) \\ &\quad + (112\ \text{kip})\left(\frac{24\ \text{in}}{2} - 2.5\ \text{in}\right) \\ &\quad + (120\ \text{kip})\left(21.5\ \text{in} - \frac{24\ \text{in}}{2}\right) \\ &= (6986\ \text{in-kip})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &= 582\ \text{ft-kip} \end{aligned}$$

C. In the case of compression failure with strain in the leftmost longitudinal steel equal to zero, the strain in the compression steel is greater than in case B. Therefore, the compression steel yields, as shown earlier. In this case, the neutral axis coincides with the effective depth.

$$c = d = 21.5\ \text{in}$$



The force components are

$$\begin{aligned} C_c &= 0.85 f_c' b(\beta_1 c) \\ &= (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)(16\ \text{in})(0.8)(21.5\ \text{in}) \\ &= 1170\ \text{kip} \\ C_s' &= (f_y - 0.85 f_c')A_s' \\ &= \left(60\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)\right)(2.00\ \text{in}^2) \\ &= 112\ \text{kip} \end{aligned}$$

Because the strain in the tension-side steel is zero, the steel stress is zero and so is the force in the reinforcement.

$$T = 0\ \text{kip}$$

Equilibrium gives

$$\begin{aligned} P_\text{C} &= C_c + C_s' - T \\ &= 1170\ \text{kip} + 112\ \text{kip} - 0\ \text{kip} \\ &= 1282\ \text{kip} \\ M_\text{C} = P_\text{C} e_\text{C} &= C_c\left(\frac{h}{2} - \frac{\beta_1 c}{2}\right) + C_s'\left(\frac{h}{2} - 2.5\ \text{in}\right) \\ &\quad + T\left(d - \frac{h}{2}\right) \\ &= (1170\ \text{kip})\left(\frac{24\ \text{in}}{2} - \frac{(0.8)(21.5\ \text{in})}{2}\right) \\ &\quad + (112\ \text{kip})\left(\frac{24\ \text{in}}{2} - 2.5\ \text{in}\right) \\ &\quad + (0\ \text{kip})\left(21.5\ \text{in} - \frac{24\ \text{in}}{2}\right) \\ &= (5042\ \text{in-kip})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &= 420\ \text{ft-kip} \end{aligned}$$

D. In the case of tension failure with strain in the left-most longitudinal steel equal to 0.01, the position of the neutral axis is found by using similar triangles.

$$\frac{c}{0.003} = \frac{d}{0.003 + 0.01}$$

$$c = \frac{0.003d}{0.003 + 0.01}$$

$$c = \frac{(0.003)\,(24 \text{ in} - 2.5 \text{ in})}{0.013} = 4.96 \text{ in}$$

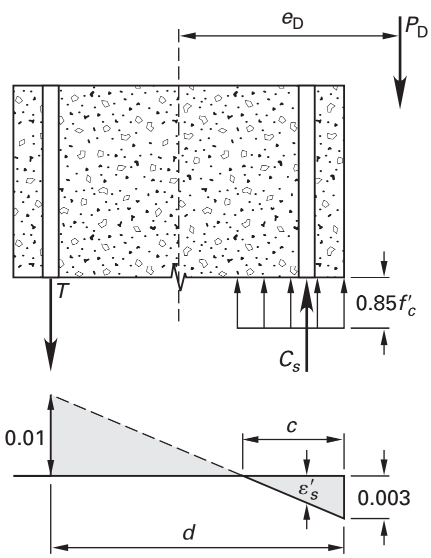
The strain in the compression side reinforcement is similarly found.

$$\frac{\varepsilon_s'}{c - 2.5 \text{ in}} = \frac{0.003}{c}$$

$$\varepsilon_s' = \frac{(0.003)(4.96 \text{ in} - 2.5 \text{ in})}{4.96 \text{ in}} = 0.00149 < \varepsilon_y = 0.00207$$

Strain is below yield, so

$$f_s' = E_s\varepsilon_s' = \left(29{,}000\ \frac{\text{kip}}{\text{in}^2}\right)(0.00149) = 43.2 \text{ ksi}$$



The force components are

$$\begin{aligned} C_c &= 0.85f_c'b(\beta_1 c) \\ &= (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)(16 \text{ in})(0.8)(4.96 \text{ in}) \\ &= 270 \text{ kip} \end{aligned}$$

$$\begin{aligned} C_s' &= (f_s' - 0.85f_c')A_s' \\ &= \left(43.2\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)\right)(2.00 \text{ in}^2) \\ &= 78 \text{ kip} \end{aligned}$$

$$\begin{aligned} T &= f_y A_{s1} \\ &= \left(60\ \frac{\text{kip}}{\text{in}^2}\right)(2.00 \text{ in}^2) \\ &= 120 \text{ kip} \end{aligned}$$

Equilibrium gives

$$\begin{aligned} P_\text{D} &= C_c + C_s' - T \\ &= 270 \text{ kip} + 78 \text{ kip} - 120 \text{ kip} \\ &= 228 \text{ kip} \end{aligned}$$

$$\begin{aligned} M_\text{D} &= P_\text{D}e_\text{D} = C_c\left(\frac{h}{2} - \frac{\beta_1 c}{2}\right) + C_s'\left(\frac{h}{2} - 2.5 \text{ in}\right) \\ &\quad + T\left(d - \frac{h}{2}\right) \\ &= (270 \text{ kip})\left(\frac{24 \text{ in}}{2} - \frac{(0.8)(4.96 \text{ in})}{2}\right) \\ &\quad + (78 \text{ kip})\left(\frac{24 \text{ in}}{2} - 2.5 \text{ in}\right) \\ &\quad + (120 \text{ kip})\left(21.5 \text{ in} - \frac{24 \text{ in}}{2}\right) \\ &= (4585 \text{ in-kip})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \\ &= 382 \text{ ft-kip} \end{aligned}$$

An interaction diagram is a plot of M against P for all possible strain variations. Figure 6.2 shows the interaction diagram for the column of Ex. 6.2. Theoretically, any combination of P and M that falls within the region bounded by the axes and the curves is safe; those outside the region cause failure. To account for variations in materials and workmanship, and the relative significance of a column failure, ACI applies a capacity reduction factor, ϕ, that depends on column type and failure mode, as defined in Secs. 9.3 and 10.3 of ACI 318. The criteria are as follows.

- When the strain in the extreme tension steel equals or exceeds 0.005 at failure (tension controlled), ϕ is equal to 0.9.
- When the strain in extreme tension-side steel is less than 0.002 (compression controlled), ϕ is equal to 0.65 for a tied column and 0.7 for a spiral column.
- When the net tension strain is between the limits of 0.005 and 0.002, ϕ varies linearly from 0.9 to its limiting value for a compression failure.

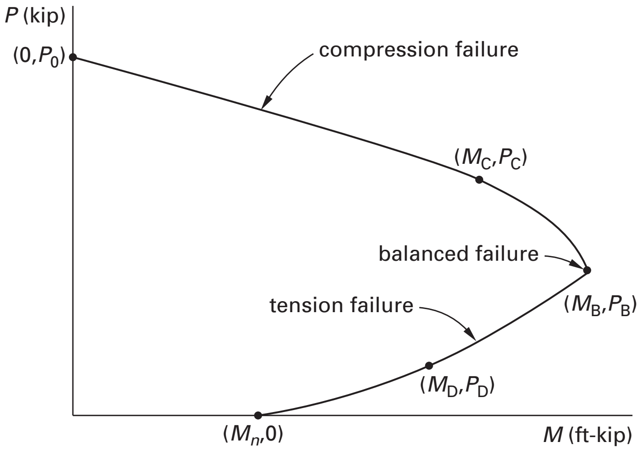


Figure 6.2 Interaction Diagram for Column of Ex. 6.2

2. Concentrically Loaded Stocky Columns

In many practical situations, the axial compressive force theoretically acts through the plastic centroid, and the column is said to be *concentrically loaded*. However, it is virtually impossible to apply a concentric load, and ACI requires that an accidental eccentricity (nominally about one-tenth the thickness of the member) must act simultaneously with the applied load. The current code treats the effect of the accidental eccentricity in a simple but indirect manner. That is, the nominal theoretical concentric load capacity is reduced by a factor to account for the accidental eccentricity. This factor is 0.8 for a tied column and 0.85 for a spiral column. The reduction is in addition to multiplication by the factor ϕ, as shown in Eqs. 10-1 and 10-2 of ACI 318.

Example 6.3
Design of a Concentrically Loaded Tied Column

Design a square tied column to support a factored concentric axial force of 1300 kip using concrete with a compressive strength of 5000 psi, grade 60 reinforcement, a steel ratio of approximately 0.02, and column dimensions rounded to the nearest 2 in increment.

Solution:
To establish the dimensions of the column, set the longitudinal steel to $0.02A_g$ and solve ACI Eq. 10-2 for the gross area needed.

$$\phi P_{n,\max} = P_u = 0.8\phi(0.85f'_c(A_g - A_{st}) + f_y A_{st})$$

$$1300 \text{ kip} = (0.8)(0.65) \times \left(\begin{array}{l} (0.85)\left(5 \ \dfrac{\text{kip}}{\text{in}^2}\right)(A_g - 0.02A_g) \\ \quad + \left(60 \ \dfrac{\text{kip}}{\text{in}^2}\right)(0.02A_g) \end{array} \right)$$

$$A_g = 466 \text{ in}^2$$

The dimensions of the column, then, are

$$b = h = \sqrt{A_g} = \sqrt{466 \text{ in}^2} = 21.6 \text{ in}$$

The 21.6 in dimension is impractical, so round b and h upward to 22 in. This increases the actual gross area to 484 in^2, which reduces the percentage of steel required to slightly lower than 2%. Recompute the steel needed, using ACI Eq. 10-2 and dimensions of 22 in.

$$\phi P_{n,\max} = P_u = 0.8\phi(0.85f'_c(A_g - A_{st}) + f_y A_{st})$$

$$1300 \text{ kip} = (0.8)(0.65) \times \left(\begin{array}{l} (0.85)\left(5 \ \dfrac{\text{kip}}{\text{in}^2}\right)(484 \text{ in}^2 - A_{st}) \\ \quad + \left(60 \ \dfrac{\text{kip}}{\text{in}^2}\right)(A_{st}) \end{array} \right)$$

$$A_{st} = 7.94 \text{ in}^2$$

Use eight no. 9 bars, for a nominal area of 8.00 in^2.

3. Lateral Reinforcement

For non-seismic applications, the design of lateral reinforcement for columns is straightforward and is covered in ACI Secs. 7.10.5 and 10.9.3. For ties, either deformed bars or welded wire fabric with equivalent area are acceptable.

- No. 3 ties are permitted for longitudinal bars no. 10 or smaller.
- No. 4 ties are required for bars no. 11 and larger, and for bundled bars of any size.
- Spacing shall not exceed 48 tie diameters, 16 longitudinal bar diameters, or the least dimension of the cross section.
- The tie pattern must support every corner bar and alternate bar, with the clear spacing between longitudinal bars not greater than 6 in.

For spiral reinforcement, smooth bar is formed into a helix, with these requirements.

- The bar diameter of the spiral must not be less than 3/8 in.
- The clear spacing between spirals must be at least 1 in and no more than 3 in.
- From ACI Eq. 10-5, the volumetric spiral reinforcement must be at least

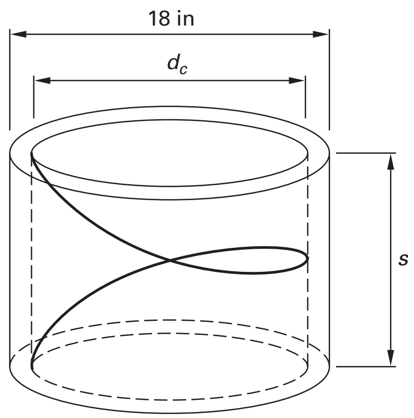
$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yt}} \tag{6.2}$$

A_{ch} is the area of concrete core measured to the outside of the spiral.

Example 6.4
Design Spiral Reinforcement

A circular column with an outside diameter of 18 in is spirally reinforced and contains six no. 9 longitudinal bars. The spiral steel has a yield strength of 60,000 psi, and the concrete has a compressive strength of 5000 psi. The cover is 1.5 in. Determine the acceptable diameter and pitch of spiral reinforcement.

Solution:



From Eq. 6.2, the required volumetric ratio is

$$\begin{aligned}\rho_s &= 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yt}}\\ &= 0.45\left(\left(\frac{18\text{ in}}{15\text{ in}}\right)^2 - 1\right)\left(\frac{5000\ \frac{\text{lbf}}{\text{in}^2}}{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right)\\ &= 0.0165\end{aligned}$$

For purposes of computing the spiral steel ratio, the length of one wound of the spiral is approximated as πd_c. The volume of confined core concrete is $(\pi d_c^2/4)s$. Thus,

$$\begin{aligned}\rho_s &= \frac{\text{volume of steel in one wound}}{\text{volume of concrete in one wound}}\\ &= \frac{\pi d_c\left(\frac{\pi d_s^2}{4}\right)}{\left(\frac{\pi d_c^2}{4}\right)s} = \frac{\pi d_s^2}{d_c s} = 0.0165\end{aligned}$$

Try a 0.375 in diameter spiral, the smallest size available. The required pitch is

$$s = \frac{\pi d_s^2}{d_c\rho_s} = \frac{\pi\,(0.375\text{ in})^2}{(15\text{ in})(0.0165)} = 1.8\text{ in}$$

This gives a feasible clear spacing of 1.125 in (1.5 in − 0.375 in). However, a larger diameter spiral might be a better choice, as larger clear spacing improves the consolidation of the concrete. Try a diameter of 0.5 in.

$$s = \frac{\pi d_s^2}{d_c\rho_s} = \frac{\pi\,(0.50\text{ in})^2}{(15\text{ in})(0.0165)} = 3.2\text{ in}$$

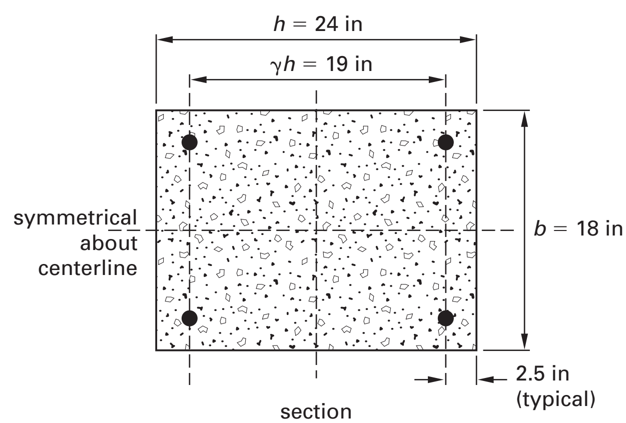
Use a ½ in diameter bar with 3 in pitch.

4. Design for Combined Axial Compression plus Bending

For the commonly encountered case of axial compression plus bending about a principal axis, design aids in the form of spreadsheets, tables, and nondimensional interaction equations are usually desirable. In what follows, nondimensional interaction diagrams facilitate design.

Example 6.5
Design Using Interaction Diagrams

The stocky column shown is subjected to a factored axial compressive force of 750 kip and a simultaneous bending moment about its strong axis equal to 350 ft-kip. The distance from the edge to the center of the longitudinal steel is 2.5 in. The compressive strength of the concrete is 4000 psi and the yield strength of the steel is 60,000 psi. Design the longitudinal steel and select an appropriate size and spacing of ties. The interaction curve shown applies for columns with material strengths and dimensions that correspond to a parameter γ, which locates the longitudinal steel with respect to the column dimension h.



Solution:

$$\begin{aligned}\gamma h &= h - 2d'\\ \gamma &= \frac{h - 2d'}{h}\\ &= \frac{24\text{ in} - (2)(2.5\text{ in})}{24\text{ in}}\\ &= 0.8\end{aligned}$$

The gross area of the column is

$$A_g = bh = (18\text{ in})(24\text{ in}) = 432\text{ in}^2$$

Assume a compression-controlled failure, with strength reduction factor ϕ equal to 0.65. The parameters needed for use of the interaction diagram are

$$\begin{aligned} K_n &= \frac{P_u}{\phi f'_c A_g} \\ &= \frac{750\text{ kip}}{(0.65)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(432\text{ in}^2)} \\ &= 0.67 \\ R_n &= \frac{M_u}{\phi f'_c A_g h} \\ &= \frac{(350\text{ ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{(0.65)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(432\text{ in}^2)(24\text{ in})} = 0.16 \end{aligned}$$

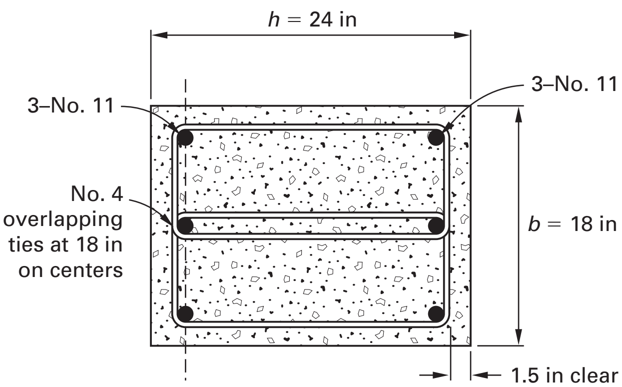
Using these values of K_n and R_n, and reading from the interaction diagram (which is printed as App. 52.N in the *Civil Engineering Reference Manual* published by PPI), gives a value for ρ_g within the compression failure region, as assumed.

$$\begin{aligned} \rho_g &= 0.02 = \frac{A_{\text{st}}}{A_g} \\ A_{\text{st}} &= \rho_g A_g = (0.02)(432\text{ in}^2) = 8.64\text{ in}^2 \end{aligned}$$

Use six no. 11 bars, three on each side. For no. 11 longitudinal bars, ACI Sec. 7.10.5 requires no. 4 ties with a spacing of

$$s \le \begin{cases} 48 \times \text{ tie diameter} = (48)\left(\frac{4\text{ in}}{8}\right) = 24\text{ in} \\ 16 \times \text{ longitudinal bar diameter} \\ \quad = (16)\left(\frac{11\text{ in}}{8}\right) = 22\text{ in} \\ \text{least column dimension} = 18\text{ in} \quad \text{[controls]} \end{cases}$$

Use no. 4 ties spaced 18 in o.c.



5. Design of Slender Columns

Fortunately, most reinforced concrete columns are stocky, and the secondary effects caused by axial compression acting on a bent column are negligible. There are cases, however, in which columns are slender, and the so-called second-order effects must be considered.

For slenderness ratios less than 100, ACI permits an approximate method to analyze the slenderness effects, which is summarized in the next section. In the approximate method, moments are calculated by an elastic (first-order) analysis for factored loadings, and a magnification factor, δ, is applied to increase the first-order moments for design. Distinction is made between columns that are restrained laterally against sidesway (for example, by stiff shear walls) and columns with sidesway (for example, rigid frames that depend on the rigidity of beams and columns to resist lateral forces).

As a practical alternative to account for the effect of cracking on the rigidity of reinforced concrete members, ACI Sec. 10.10.4.1 permits a first-order analysis using approximate moments of inertia ($0.35I_g$ for beams; $0.70I_g$ for columns, and so forth, where I_g is the moment of inertia of the gross concrete cross section).

A. Magnified Moments for Columns Without Sidesway

Factored axial compression force is denoted by P_u. M_1 and M_2 denote factored first-order moments at the ends of the column, with M_1 the smaller. The design moment, M_c, is the product of the magnification factor and the larger moment.

$$M_c = \delta M_2 \qquad 6.3$$

Reinforcement is selected to resist the combined action of P_u and M_c, as illustrated in Ex. 6.6. The flowchart of Fig. 6.3 summarizes the magnified moment procedure for a slender column restrained against sidesway.

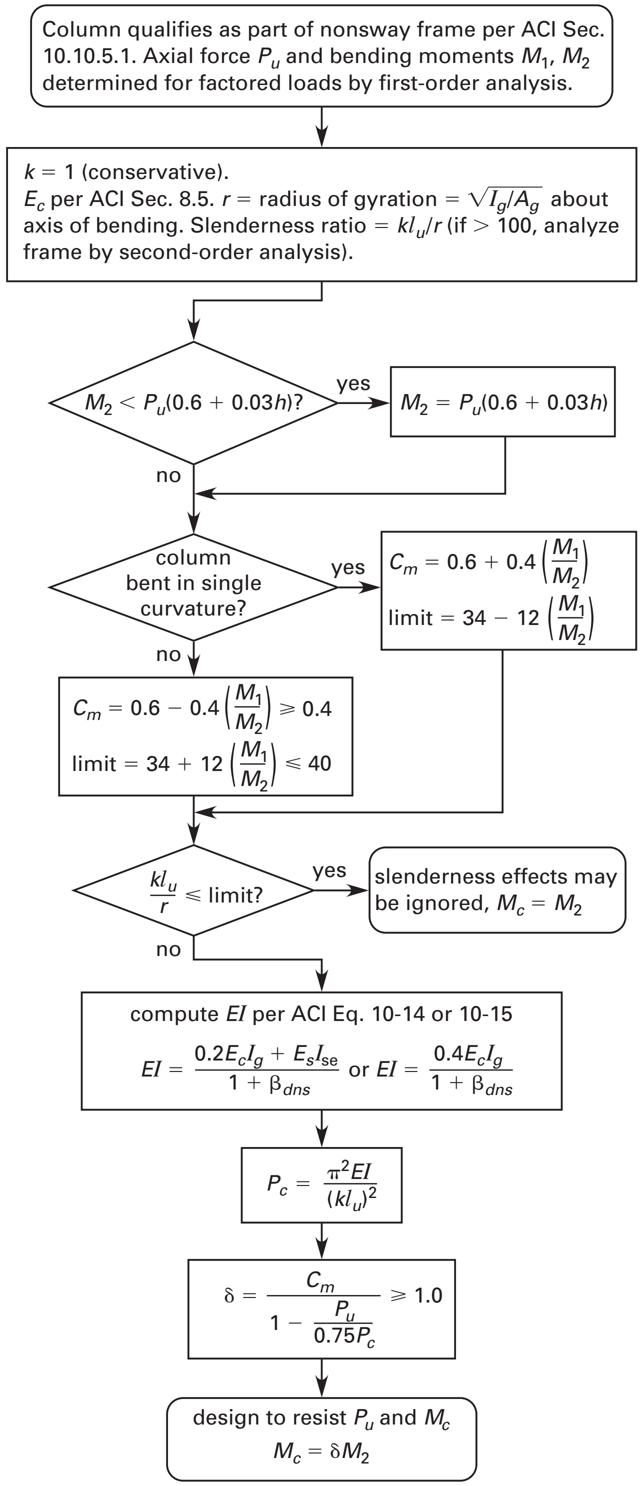
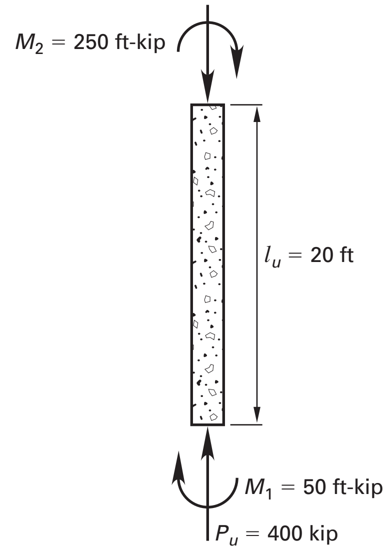


Figure 6.3 Flowchart for Computing Magnified Moment for Column Without Sway

Example 6.6
Slenderness Effects for a Column Without Sway

A first-order analysis gives factored axial force and moments on a column as shown. The column, part of a braced frame, has cross-sectional dimensions 12 in by 16 in. Bending moments produce reverse curvature bending about the strong axis. The ratio of factored dead to total load, β_{dns}, is 0.6. The concrete is of normal weight with a compressive strength of 5000 psi. Determine the magnitude of moment for which the column should be designed, using the simpler ACI Eq. 10-15 to compute EI.



Solution:
The minimum eccentricity provision of ACI Sec. 10.10.6.5 gives

$$\begin{aligned} M_{2,\min} &= P_u(0.6 + 0.03h) \\ &= (500 \text{ kip})(0.6 \text{ in} + (0.03)(16 \text{ in})) \\ &= 540 \text{ in-kip} \end{aligned}$$

$$M_2 = (250 \text{ ft-kip})\left(12 \ \frac{\text{in}}{\text{ft}}\right) = 3000 \text{ in-kip}$$

$$M_{2,\min} < M_2$$

Use $M_2 = 250$ ft-kip. The modulus of elasticity for normal weight concrete is

$$\begin{aligned} E_c &= 57{,}000\sqrt{f'_c} \\ &= 57{,}000\sqrt{5000 \ \frac{\text{lbf}}{\text{in}^2}} \\ &= (57{,}000)\left(70.71 \ \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{1 \text{ kip}}{1000 \text{ lbf}}\right) \\ &= 4030 \text{ ksi} \end{aligned}$$

The moment of inertia of the gross concrete cross section is

$$I_g = \frac{bh^3}{12} = \frac{(12 \text{ in})(16 \text{ in})^3}{12} = 4096 \text{ in}^4$$

The area of the gross concrete cross section is

$$A_g = bh = (12 \text{ in})(16 \text{ in}) = 192 \text{ in}^2$$

The radius of gyration is

$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{4096 \text{ in}^4}{192 \text{ in}^2}} = 4.62 \text{ in}$$

Calculate the slenderness ratio.

$$\frac{kl_u}{r} = \frac{(1.0)(20 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{4.62 \text{ in}} = 52$$

By ACI Sec. 10.10.1, slenderness effects may be ignored if

$$\frac{kl_u}{r} \leq \begin{cases} 34 - 12\left(-\frac{M_1}{M_2}\right) \\ 40 \end{cases}$$

$$52 \leq \begin{cases} 34 - 12\left(-\frac{50 \text{ ft-kip}}{250 \text{ ft-kip}}\right) = 36.4 \\ 40 \end{cases}$$

The condition is not satisfied, so include slenderness effects per ACI Sec. 10.10. Using ACI Eq. 10-15, the stiffness is

$$\begin{aligned} EI &= \frac{0.4E_C I_g}{1+\beta_{dns}} \\ &= \frac{(0.4)\left(4030 \ \frac{\text{kip}}{\text{in}^2}\right)(4096 \text{ in}^4)}{1+0.6} \\ &= 4{,}130{,}000 \text{ in}^2\text{-kip} \end{aligned}$$

From ACI Eq. 10-13, the critical load is

$$\begin{aligned} P_c &= \frac{\pi^2 EI}{(kl_u)^2} \\ &= \frac{\pi^2(4{,}130{,}000 \text{ in}^2\text{-kip})}{(1.0)\,(20 \text{ ft})^2\left(12 \ \frac{\text{in}}{\text{ft}}\right)^2} \\ &= 708 \text{ kip} \end{aligned}$$

For reverse-curvature bending, the correction factor C_m from ACI Eq. 10-16 is

$$\begin{aligned} C_m &= 0.6 - \frac{0.4M_1}{M_2} \\ &= 0.6 - \frac{(0.4)(50 \text{ ft-kip})}{250 \text{ ft-kip}} \\ &= 0.52 \end{aligned}$$

From ACI Eq. 10-12, the moment magnification factor δ is

$$\delta \geq \begin{cases} \frac{C_m}{1-\frac{P_u}{0.75P_c}} = \frac{0.52}{1-\frac{400 \text{ kip}}{(0.75)(708 \text{ kip})}} \\ \qquad = 2.11 \quad \text{[controls]} \\ 1.0 \end{cases}$$

Using ACI Eq. 10-11, the column should be designed for a moment of

$$M_c = \delta M_2 = (2.11)(250 \text{ ft-kip}) = 528 \text{ ft-kip}$$

B. Magnified Moments for Columns with Sidesway

For slender columns in a frame subjected to sidesway, the ACI code magnifies the end moments due to factored forces associated with the sway, $\delta_s M_{1s}$ and $\delta_s M_{2s}$, and adds these magnified moments to the corresponding non-magnified, non-sway end moments.

$$M_1 = M_{1\text{ns}} + \delta_s M_{1s} \qquad \text{6.4}$$

$$M_2 = M_{2\text{ns}} + \delta_s M_{2s} \qquad \text{6.5}$$

Any of three alternative methods may be used to determine $\delta_s M_{1s}$ and $\delta_s M_{2s}$.

- an "exact" second-order elastic analysis
- an iterative approximate second-order analysis
- an approximation of the factor δ_s using ACI Eq. 10-18 applied to M_1 and M_2 calculated by first-order elastic analysis

The last method is conservative and is the one most easily adapted to manual calculations. This approach is the one summarized in this section.

In a frame subject to sidesway, the effective length factors are greater than 1.0 and can be determined using the procedure in ACI Sec. 10.10.7.2. The rigidity, EI, is calculated using ACI Eq. 10-14 or Eq. 10-15, where β_{dns} is 1.0 for lateral loadings of short duration, such as from wind, blast, or earthquake. For sidesway instability, every column in a particular story buckles simultaneously; thus, the moment magnifier for sway moments involves the summation of the factored axial forces and critical loads for every column in the story.

$$\delta_s M_s \geq \begin{cases} \frac{M_s}{1-\frac{\sum P_u}{0.75\sum P_c}} \\ M_s \end{cases} \qquad \text{6.6}$$

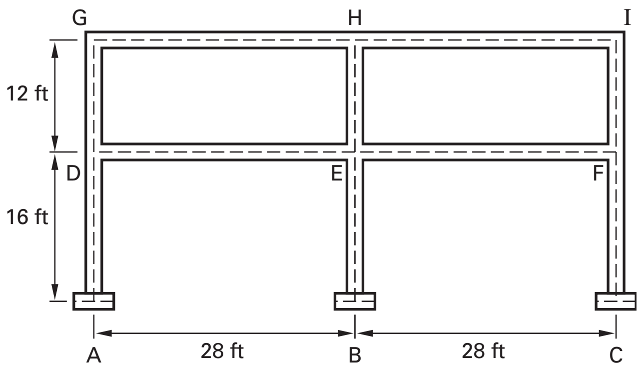
A different method applies if the column is exceptionally slender such that

$$\frac{l_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f'_c A_g}}} \qquad \text{6.7}$$

In this case, the values of M_1 and M_2 are computed for the sway condition, and then used to compute the design moment, M_c, by following the flowchart for the stocky column in Fig. 6.3.

Example 6.7
Effective Length Factor for a Column in a Sway Frame

In the frame shown, columns are 16 in by 16 in and made of normal weight concrete with a compression strength of 5000 psi. Girders have an overall depth of 24 in and a width of 30 in, and are of normal weight 4000 psi concrete. For purposes of analysis, assume the spread footings at A, B, and C behave as ideal hinges; use the centerline-to-centerline dimensions shown. Determine the effective length factor for member BE.



Solution:

For normal weight concrete, ACI Sec. 10.10.4.1 defines member rigidity as

$$\begin{aligned} EI_{\text{col}} &= (E_c)(0.7I_g) \\ &= \left(57{,}000\sqrt{f'_c}\right)(0.7)\left(\frac{bh^3}{12}\right) \\ &= 57{,}000\sqrt{5000\ \frac{\text{lbf}}{\text{in}^2}}(0.7)\left(\frac{(16\text{ in})(16\text{ in})^3}{12}\right) \\ &= 15.4\times 10^9\ \text{in}^2\text{-lbf} \end{aligned}$$

$$\begin{aligned} EI_{\text{bm}} &= 57{,}000\sqrt{f'_c}(0.35)\left(\frac{bh^3}{12}\right) \\ &= 57{,}000\sqrt{4000\ \frac{\text{lbf}}{\text{in}^2}}(0.35)\left(\frac{(30\text{ in})(24\text{ in})^3}{12}\right) \\ &= 43.6\times 10^9\ \text{in}^2\text{-lbf} \end{aligned}$$

The alignment chart of ACI R10.10.1.1 requires parameters that relate the summation of the column stiffness at a joint to the summation of the girder stiffness framed to the joint, calculated at the top and bottom ends of the column under consideration—in this case at joints B and E.

In the case of a column supported by a hinged connection, such as at joint B, this ratio approaches infinity; however, ACI permits a value of 10 for ψ_B, as representative of the actual restraint to rotation at such a support.

For joint E at the upper end of column BE,

$$\begin{aligned} \psi_E &= \frac{\sum\left(\frac{EI}{L}\right)_{\text{col}}}{\sum\left(\frac{EI}{L}\right)_{\text{bm}}} \\ &= \frac{\frac{15.4\times 10^9\ \text{in}^2\text{-lbf}}{16\text{ ft}} + \frac{15.4\times 10^9\ \text{in}^2\text{-lbf}}{12\text{ ft}}}{\frac{43.6\times 10^9\ \text{in}^2\text{-lbf}}{28\text{ ft}} + \frac{43.6\times 10^9\ \text{in}^2\text{-lbf}}{28\text{ ft}}} \\ &= 0.72 \end{aligned}$$

Use the appropriate alignment chart from ACI 318 R10.10.1.1. The values of ψ at the two column ends are found in the left and right scales, and a line is drawn connecting the two points. The intersection of this line with the center scale gives the appropriate value of the effective length for the column: $k = 1.8$.

The effective lengths determined by the procedure in Ex. 6.6 permit the calculation of critical load, P_c. Summing the ratios of factored axial force to critical load for every column in a story makes possible an evaluation of the magnified moment for the column, as given by ACI Eq. 10-18.

6. Concrete Bearing Strength

ACI Sec. 10.14 gives the bearing strength of a concrete support as

$$\phi P_n = \phi(0.85f'_c)A_1 \qquad 6.8$$

A_1 is the loaded area. In many cases, the area of the concrete support is effectively larger than the area A_1, which confines the concrete and makes it significantly stronger in compression. In these cases, a factor may be multiplied to the design bearing strength.

$$\text{factor} \le \begin{cases} \sqrt{\dfrac{A_2}{A_1}} \\ 2.0 \end{cases} \qquad 6.9$$

A_2 is the area of a surface defined by projecting from the edges of the loaded area on slopes of 1 vertical to 2 horizontal, to intersect either the edge or bottom of the support.

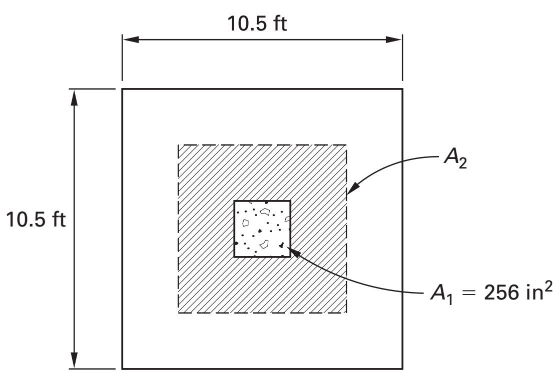
ACI Sec. 15.8 treats a closely related design consideration, that of transferring compressive force from the base of a column into a supporting wall, pedestal, or footing. In this case, the bearing strengths on the gross area of the column and on the footing must both be checked, and the smaller value controls. The design bearing strength can be augmented by properly developed dowels that project from the support and are lap spliced to the longitudinal column reinforcement. In this case, the minimum area of dowels is $0.005A_g$, where A_g is the gross area of the column.

A related issue is the transmission of column loads through a floor system, which ACI Sec. 10.12 addresses. The lateral confinement provided by the floor system makes it possible for compressive strength in the floor to be significantly higher than its uniaxial compressive strength. Thus, the loads in a column having a specified compressive strength larger than that used for the floor system are feasible.

Example 6.8
Bearing of a Column on a Footing

A 16 in by 16 in column of concrete with a compressive strength of 5000 psi is reinforced with four no. 10 grade 60 rebars. The column is loaded concentrically in compression with a factored force of 700 kip. The column bears on a square spread footing of normal weight concrete with a compressive strength of 3000 psi. The footing has an effective depth of 22 in and plan dimensions as shown. Determine the minimum area of dowels required from the footing into the column.

Solution:



Because the longitudinal column steel is not developed at the column-footing interface, only the concrete bearing strength of the column is effective. From Eq. 6.8, the bearing strength is

$$\begin{aligned}\phi P_{n,\text{col}} &= \phi 0.85 f'_{c,\text{col}} A_1 \\ &= (0.65)(0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)(256\ \text{in}^2) \\ &= 707\ \text{kip}\end{aligned}$$

For the footing, the projected area A_2 is

$$A_2 \leq \begin{cases} B^2 = (10.5\ \text{ft})^2\left(12\ \frac{\text{in}}{\text{ft}}\right)^2 \\ \quad = 15{,}876\ \text{in}^2 \\ (h+4d)^2 = (16\ \text{in} + (4)(22\ \text{in}))^2 \\ \quad = 10{,}816\ \text{in}^2 \quad \text{[controls]} \end{cases}$$

From Eq. 6.9,

$$\text{factor} \leq \begin{cases} \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{10{,}816\ \text{in}^2}{256\ \text{in}^2}} = 6.5 \\ 2.0 \quad \text{[controls]} \end{cases}$$

The bearing strength of the footing is

$$\begin{aligned}\phi P_{n,\text{footing}} &= \phi(\text{factor})(0.85 f'_{c,\text{footing}} A_1) \\ &= (0.65)(2.0)(0.85)\left(3\ \frac{\text{kip}}{\text{in}^2}\right)(256\ \text{in}^2) \\ &= 848\ \text{kip}\end{aligned}$$

Because the bearing strength of the footing is greater than the bearing strength of the column, the column strength controls. The factored axial force in the column is smaller than the bearing strength of the connection and there is theoretically no need for dowels. However, ACI Sec. 15.8.2.1 requires a minimum area of

$$\begin{aligned}A_{s,\min} &= 0.005A_g \\ &= (0.005)(256\ \text{in}^2) \\ &= 1.28\ \text{in}^2\end{aligned}$$

Four no. 6 dowel bars will extend into the footing 22 in and furnish the required dowel reinforcement.

7

Continuous One-Way Systems

Most cast-in-place concrete structures are monolithic, cast in a single piece. This is an economical way to achieve continuity among framing elements such as columns, girders, beams, and slabs.

This capability is a major advantage of reinforced concrete. In designing steel and timber structures, achieving continuity requires elaborate and expensive joint connections to transfer moments and forces. But in a reinforced concrete system, placing reinforcing steel where it is most effective causes the elements themselves to behave as rigid joints when the concrete hardens.

1. Advantages and Disadvantages

Figure 7.1 compares two structures, each consisting of a series of three equal span beams that support uniformly distributed loads. In one case, the beams behave as a series of simple spans such that no moment transfers from one member to another. In the second case, the beams are continuous over the interior supports.

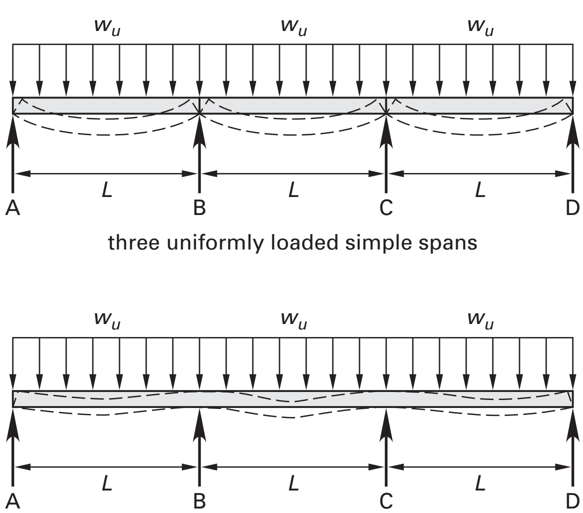


Figure 7.1 Simply Supported Versus Continuous Beams with Same Spans and Loadings

With the spans, loadings, and materials the same in the both cases, the continuous spans provide several advantages.

- The magnitude of the maximum bending moment in a continuous span is smaller than in the simply supported span.
- The continuous spans are much stiffer than the simple spans, which results in smaller deflections and better dynamic behavior.
- The continuous spans provide redundancy, which can prevent collapse of the system if there is a local failure of an element. In the simple system, for example, if the support at B were removed, spans AB and BC would collapse. But with proper detailing, these spans in the continuous system could redistribute forces to other supports. Criteria necessary to achieve integrity under these conditions are in ACI Sec. 7.13.

The advantages of continuity generally outweigh the disadvantages in cast-in-place structures. But there are nevertheless several disadvantages worth noting.

- Structural analysis is far more complicated for the continuous beams than for the simple spans. Not only is the continuous structure statically indeterminate, but multiple loading patterns must be analyzed to determine the design shears and moments at critical sections, as illustrated in Fig. 7.2. For example, in designing span BC for uniformly distributed live and dead loads, the design moment for the simple span is easily calculated using the equation $w_u L^2/8$. But for the continuous span, three loading conditions must be analyzed: one producing critical negative bending over support B, another giving maximum positive bending near midspan, and a third giving critical negative moment over support C. Moment diagrams corresponding to each of these loadings must be superimposed, and the controlling magnitude at every point on the span gives the *moment envelope*.

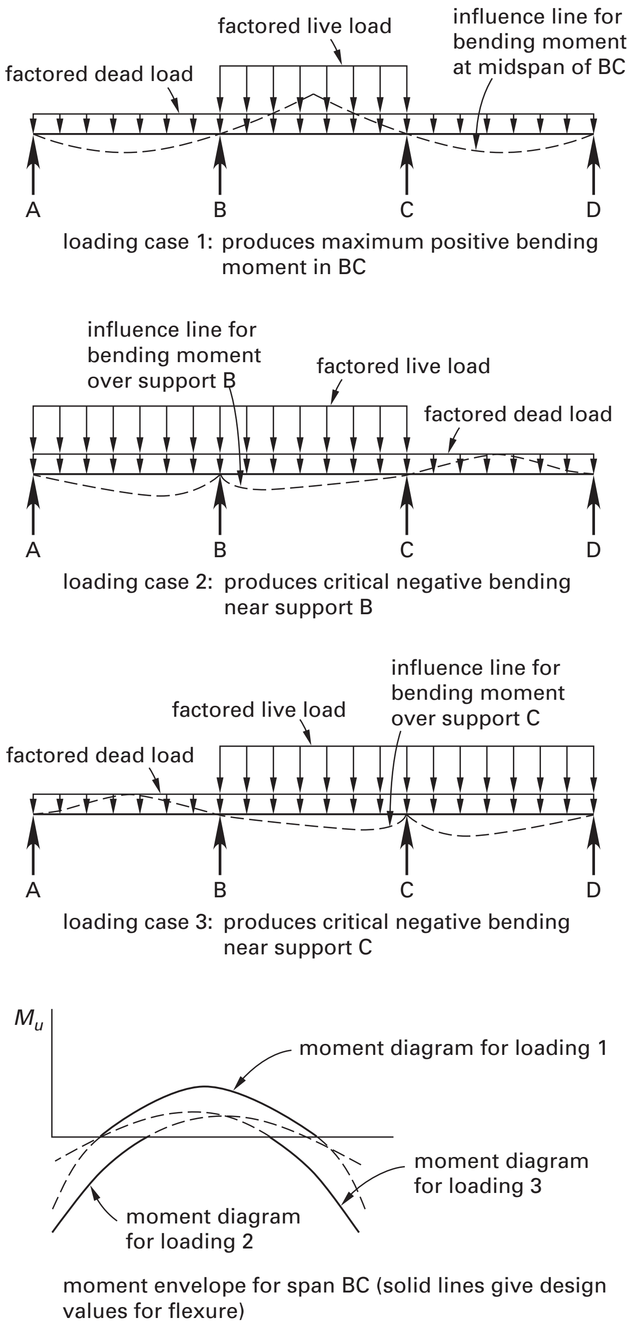


Figure 7.2 Moment Envelope for a Three-Span Continuous Beam

- The design shear force in the exterior span of the continuous beams is larger than it is for the simple spans. For the simple spans, the maximum shear is $w_uL/2$ at each end. For the continuous exterior span, the negative moment at the continuous end creates a shear equal to $w_uL/2+M_u^-/L$, where M_u^- denotes the magnitude of the negative moment at the continuous end. There is a corresponding reduction of shear at the exterior end, but the larger value at the continuous end generally controls design. This is particularly important for joists in one-way ribbed slabs, as will be discussed in a later section.

- Changes in volume due to creep, shrinkage, temperature change, and support settlements create stresses in continuous systems, while simple systems accommodate these changes without stresses. There is a trade-off in this regard, however. Although there are no stresses in the simple system, control joints are necessary to eliminate the stresses and these joints are expensive to construct and maintain.

2. ACI Gravity Load Analysis

In resisting lateral forces, such as from wind or earthquake, the entire structural system is effective. By contrast, gravity loads due to self-weight apply only to portions of the structure. The effects of creep, shrinkage, changes to rigidity due to concrete cracking, and redistribution of internal loads make it impossible to compute internal stresses accurately.

For these reasons, ACI 318, in Secs. 8.4 to 8.11, simplifies the gravity load analysis of concrete frames. In lieu of multiple analyses of entire frames, only the beams at a particular level and the adjacent columns above and below are considered. Live load patterns are limited to alternate spans loaded, which give the positive bending moments, and a sequence of analyses with live loads on adjacent spans, which give the critical negative moments over each interior support. If the beams meet certain ductility limits, ACI Sec. 8.4 permits redistribution of the negative moments computed by elastic analysis.

Many reinforced concrete systems qualify for further simplification in analysis. These are characterized by uniformly loaded, prismatic continuous members of two or more spans for which live loads are less than three times the dead load and span lengths are approximately equal (specifically, no span is more than 20% greater than an adjacent span). In these cases moments and shears can be approximated by formulas given in ACI Sec. 8.3. These formulas are summarized in Fig. 7.3. This approximate method is the basis of analysis in the remainder of this chapter.

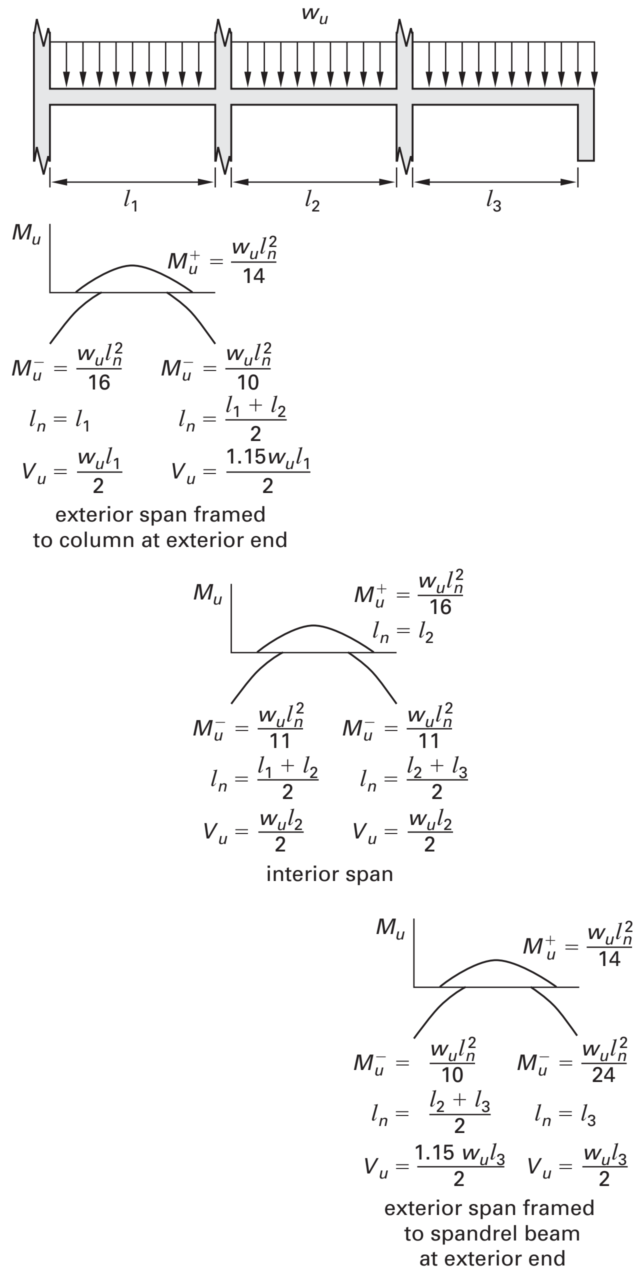


Figure 7.3 ACI Approximate Expressions for Shears and Moments in Continuous Beams

3. Solid One-Way Slabs

Slabs are among the most important applications of reinforced concrete. There are three general categories of slabs: slabs on grade, elevated one-way slabs, and elevated two-way slabs.

Slabs cast directly on a properly prepared sub base are called *slabs on grade*. These usually require only detailing to control cracking due to shrinkage and settlement.

This section summarizes the design criteria for one-way solid slabs, and the following chapter gives similar criteria for two-way slabs. Loading and dimensions are such that the approximate analysis of ACI Sec. 8.3 applies with two or more continuous spans, which is the usual case. For convenience, the design is on a per-foot width basis; that is, $b = 12$ in/ft. The applicable criteria and code sections are as follows.

- The minimum thickness, h, must satisfy serviceability (ACI Table 9.5(a)).

$$h \geq \begin{cases} \lambda_1 \lambda_2 \left(\dfrac{l_{\text{ext}}}{24}\right) \\ \lambda_1 \lambda_2 \left(\dfrac{l_{\text{int}}}{28}\right) \end{cases} \qquad 7.1$$

  In this formula,

$$\lambda_1 = 1 \quad \text{[for normal weight]} \qquad 7.2$$

$$\lambda_1 \geq \begin{cases} 1.65 - 0.005 w_c \\ 1.09 \end{cases} \quad \begin{bmatrix}\text{for structural} \\ \text{lightweight}\end{bmatrix} \qquad 7.3$$

$$\lambda_2 = 1 \quad [\text{for } f_y = 60{,}000 \text{ psi}] \qquad 7.4$$

$$\lambda_2 = 0.4 + \frac{f_y}{100{,}000} \quad [\text{for } f_y \neq 60{,}000 \text{ psi}] \qquad 7.5$$

- According to ACI Sec. 7.7.1, the minimum cover is 3/4 in. The effective depth, d, can be reasonably approximated as $h - 1.0$ in to allow for minimum cover plus an estimate of the additional distance to the centroid of main steel.
- According to ACI Sec. 7.12, the minimum steel required to control cracking due to restraint to shrinkage and temperature changes is

$$A_{s,\min} = 0.0018bh \quad [\text{for } f_y = 60{,}000 \text{ psi}] \qquad 7.6$$

$$A_{s,\min} = 0.002bh \quad \begin{bmatrix}\text{for } f_y = 40{,}000 \\ \text{psi or } 50{,}000 \text{ psi}\end{bmatrix} \qquad 7.7$$

$$A_{s,\min} = \frac{0.0018 f_y bh}{60{,}000} \quad [\text{for } f_y > 60{,}000 \text{ psi}] \qquad 7.8$$

- ACI Secs. 11.2.1 and 11.4.6 control shear strength.

$$V_{u,\max} \leq 2\phi\lambda\sqrt{f'_c}bd \qquad 7.9$$

  It is usually impractical to provide shear reinforcement in a solid one-way slab and if shear is critical (it rarely is in a one-way slab) it is usually best to increase the slab thickness to furnish the required shear strength.
- Flexural steel is computed based on design moments at supports and midspan, and is limited by $A_{s,\min}$. Wire fabric is often used in slabs 4 in or thinner, but the use of either reinforcing bars or wire fabric is permitted. The bar spacing, s, must not exceed limits of ACI Sec. 7.6.5 for main

steel or ACI Sec. 7.12.2.2 for the temperature and shrinkage steel. For main steel,

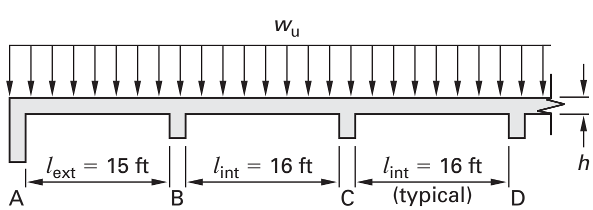
$$s \le \begin{cases} 3h \\ 18 \text{ in} \end{cases} \quad 7.10$$

For temperature and shrinkage steel,

$$s \le \begin{cases} 5h \\ 18 \text{ in} \end{cases} \quad 7.11$$

Example 7.1
Design of a Solid One-Way Reinforced Concrete Slab

A solid one-way slab spans between integral spandrel beams at the exterior ends and interior beams as shown. Concrete is sand-lightweight with a unit weight of 110 lbf/ft^3 and compressive strength of 3500 psi. The reinforcing steel is grade 60. The spandrel beam is integral with the slab, but its torsional stiffness is so small that the exterior span is unrestrained at one end. Fire rating requires a slab thickness of 5 in or greater. Service loads are 100 lbf/ft^2 (nonreducible) live load and superimposed dead load of 15 lbf/ft^2. Assuming continuity for three or more typical interior spans, determine the slab thickness and calculate the area of main steel required over the first interior support per foot of width.



Solution:

From Eq. 7.4, for grade 60 reinforcing steel, λ_2 is 1. From Eq. 7.3, for lightweight concrete with a unit weight of 110 lbf/ft^3,

$$\lambda_1 \ge \begin{cases} 1.65 - 0.005w_c \\ \quad = 1.65 - \left(0.005 \ \frac{\text{ft}^3}{\text{lbf}}\right)\left(110 \ \frac{\text{lbf}}{\text{ft}^3}\right) \\ \quad = 1.1 \quad \text{[controls]} \\ 1.09 \end{cases}$$

From Eq. 7.1, to satisfy serviceability,

$$h \ge \begin{cases} \lambda_1\lambda_2\left(\frac{l_{\text{ext}}}{24}\right) \\ \quad = (1.1)(1.0)\left(\frac{15 \text{ ft}}{24}\right)\left(12 \ \frac{\text{in}}{\text{ft}}\right) \\ \quad = 8.25 \text{ in} \quad \text{[controls]} \\ \lambda_1\lambda_2\left(\frac{l_{\text{int}}}{28}\right) \\ \quad = (1.1)(1.0)\left(\frac{16 \text{ ft}}{28}\right)\left(12 \ \frac{\text{in}}{\text{ft}}\right) \\ \quad = 7.5 \text{ in} \end{cases}$$

Choose a solid one-way slab $8^1\!/\!_4$ in thick throughout. Calculate the factored load.

$$\begin{aligned} w_d &= (\text{superimposed dead load} + h_s w_c)(\text{unit width}) \\ &= \left(15 \ \frac{\text{lbf}}{\text{ft}^2} + \left(\frac{8.25 \text{ in}}{12 \ \frac{\text{in}}{\text{ft}}}\right)\left(110 \ \frac{\text{lbf}}{\text{ft}^3}\right)\right)(1 \text{ ft}) \\ &= 91 \text{ lbf/ft} \\ w_u &= 1.2w_d + 1.6w_l \\ &= (1.2)\left(91 \ \frac{\text{lbf}}{\text{ft}}\right) + (1.6)\left(100 \ \frac{\text{lbf}}{\text{ft}}\right) \\ &= 269 \text{ lbf/ft} \end{aligned}$$

Shear is critical at distance d from face of interior support or at distance d from face of the longer interior supports.

$$\begin{aligned} d &= h - 1 \text{ in} \\ &= (8.25 \text{ in} - 1 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \\ &= 0.60 \text{ ft} \end{aligned}$$

The shear force is

$$V_u \ge \begin{cases} \frac{1.15w_u l_{\text{ext}}}{2} - w_u d \\ \quad = \frac{(1.15)\left(269 \ \frac{\text{lbf}}{\text{ft}}\right)(15 \text{ ft})}{2} - \left(269 \ \frac{\text{lbf}}{\text{ft}}\right)(0.60 \text{ ft}) \\ \quad = 2159 \text{ lbf} \quad \text{[controls]} \\ \frac{w_u l_{\text{int}}}{2} - w_u d \\ \quad = \frac{\left(269 \ \frac{\text{lbf}}{\text{ft}}\right)(16 \text{ ft})}{2} - \left(269 \ \frac{\text{lbf}}{\text{ft}}\right)(0.60 \text{ ft}) \\ \quad = 1991 \text{ lbf} \end{cases}$$

From Eq. 7.9,

$$
\begin{aligned}
V_u &\le \phi V_c \\
&\le 2\phi\lambda\sqrt{f'_c}bd \\
&\le (2)(0.75)(0.85)\sqrt{3500\ \frac{\text{lbf}}{\text{in}^2}}(12\ \text{in})(7.25\ \text{in}) \\
2159\ \text{lbf} &\le 6562\ \text{lbf}
\end{aligned}
$$

Therefore, 8¼ in slab is adequate for the given fire rating, serviceability, and shear. Reinforce the slab for flexure and temperature and shrinkage.

$$
\begin{aligned}
A_{s,\min} &= 0.0018bh \\
&= (0.0018)\left(12\ \frac{\text{in}}{\text{ft}}\right)(8.25\ \text{in}) \\
&= 0.18\ \text{in}^2/\text{ft}
\end{aligned}
$$

No. 4 bars at 12 in spacing will satisfy the steel quantity and spacing limits for temperature and shrinkage.

For complete flexural design, calculate the design moment at top, at midspan, and over each interior support. In this example, only the design moment over the first interior support is required.

From the approximate equations of ACI Sec. 8.3,

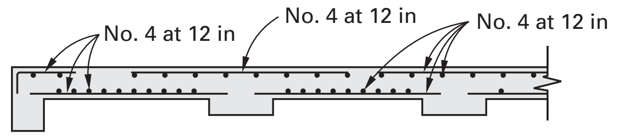
$$
\begin{aligned}
M_u^- &= \frac{w_u l_n^2}{10} \\
&= \left(\frac{\left(269\ \frac{\text{lbf}}{\text{ft}}\right)\left(\frac{15\ \text{ft} + 16\ \text{ft}}{2}\right)^2}{10}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right) \\
&= 77{,}550\ \text{in-lbf}
\end{aligned}
$$

Equating the design bending moment to the design moment strength gives

$$
\begin{aligned}
M_u^- &= \phi M_n \\
&= 0.9A_s f_y\left(d - 0.59\left(\frac{A_s f_y}{bf'_c}\right)\right) \\
77{,}550\ \text{in-lbf} &= 0.9A_s\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right) \\
&\quad \times \left(7.25\ \text{in} - (0.59)\left(\frac{A_s\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)}{(12\ \text{in})\left(3500\ \frac{\text{lbf}}{\text{in}^2}\right)}\right)\right) \\
A_s &= 0.20\ \text{in}^2/\text{ft} \quad [> A_{s,\min}]
\end{aligned}
$$

Use no. 4 bars at 12 in centers over support B.

Similarly, flexural steel is calculated at each critical location, and if the steel needed is less than $A_{s,\min}$, the minimum steel is used instead. The cross section below shows the slab steel required in both directions.



section showing top and bottom slab steel

4. Ribbed One-Way Slabs

Solid one-way slabs are usually uneconomical for clear spans greater than about 20 ft. For these longer spans, a ribbed slab is often a better choice for one-way systems. The ribbed slab consists of a series of closely spaced ribs (called joists) that in turn support a thin one-way slab, as shown in Fig. 7.4. ACI Table 9.5(a) gives the following criteria for the total thickness of a ribbed slab.

$$
h \ge \begin{cases} \lambda_1\lambda_2\left(\dfrac{l_{\text{ext}}}{18.5}\right) \\ \lambda_1\lambda_2\left(\dfrac{l_{\text{int}}}{21}\right) \end{cases} \tag{7.12}
$$

In this formula,

$$
\lambda_1 = 1 \quad [\text{for normal weight}] \tag{7.13}
$$

$$
\lambda_1 \ge \begin{cases} 1.65 - 0.005w_c \\ 1.09 \end{cases} \quad \begin{bmatrix}\text{for structural} \\ \text{lightweight}\end{bmatrix} \tag{7.14}
$$

$$
\lambda_2 = 1 \quad [\text{for } f_y = 60{,}000\ \text{psi}] \tag{7.15}
$$

$$
\lambda_2 = 0.4 + \frac{f_y}{100{,}000} \quad [\text{for } f_y \ne 60{,}000\ \text{psi}] \tag{7.16}
$$

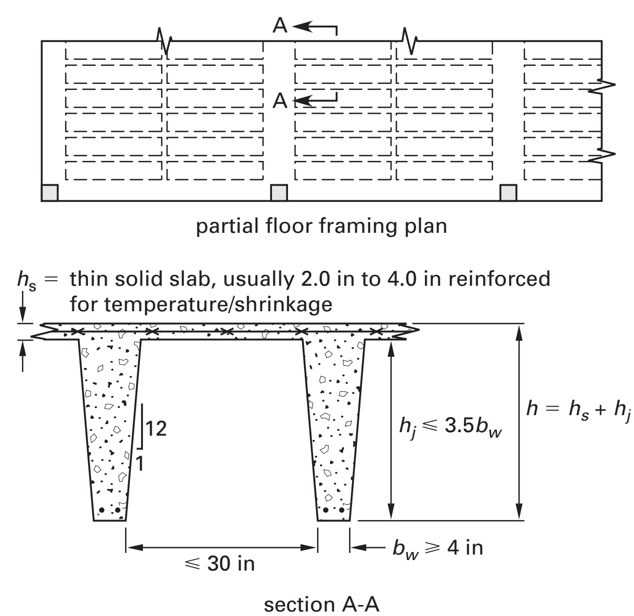


Figure 7.4 Typical One-Way Ribbed Slab System

For the most common case of ribs formed with removable fillers, the design criteria in ACI Sec. 8.13 can be summarized as follows.

- The maximum clear spacing between ribs is 30 in.
- The width of the joist at bottom must be at least 4 in and the height of joist must not exceed 3.5 times the width at bottom.
- The solid one-way slab spanning between joists must be at least 2 in thick and no less than one-twelfth the clear distance between ribs. The slab must have adequate strength and reinforcement must not be less than that required for a solid one-way slab of the same thickness.
- ACI Sec. 8.13.8 permits taking the shear strength as

$$\phi V_c = 2.2\phi\lambda\sqrt{f'_c}b_w d \qquad 7.17$$

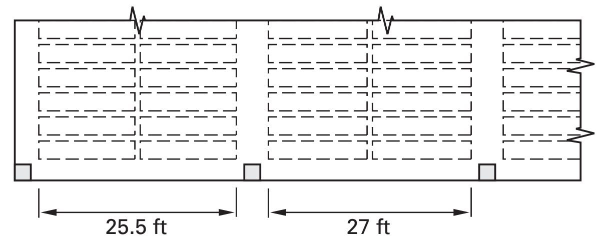
b_w is the width at the bottom of joists.

For interior exposures, ACI Sec. 7.7.2 requires a minimum cover for joist reinforcement of 3/4 in. A reasonable approximation to the effective depth of the joist is the slab thickness plus the rib height minus 1 in. The standard joist module has a joist width of 5 in and clear spacing between joists of 20 in; thus, the joists are spaced at 25 in (2.08 ft) on centers. For this case, the standard filler forms create 8 in, 10 in, 12 in, 14 in, or 16 in joist heights. The widest module is 30 in clear spacing with 6 in joist width, giving a more convenient 3 ft center-to-center spacing of joists.

Example 7.2
Design of a Ribbed One-Way Reinforced Concrete Slab

A ribbed one-way slab spans between integral spandrel beams at the exterior ends and interior beams as shown. Concrete is normal weight with a compressive strength of 4000 psi, and the reinforcing steel is grade 60. The spandrel beam is integral with the slab but its torsional stiffness is so small that the exterior span is unrestrained at one end. Construction considerations dictate a slab thickness of 3 in over the joists. Service loads are 100 lbf/ft^2 (nonreducible) live load and superimposed dead load of 15 lbf/ft^2. Assuming a standard module with continuity for three or more typical interior spans, determine the joist thickness and calculate the area of main steel required in the typical interior span positive region.

Solution:

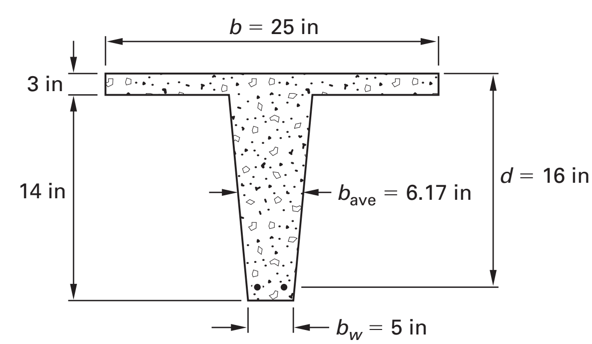


partial floor framing plan

From Eq. 7.12, for normal weight concrete and grade 60 reinforcement, the overall depth to satisfy serviceability is

$$h \geq \begin{cases} \lambda_1\lambda_2\left(\dfrac{l_{\text{ext}}}{18.5}\right) \\ \quad = (1.0)(1.0)\left(\dfrac{25.5\text{ ft}}{18.5}\right)\left(12\ \dfrac{\text{in}}{\text{ft}}\right) \\ \quad = 16.5\text{ in} \quad \text{[controls]} \\ \lambda_1\lambda_2\left(\dfrac{l_{\text{int}}}{21}\right) \\ \quad = (1.0)(1.0)\left(\dfrac{27.0\text{ ft}}{21}\right)\left(12\ \dfrac{\text{in}}{\text{ft}}\right) \\ \quad = 15.4\text{ in} \end{cases}$$

Try a 14 in joist plus 3 in slab giving 17 in overall depth and 16 in effective depth. For a standard module joist, the cross section of a typical joist is



(not to scale)

The cross-sectional area of the typical joist is

$$\begin{aligned} A &= bh_s + b_{\text{ave}}h_j \\ &= (25\text{ in})(3\text{ in}) + (6.17\text{ in})(14\text{ in}) \\ &= (161\text{ in}^2)\left(\frac{1\text{ ft}^2}{144\text{ in}^2}\right) \\ &= 1.12\text{ ft}^2 \end{aligned}$$

The weight of the typical joist per foot is

$$\begin{aligned} w_j &= A w_c \\ &= (1.12\ \text{ft}^2)\left(150\ \frac{\text{lbf}}{\text{ft}^3}\right) \\ &= 168\ \text{lbf/ft} \end{aligned}$$

The factored load on a typical joist is

$$\begin{aligned} w_u &= 1.2 w_d + 1.6 w_l \\ &= 1.2(w_j + w_{ds} b) + 1.6 w_{ls} b \\ &= (1.2)\left(168\ \frac{\text{lbf}}{\text{ft}} + \left(15\ \frac{\text{lbf}}{\text{ft}^2}\right)(2.08\ \text{ft})\right) \\ &\quad + (1.6)\left(100\ \frac{\text{lbf}}{\text{ft}^2}\right)(2.08\ \text{ft}) \\ &= 572\ \text{lbf/ft} \end{aligned}$$

Check shear in a typical joist.

$$V_u \geq \begin{cases} \dfrac{1.15 w_u l_{\text{ext}}}{2} - w_u d \\ \quad = \dfrac{(1.15)\left(572\ \dfrac{\text{lbf}}{\text{ft}}\right)(25.5\ \text{ft})}{2} \\ \qquad - \left(572\ \dfrac{\text{lbf}}{\text{ft}}\right)(1.33\ \text{ft}) \\ \quad = 7626\ \text{lbf} \quad \text{[controls]} \\ \dfrac{w_u l_{\text{int}}}{2} - w_u d \\ \quad = \dfrac{\left(572\ \dfrac{\text{lbf}}{\text{ft}}\right)(27.0\ \text{ft})}{2} \\ \qquad - \left(572\ \dfrac{\text{lbf}}{\text{ft}}\right)(1.33\ \text{ft}) \\ \quad = 6960\ \text{lbf} \end{cases}$$

From Eq. 7.17,

$$\begin{aligned} \phi V_c &= 2.2\phi\lambda\sqrt{f'_c}\, b_w d \\ &= (2.2)(0.75)(1.0)\sqrt{4000\ \frac{\text{lbf}}{\text{in}^2}}(5\ \text{in})(16\ \text{in}) \\ &= 8350\ \text{lbf} > V_u \end{aligned}$$

Thus, the joist satisfies serviceability and shear strength requirements. Design flexural steel for the positive moment region of a typical interior span.

$$\begin{aligned} l_n &= l_{\text{int}} = 27.0\ \text{ft} \\ M_u^+ &= \frac{w_u l_n^2}{16} \\ &= \frac{\left(572\ \dfrac{\text{lbf}}{\text{ft}}\right)(27.0\ \text{ft})^2\left(\dfrac{1\ \text{kip}}{1000\ \text{lbf}}\right)}{16} \\ &= 26\ \text{ft-kip} \end{aligned}$$

The section resisting bending in the positive region is a T-section with $b = 25$ in. Assume that the neutral axis is within the flange, compute flexural steel on that basis, then verify the assumption. From Eqs. 3.12 and 3.14,

$$\begin{aligned} \phi M_n &= M_u \\ &= \phi\rho f_y b d^2\left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right) \\ (26\ \text{ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right) &= 0.9\rho\left(60\ \frac{\text{kip}}{\text{in}^2}\right)(25\ \text{in})\,(16\ \text{in})^2 \\ &\quad \times\left(1 - 0.59\rho\left(\frac{60\ \dfrac{\text{kip}}{\text{in}^2}}{4\ \dfrac{\text{kip}}{\text{in}^2}}\right)\right) \\ \rho &= 0.0009 \end{aligned}$$

From Eq. 3.13, the area of tension reinforcement is

$$\begin{aligned} A_s &= \rho b d \\ &= (0.0009)(25\ \text{in})(16\ \text{in}) \\ &= 0.36\ \text{in}^2 \end{aligned}$$

ACI Sec. 10.5.1 gives the minimum steel area as

$$\begin{aligned} A_{s,\min} &= \frac{200 b_w d}{f_y} \\ &= \frac{\left(200\ \dfrac{\text{lbf}}{\text{in}^2}\right)(5\ \text{in})(16\ \text{in})}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}} \\ &= 0.27\ \text{in}^2 \\ &< A_s = 0.36\ \text{in}^2 \end{aligned}$$

Verify that the neutral axis is within the flange as assumed. From Eq. 3.4,

$$\begin{aligned} c &= \frac{a}{\beta_1} \\ &= \frac{A_s f_y}{0.85 f'_c b \beta_1} \\ &= \frac{(0.36\ \text{in}^2)\left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(4\ \dfrac{\text{kip}}{\text{in}^2}\right)(25\ \text{in})(0.85)} \\ &= 0.30\ \text{in} \\ &< h_s = 3\ \text{in} \end{aligned}$$

Thus, the assumption is correct. Use two no. 4 bottom steel bars in the interior joists.

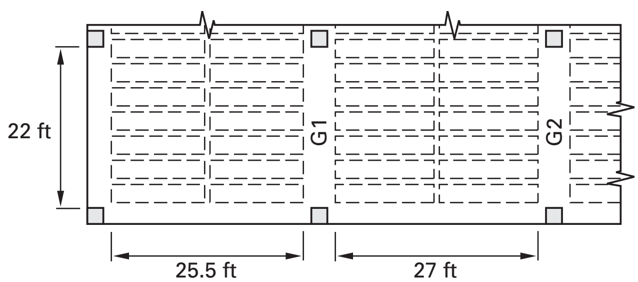
---

5. One-Way Beams and Girders

The one-way solid and ribbed slabs transfer their end reactions into either beams or girders, which transfer the loads in turn to columns (or bearing walls, in some cases). For uniformly loaded beams and girders meeting the requirements of the ACI approximate method, design involves calculating the critical shears and moments and proportioning the section and reinforcement to resist those actions, as explained in previous chapters.

Example 7.3
Design of a Girder Supporting a Ribbed Slab

For the joist system of Ex. 7.2, assume that the exterior span of girder G1, denoted as shown here, spans 22 ft from face to face between supporting columns. The overall girder width is 24 in, and the overall depth is 30 in. Compute the design moment at the interior face of the exterior support for this girder.



partial floor framing plan

Solution:

The girder is designed to support the reactions from joists to either side plus the additional loads due to its own weight and superimposed loadings. The reactions from the closely spaced joists can be treated as uniformly distributed.

$$
\begin{aligned}
w_u &= \frac{\dfrac{1.15 w_{uj} l_{\text{ext}}}{2} + \dfrac{w_{uj} l_{\text{int}}}{2}}{s} \\
&\quad + 1.2(bhw_c + w_{ds}b) + 1.6w_{ls}b \\
&= \frac{\dfrac{(1.15)\left(572\ \dfrac{\text{lbf}}{\text{ft}}\right)(25.5\ \text{ft})}{2} + \dfrac{\left(572\ \dfrac{\text{lbf}}{\text{ft}}\right)(27.0\ \text{ft})}{2}}{2.08\ \text{ft}} \\
&\quad + (1.2)\left(\begin{array}{c}(2\ \text{ft})(2.5\ \text{ft})\left(150\ \dfrac{\text{lbf}}{\text{ft}^3}\right) \\ + \left(15\ \dfrac{\text{lbf}}{\text{ft}^2}\right)(2\ \text{ft})\end{array}\right) \\
&\quad + (1.6)\left(100\ \frac{\text{lbf}}{\text{ft}^2}\right)(2\ \text{ft}) \\
&= \left(9000\ \frac{\text{lbf}}{\text{ft}}\right)\left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) \\
&= 9.0\ \text{kip/ft}
\end{aligned}
$$

The design moment at the interior face of the exterior support is

$$
\begin{aligned}
M_u^- &= \frac{w_u l_n^2}{10} \\
&= \frac{\left(9.0\ \dfrac{\text{kip}}{\text{ft}}\right)(22\ \text{ft})^2}{10} \\
&= 436\ \text{ft-kip}
\end{aligned}
$$

8

Two-Way Slab Systems

Two-way slabs are parts of floor or roof systems in which slab elements transfer forces in two directions simultaneously. ACI Sec. 9.5 distinguishes slabs as one-way if the ratio of longer to shorter side length is greater than two; otherwise, the slab behaves as a two-way slab. Figure 8.1 shows several typical two-way systems and the terminology that describes them.

1. Variations of Two-Way Slabs

The simplest and most widely used type of two-way slab for new construction is the *flat plate*, which consists of a slab of uniform thickness supported directly by columns as shown in the first part of Fig. 8.1. The critical region of such a slab is the area near a perimeter taken $d/2$ from the column faces. This critical region is the area in which the shear stress, called *punching shear stress*, is high.

In older construction, it was common to increase the punching shear stress resistance in one of two ways, either increasing the slab thickness by the use of a drop panel, or by adding a flare-out at the top of the column called a capital. A *flat slab* is a two-way slab containing drop panels and/or column capitals. The costs of formwork in modern construction generally make flat slabs uneconomical, and it is common practice today to resist punching shear stress by embedding shear reinforcement over the columns in flat plates. The embedded items are called shearheads and their design and detailing is specified in ACI Sec. 11.11. Another widely used approach is to post-tension flat plates, which transfers force into columns mechanically and increases punching shear resistance by reducing principal tension stresses. Chapter 10 covers basic concepts of post-tensioning.

There are many variations to the two-way systems shown in Fig. 8.1. One approach is to make waffle slabs, as shown in Fig. 8.2. This is done by using filler forms to create a system of intersecting joists in regions away from the column, where shear and negative bending moments are relatively small. Another approach is to form beams over column centerlines and produce two-way

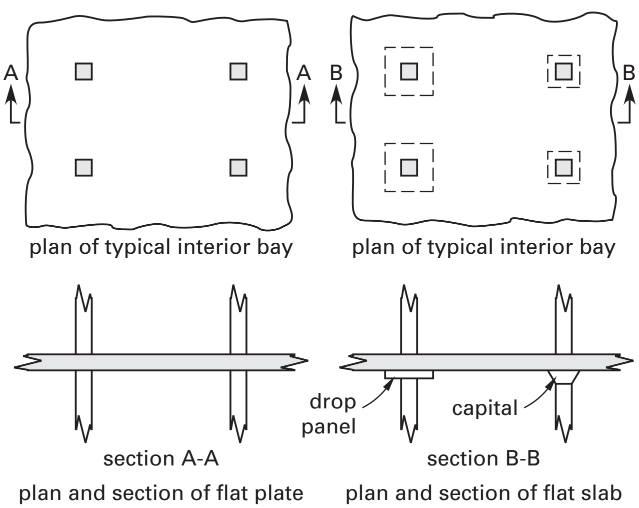


Figure 8.1 Representative Two-Way Reinforced Concrete Floor Systems

beam-slab systems. Spandrel beams are frequently used around the perimeters of all types of two-way systems, to stiffen and strengthen those regions and to support the weight of exterior walls and floor loading.

Three sections of ACI 318 are devoted exclusively to two-way slab systems.

- ACI Sec. 9.5.3 gives the minimum thickness required for serviceability.
- ACI Secs. 11.10 and 11.11 cover the transfer of shear and moment from slab to column.
- ACI Sec. 13 is devoted to analysis, design, and detailing of two-way systems.

Of several analysis methods covered in ACI Sec. 13, only the direct design method of ACI Sec. 13.6 is amenable to manual calculations, and this is the only method covered in this book.

In addition to the sections that are specific to two-way systems, ACI Sec. 7.7 gives the minimum cover requirement (3/4 in) and ACI Sec. 7.12 gives the minimum requirements for steel (for temperature and shrinkage) and spacing, which are the same as for the one-way slabs discussed earlier.

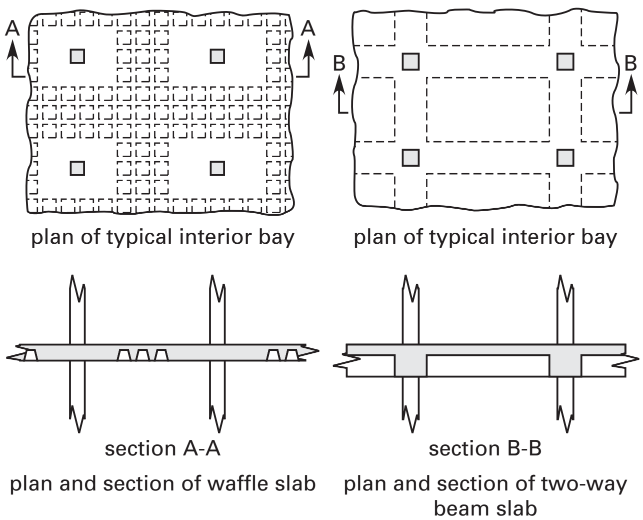


Figure 8.2 Two-Way Waffle Slab and Beam-Slab Floor Systems

2. Minimum Thickness of Two-Way Slabs

Specifying a minimum overall thickness, h, ensures the serviceability of two-way slabs for immediate and long-term deflections, crack control, and vibration. ACI Sec. 9.5.3 gives empirical relationships for the minimum slab thickness that depend on the longer clear spans in the exterior and interior spans of the system. The following sections summarize the requirements for three basic slab types.

A. Flat Plates

For flat plates, the minimum thickness, h, must be

- at least 5 in
- at least the minimum shown in Table 8.1, where l_n denotes the larger clear span measured between column faces in each panel

Table 8.1 Minimum Slab Thickness for Flat Plates

	exterior panels		interior panels
f_y (psi)	without edge beams	with edge beams	
40,000	$h \geq \frac{l_n}{33}$	$h \geq \frac{l_n}{36}$	$h \geq \frac{l_n}{36}$
60,000	$h \geq \frac{l_n}{30}$	$h \geq \frac{l_n}{33}$	$h \geq \frac{l_n}{33}$
75,000	$h \geq \frac{l_n}{28}$	$h \geq \frac{l_n}{31}$	$h \geq \frac{l_n}{31}$

B. Flat Slabs with Drop Panels

For flat slabs with drop panels conforming to the limitations of ACI Sec. 13.2.5, the minimum thickness, h, must be

- at least 4 in
- at least the minimum given in ACI Table 9.5(c) as follows, where l_n denotes the larger clear span measured between column faces in each panel

Table 8.2 Minimum Slab Thickness for Flat Slabs with Drop Panels

	exterior panels		interior panels
f_y (psi)	without edge beams	with edge beams	
40,000	$h \geq \frac{l_n}{36}$	$h \geq \frac{l_n}{40}$	$h \geq \frac{l_n}{40}$
60,000	$h \geq \frac{l_n}{33}$	$h \geq \frac{l_n}{36}$	$h \geq \frac{l_n}{36}$
75,000	$h \geq \frac{l_n}{31}$	$h \geq \frac{l_n}{34}$	$h \geq \frac{l_n}{34}$

C. Two-Way Beam-Slab Systems

For two-way beam slabs, ACI Sec. 13.2 defines the beam element as containing a portion of the slab extending a distance equal to either the depth of the beam below the slab or four times the overall slab thickness, whichever is smaller. A parameter β denotes the ratio of the longer to the shorter clear span in each panel. l_n denotes the larger clear span measured between column faces in each panel. In addition, the flexural stiffness of the slab measured from center to center of adjacent column lines is divided by the stiffness of the beam to give a parameter α_f for each beam line bounding a panel. The minimum thickness depends on the ratio β and the average of the ratios α_f, which is denoted by α_{fm}. When α_{fm} is less than 0.2, the system is treated as a flat plate and the criteria above apply. When α_{fm} exceeds 0.2 but is less than 2.0, the minimum thickness is

$$h \geq \begin{cases} \dfrac{l_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \\ 5 \text{ in} \end{cases} \qquad 8.1$$

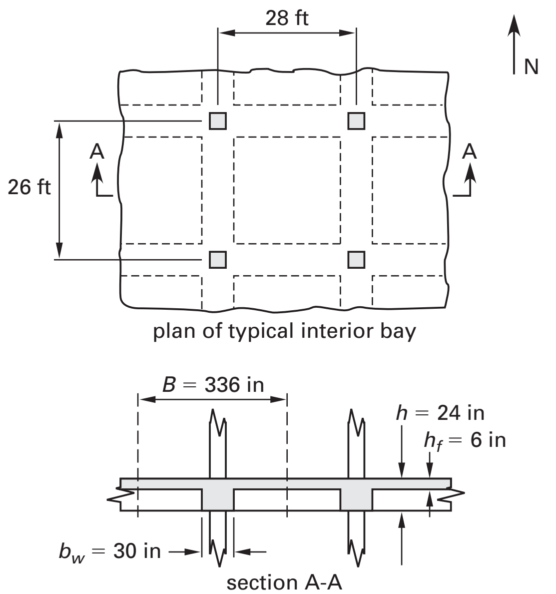
When α_{fm} exceeds 2.0, the minimum thickness is

$$h \geq \begin{cases} \dfrac{l_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 9\beta} \\ 3.5 \text{ in} \end{cases} \qquad 8.2$$

ACI Sec. 9.5.3.3 requires that at a discontinuous edge, one of two conditions must be met: Either an edge beam with stiffness ratio α of at least 0.8 must exist, or the slab thickness must be at least 10% greater than required by the previous criteria.

Example 8.1
Minimum Slab Thickness for a Two-Way Beam-Slab System

The two-way beam-slab floor shown here consists of rectangular panels spanning 26 ft in the north-south direction and 28 ft in the east-west direction. Columns are square and 16 in on a side. Preliminary design gives beams 30 in wide by 24 in overall depth with a trial slab thickness of 6 in. Use grade 60 rebars. Verify whether a 6 in slab satisfies ACI serviceability requirements for an interior panel.



Solution:
ACI 318 defines the parameter β as the ratio of longer-to-shorter clear spans.

$$\beta = \frac{l_{n,\text{long}}}{l_{n,\text{short}}} = \frac{l_{1,\text{long}} - b_w}{l_{1,\text{short}} - b_w} = \frac{(28 \text{ ft} - 2.5 \text{ ft})}{(26 \text{ ft} - 2.5 \text{ ft})} = 1.09$$

The beams are T-shaped, with an effective width given by ACI Sec. 13.2 as

$$b \le \begin{cases} b_w + 8h_f \\ \quad = 30 \text{ in} + (8)(6 \text{ in}) \\ \quad = 78 \text{ in} \\ b_w + 2(h - h_f) \\ \quad = 30 \text{ in} + (2)(24 \text{ in} - 6 \text{ in}) \\ \quad = 66 \text{ in} \quad [\text{controls}] \end{cases}$$

Calculate the moment of inertia of the beam for the gross section about its centroid.

$$\Delta\bar{y} = \frac{b_w(h - h_f)\left(\frac{h}{2}\right)}{bh_f + b_w(h - h_f)} = \frac{(30 \text{ in})(24 \text{ in} - 6 \text{ in})\left(\frac{24 \text{ in}}{2}\right)}{(66 \text{ in})(6 \text{ in}) + (30 \text{ in})(24 \text{ in} - 6 \text{ in})} = 6.92 \text{ in}$$

$$I_b = \frac{bh_f^3}{12} + bh_f\,(\Delta\bar{y})^2 + \frac{b_w\,(h - h_f)^3}{12} + b_w(h - h_f)\left(\frac{h}{2} - \Delta\bar{y}\right)^2$$
$$= \frac{(66 \text{ in})\,(6 \text{ in})^3}{12} + (66 \text{ in})(6 \text{ in})\,(6.92 \text{ in})^2 + \frac{(30 \text{ in})\,(24 \text{ in} - 6 \text{ in})^3}{12} + (30 \text{ in})(24 \text{ in} - 6 \text{ in})\left(\frac{24 \text{ in}}{2} - 6.92 \text{ in}\right)^2$$
$$= 48{,}667 \text{ in}^4 \quad (48{,}700 \text{ in}^4)$$

The width of the slab depends on the direction of span. For the north-south direction, slab width extends to the centerlines of the panels on each side, a distance of 336 in (28 ft). The stiffness ratio in that direction is

$$I_s = \frac{Bh_f^3}{12} = \frac{(336 \text{ in})\,(6 \text{ in})^3}{12} = 6048 \text{ in}^4$$
$$\alpha_{f1} = \frac{(EI)_b}{(EI)_s} = \frac{48{,}700 \text{ in}^4}{6048 \text{ in}^4} = 8.0$$

For the east-west span, the width of support is 312 in (26 ft).

$$I_s = \frac{Bh_f^3}{12} = \frac{312 \text{ in}\,(6 \text{ in})^3}{12} = 5620 \text{ in}^4$$
$$\alpha_{f1} = \frac{EI_b}{EI_s} = \frac{48{,}700 \text{ in}^4}{5620 \text{ in}^4} = 8.7$$

The average flexural stiffness ratio is

$$\alpha_{fm} = \frac{\alpha_{f1} + \alpha_{f2}}{2} = \frac{8.0 + 8.7}{2} = 8.35 > 2.0$$

For α_{fm} greater than 2.0, ACI Eq. 9-13 applies and the minimum slab thickness is

$$h \geq \begin{cases} \dfrac{l_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 9\beta} \\ = \dfrac{(25.5 \text{ ft})\left(12 \dfrac{\text{in}}{\text{ft}}\right)\left(0.8 + \dfrac{60{,}000 \dfrac{\text{lbf}}{\text{in}^2}}{200{,}000 \dfrac{\text{lbf}}{\text{in}^2}}\right)}{36 + (9)(1.09)} \\ = 7.4 \text{ in} \quad [\text{controls}] \\ 3.5 \text{ in} \end{cases}$$

Thus, the trial thickness of 6 in is inadequate; use a 7.5 in slab.

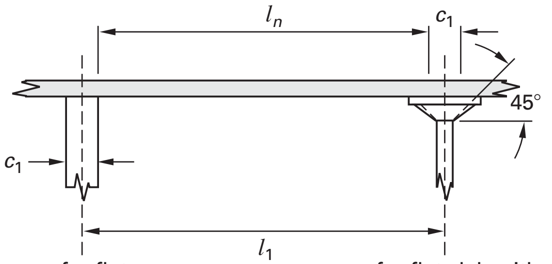


Figure 8.3 Dimensions in Span Direction of Two-Way Reinforced Concrete Floor Systems

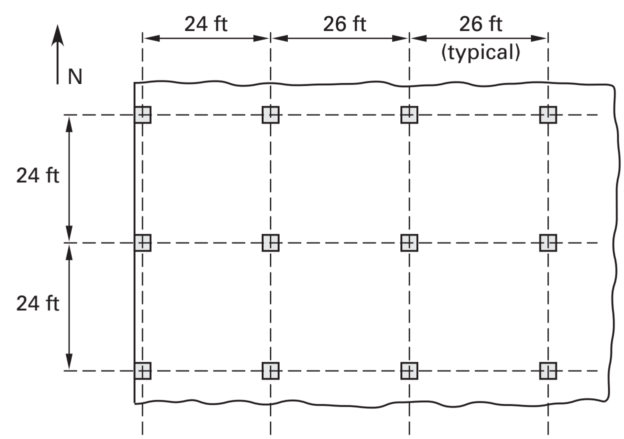
3. Flexural Design of Two-Way Slabs by Direct Design

Flexural design of two-way systems makes use of slab strips in orthogonal directions acting independently. The strips consist of slab regions extending halfway to adjacent column lines to each side for interior column lines, or to the edge of slab on the exterior strips. Column strips and middle strips, as defined in ACI Sec. 13.1, divide the column lines into design regions. The column strips contain the beams, if present, and extend a distance equal to 25% of the least centerline-to-centerline span in the panel to each side of the column centerline. Middle strips consist of the slab portions that are between the column strips.

When designing strips, l_1 denotes the center-to-center span length, l_2 denotes the width perpendicular to the span, and c_1 is the effective column width in the direction of span as defined in ACI Sec. 13.1.2. c_1 is usually the dimension of a square or rectangular column, but in the case of a slab supported by a column capital, an effective dimension is defined by a line projected from the bottom of the capital to the bottom of the slab or drop panel, at a 45° angle to the slab or panel, as shown in Fig. 8.3. The clear span for the direction under consideration is $l_n = l_1 - c_1$.

Example 8.2
Column and Middle Strips for a Flat Plate System

A two-way flat plate consists of rectangular panels as shown in partial plan. Columns are 16 in by 16 in in cross section. Subdivide the slab into column and middle strips.



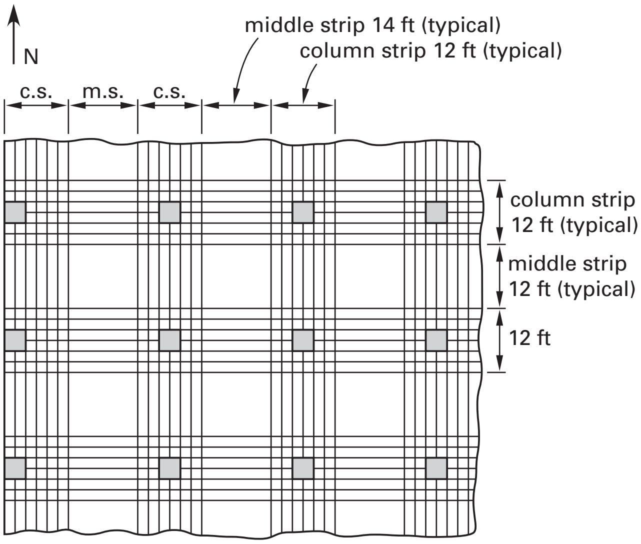
partial floor framing plan

Solution:

For an interior panel, the width of the column strip to either side of the column centerline is

$$c_s \leq \begin{cases} 0.25 l_1 \\ 0.25 l_2 \end{cases}$$
$$\leq \begin{cases} (0.25)(26 \text{ ft}) = 6.5 \text{ ft} \\ (0.25)(24 \text{ ft}) = 6 \text{ ft} \quad [\text{controls}] \end{cases}$$

The pattern of column and middle strips for the partial floor plan is as shown.



partial floor framing plan

ACI Sec. 13.6 defines the direct design method for two-way slabs. This method is applicable under the following conditions.

- There are three or more continuous spans in each direction.
- Panels are approximately rectangular.
- Columns are offset no more than 10% of the span length.
- The ratio of longer to shorter center-to-center spans is less than or equal to 2.0.
- Two successive span lengths differ by no more than one-third the longer span.
- Factored live loads are no more than two times the factored dead load, and all loads are uniformly distributed.
- If beams are present on all sides, the following condition must be met.

$$0.2 \le \frac{\alpha_{f1} l_2^2}{\alpha_{f2} l_1^2} \le 5.0 \qquad 8.3$$

α_f is the ratio of the flexural rigidity of the beam, $E_{cb} I_b$, to the flexural rigidity of the slab, $E_{cs} I_s$, for the direction under consideration.

The direct design method uses the static moment, M_o, computed in each direction. This moment is distributed to the positive and negative regions of column and middle strips based on the flexural and torsional stiffness of the elements. ACI Eq. 13-4 gives the static moment for a panel as

$$M_o = \frac{q_u l_2 l_n^2}{8} \qquad 8.4$$

q_u is the factored gravity load on the slab under consideration, l_2 is the width of the strip, and l_n is the clear span but cannot be less than $0.65 l_1$. The portion of the static moment assigned to each region is

- for interior spans,

$$M_u^- = 0.65 M_o \qquad 8.5$$

$$M_u^+ = 0.35 M_o \qquad 8.6$$

- for exterior spans, as given in Table 8.3.

The factored moment in the positive and negative moment regions that is assigned to the column and middle strips is found in Table 8.4. Use linear interpolation between tabulated values.

Divide the positive moment between column and middle strips by assigning the percentage indicated in Table 8.5 to the column strips. Use linear interpolation between tabulated values. The portions of positive and negative moments not assigned to the column strips are allocated to the middle strips, half to each side. The sum of the half middle strip moments from each adjacent column strip governs the flexural design of the middle strips. ACI Sec. 13.6.7 permits redistribution of positive and negative moments from the values tabulated above, up to maximum of 10%, provided resistance of total static moment, M_o, is present in the panel.

Table 8.3 Portion of Static Moment by Region, Exterior Spans

			slabs without beams between interior supports		
location	exterior edge unrestrained	beams between all supports	without edge beam	with edge beam	exterior edge fully restrained
interior negative factored moment	$0.75M_o$	$0.70M_o$	$0.70M_o$	$0.70M_o$	$0.65M_o$
positive factored moment	$0.63M_o$	$0.57M_o$	$0.52M_o$	$0.50M_o$	$0.35M_o$
exterior negative factored moment	0	$0.16M_o$	$0.26M_o$	$0.30M_o$	$0.65M_o$

Table 8.4 Factored Moment in Column and Middle Strips

for interior column strips

$\alpha_{f1}l_2/l_1$	l_2/l_1 0.5	1.0	2.0
0	$0.75M^-$	$0.75M^-$	$0.75M^-$
≥ 1.0	$0.90M^-$	$0.75M^-$	$0.45M^-$

for exterior column strips

$\alpha_{f1}l_2/l_1$	β_t	l_2/l_1 0.5	1.0	2.0
0	0	M^-	M^-	M^-
	≥ 2.5	$0.75M^-$	$0.75M^-$	$0.75M^-$
≥ 1.0	0	M^-	M^-	M^-
	≥ 2.5	$0.90M^-$	$0.75M^-$	$0.45M^-$

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s}$$

$$C = \sum \left(1 - \frac{0.63x}{y}\right)\left(\frac{x^3y}{3}\right)$$

x and y are the shorter and longer sides, respectively, of a rectangular component comprising the beam cross section.

For slabs with beams, the portion of the column strip moment that is not resisted by the beams is resisted by the remaining slab portions.

Table 8.5 Percentage of Factored Moment in Column Strips

$\alpha_{f1}l_2/l_1$	l_2/l_1 0.5	1.0	2.0
0	$0.60M^+$	$0.60M^+$	$0.60M^+$
≥ 1.0	$0.90M^+$	$0.75M^+$	$0.45M^+$

Example 8.3
Flexural Design of a Flat Plate System

Design flexural steel in the east-west direction over the first interior support of the flat plate described in Ex. 8.2. Concrete is normal weight with a compressive strength of 3500 psi, and the steel reinforcement has a yield strength of 60,000 psi. The superimposed service dead load is 25 lbf/ft^2, and service live load is 50 lbf/ft^2 (nonreducible). Assume a 9 in overall slab depth and an average effective depth of 7.5 in for both directions.

Solution:

The factored gravity load is

$$\begin{aligned} q_u &= 1.2w_d + 1.6w_l \\ &= 1.2(w_{ds} + hw_c) + 1.6w_l \\ &= (1.2)\left(25\ \frac{\text{lbf}}{\text{ft}^2} + \left(\frac{9\text{ in}}{12\ \frac{\text{in}}{\text{ft}}}\right)\left(150\ \frac{\text{lbf}}{\text{ft}^3}\right)\right) \\ &\quad + (1.6)\left(50\ \frac{\text{lbf}}{\text{ft}^2}\right) \\ &= 245\ \text{lbf/ft}^2 \end{aligned}$$

For the east-west direction, the span length equals 24 ft. For the exterior span,

$$l_n = l_1 - c_1 = 24\text{ ft} - \frac{16\text{ in}}{12\ \frac{\text{in}}{\text{ft}}} = 22.67\text{ ft}$$

From Eq. 8.4, the static moment in the exterior span is computed using ACI Eq. 13-4.

$$\begin{aligned} M_o &= \frac{q_u l_2 l_n^2}{8} \\ &= \frac{\left(245\ \frac{\text{lbf}}{\text{ft}^2}\right)(24\text{ ft})(22.67\text{ ft})^2}{8} \\ &= 378{,}000\text{ ft-lbf} \end{aligned}$$

The portion of the exterior span moment in the column strip over the first interior support, on a per-foot width basis, is

$$\begin{aligned} m_{\text{cs}}^- &= \frac{(\%\text{neg})(\%\text{cs})M_o}{b_{\text{cs}}} \\ &= \frac{(0.70)(0.75)(378{,}000\text{ ft-lbf})}{12\text{ ft}} \\ &= 16{,}500\text{ ft-lbf/ft} \end{aligned}$$

Note that l_2/l_1 equals 1.0, and α_{f1} equals zero.

Because the first interior clear span is larger than the exterior span, the moment on the opposite face must also be computed, and the larger value must be used for flexural design. For the interior span. ACI Eq. 13-4 gives the statical moment.

$$\begin{aligned} M_o &= \frac{q_u l_2 l_n^2}{8} \\ &= \frac{\left(245\ \frac{\text{lbf}}{\text{ft}^2}\right)(24\text{ ft})(24.67\text{ ft})^2}{8} \\ &= 447{,}000\text{ ft-lbf} \end{aligned}$$

The statical moment is allocated to the negative region of the column strip using the factors tabulated in Tables 8.3 and 8.5, and expressed on a per-foot width basis by dividing by the column strip width.

$$\begin{aligned} m^-_{\text{cs}} &= \frac{(\%\text{neg})(\%\text{cs})M_o}{b_{\text{cs}}} \\ &= \frac{(0.65)(0.75)(447{,}000 \text{ ft-lbf})}{12 \text{ ft}} \\ &= 18{,}200 \text{ ft-lbf/ft} \quad \text{[controls]} \end{aligned}$$

The column strip negative region must resist 18,200 ft-lbf per foot of width. The minimum steel throughout the 9 in slab for temperature and shrinkage (per ACI Sec. 7.12) is

$$\begin{aligned} A_{s,\text{min}} &= 0.0018bh \\ &= (0.0018)\left(\frac{12 \text{ in}}{\text{ft}}\right)(9 \text{ in}) \\ &= 0.19 \text{ in}^2/\text{ft} \end{aligned}$$

For flexure, the main steel area is calculated using Eq. 3.14.

$$\begin{aligned} m^-_{\text{cs}} &= \phi M_n \\ &= \phi\rho b d^2 f_y\left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right) \end{aligned}$$

$$\begin{aligned} \left(18{,}200 \ \frac{\text{ft-lbf}}{\text{ft}}\right)\left(12 \ \frac{\text{in}}{\text{ft}}\right) &= 0.9\rho\left(12 \ \frac{\text{in}}{\text{ft}}\right)(7.5 \text{ in})^2\left(60{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right) \\ &\quad \times \left(1 - 0.59\rho\left(\frac{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}{3{,}500 \ \frac{\text{lbf}}{\text{in}^2}}\right)\right) \\ \rho &= 0.0065 \\ A_s &= \rho b d \\ &= (0.0065)\left(12 \ \frac{\text{in}}{\text{ft}}\right)(7.5 \text{ in}) \\ &= 0.59 \text{ in}^2/\text{ft} \\ &> A_{s,\text{min}} \end{aligned}$$

Thus, no. 7 bars at 12 in centers furnish the required steel and satisfy the spacing limits.

4. Shear Strength of Two-Way Slabs

The shear strength of a two-way slab is controlled by the more severe of two conditions: *wide beam shear* and *punching shear*.

In wide beam shear, the entire width of a critical section—taken at a distance of d from the face of support—gives a design resistance of

$$\phi V_c = 2\phi\lambda\sqrt{f'_c}Bd \qquad 8.7$$

Punching shear occurs around the perimeter of a support and is located at $d/2$ from the support face. The design punching shear resistance of a square column is

$$\phi V_c = 4\phi\lambda\sqrt{f'_c}b_o d \qquad 8.8$$

b_o is the critical perimeter. In the case of a rectangular column, the punching shear strength diminishes as the ratio of the column's long side to its short side, β, increases. In this case, ACI Sec. 11.11.2.1 gives the design punching shear strength as

$$\phi V_c \le \begin{cases} 4\phi\lambda\sqrt{f'_c}b_o d \\ \left(2 + \dfrac{4}{\beta}\right)\phi\lambda\sqrt{f'_c}b_o d \\ \left(\dfrac{\alpha_s d}{b_o} + 2\right)\phi\lambda\sqrt{f'_c}b_o d \end{cases} \qquad 8.9$$

The parameter α_s is 40 for an interior column, 30 for an exterior column, and 20 for a corner column.

Example 8.4
Shear Strength of a Flat Plate System

Check the shear strength at a typical interior column in the flat plate of Ex. 8.2. The transfer of moment to the column is negligible.

Solution:
From the previous example, the factored gravity load on the slab is 245 lbf/ft^2. Critical wide beam shear occurs in the longer span at distance d from the support.

$$\begin{aligned} V_u &= q_u l_2\left(\frac{l_n}{2} - d\right) \\ &= \left(245 \ \frac{\text{lbf}}{\text{ft}^2}\right)(24 \text{ ft})\left(\frac{24.67 \text{ ft}}{2} - \frac{7.5 \text{ in}}{12 \ \frac{\text{in}}{\text{ft}}}\right) \\ &= 69{,}000 \text{ lbf} \end{aligned}$$

From Eq. 8.7,

$$\begin{aligned} \phi V_c &= 2\phi\lambda\sqrt{f'_c}Bd \\ &= (2)(0.75)(1.0)\sqrt{3500 \ \frac{\text{lbf}}{\text{in}^2}}(24 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)(7.5 \text{ in}) \\ &= 192{,}000 \text{ lbf} \\ &> V_u \end{aligned}$$

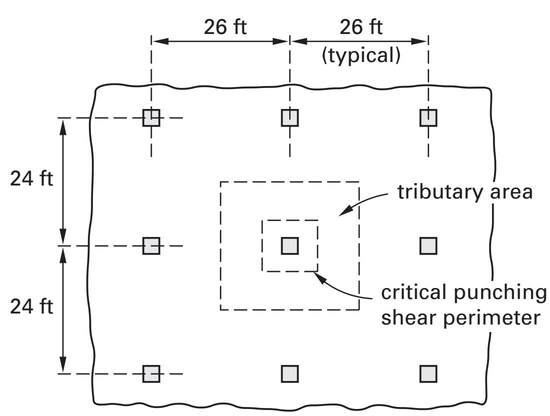
Thus, the slab has adequate wide beam shear strength. Check the punching shear. For a square interior column,

$$\begin{aligned} b_o &= 2(c_1 + d) + 2(c_2 + d) \\ &= (2)(16 \text{ in} + 7.5 \text{ in}) + (2)(16 \text{ in} + 7.5 \text{ in}) \\ &= 94 \text{ in} \end{aligned}$$

From ACI Sec. 11.11,

$$\phi V_c \leq \begin{cases} 4\phi\lambda\sqrt{f'_c}b_o d \\ \quad = (4)(0.75)(1.0)\sqrt{3500 \ \frac{\text{lbf}}{\text{in}^2}}(94 \text{ in})(7.5 \text{ in}) \\ \quad = 125{,}000 \text{ lbf} \quad \text{[controls]} \\ \left(2 + \frac{4}{\beta}\right)\phi\lambda\sqrt{f'_c}b_o d \\ \quad = \left(2 + \frac{4}{1}\right)(0.75)(1.0)\sqrt{3500 \ \frac{\text{lbf}}{\text{in}^2}} \\ \qquad \times (94 \text{ in})(7.5 \text{ in}) \\ \quad = 188{,}000 \text{ lbf} \\ \left(\frac{\alpha_s d}{b_o} + 2\right)\phi\lambda\sqrt{f'_c}b_o d \\ \quad = \left(\frac{(40)(7.5 \text{ in})}{94 \text{ in}} + 2\right)(0.75)(1.0) \\ \qquad \times \sqrt{3500 \ \frac{\text{lbf}}{\text{in}^2}}(94 \text{ in})(7.5 \text{ in}) \\ \quad = 162{,}400 \text{ lbf} \end{cases}$$

For an interior column, the tributary area for the punching shear is outside the critical perimeter and extending to the centerline of the panels in each direction, as shown.



partial floor framing plan

The punching shear force is

$$\begin{aligned} V_u &= q_u(l_1 l_2 - b_1 b_2) \\ &= 245 \ \frac{\text{lbf}}{\text{ft}^2} \\ &\quad \times \left((26 \text{ ft})(24 \text{ ft}) - \left(\frac{23.5 \text{ in}}{12 \ \frac{\text{in}}{\text{ft}}}\right)\left(\frac{23.5 \text{ in}}{12 \ \frac{\text{in}}{\text{ft}}}\right)\right) \\ &= 152{,}000 \text{ lbf} \\ &> \phi V_c \end{aligned}$$

The calculated punching shear, V_u, exceeds the punching shear resistance of the slab, ϕV_c. There are several possible remedies.

- Increase the critical perimeter by increasing the column dimensions or adding a capital.
- Increase the slab thickness either locally (by adding a drop panel) or globally.
- Add shear reinforcement or a shear head, as defined in ACI Sec. 11.11.4.
- Increase the compressive strength of the concrete locally or globally.
- Use appropriate combinations of the previous four options.

---

5. Transfer of Moment to Columns in Two-Way Slabs

The shear calculations in the previous section assume that only a direct shear force transfers from slab to column and the force distributes symmetrically around the critical perimeter. In general, both shear and moment must transfer from the slab and in this case the stress varies around the perimeter. ACI Sec. 13.5.3.2 permits a fraction of the unbalanced moment to transfer by flexure across an effective width extending $1.5h$ to either side of the column face. The fraction transferred by flexure is

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} \qquad 8.10$$

$$b_1 = c_1 + d$$
$$b_2 = c_2 + d$$

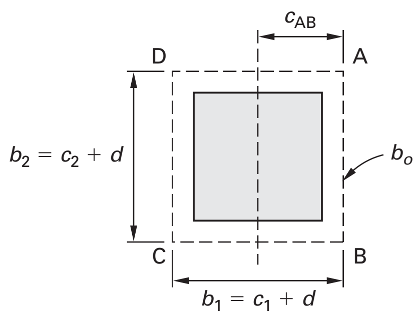
The remainder of the unbalanced moment must transfer by eccentricity of shear, as required in ACI Sec. 11.11.7. Stresses due to the action of the direct shear and eccentricity of shear are added algebraically and checked against an equivalent critical stress.

$$\phi v_c = \frac{\phi V_c}{b_o d} \qquad 8.11$$

The equivalent shear stress is

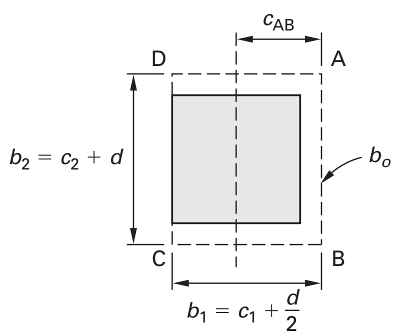
$$v_u = \frac{V_u}{b_o d} + \frac{\gamma_v M_u c_{\mathrm{AB}}}{J_c} \qquad 8.12$$

M_u is the total factored unbalanced moment at the centerline of the column. γ_v, which is equal to one minus γ_f, is the fraction of unbalanced moment to transfer by eccentricity of shear. c_{AB} is the distance from the centroid of the critical area to the extreme point on critical perimeter. J_c is a torsion property of the critical area. The commentary to ACI Sec. 11.11.7 gives an expression for J_c for the common case of an interior column without beams. Figure 8.4 gives expressions for other commonly encountered cases.



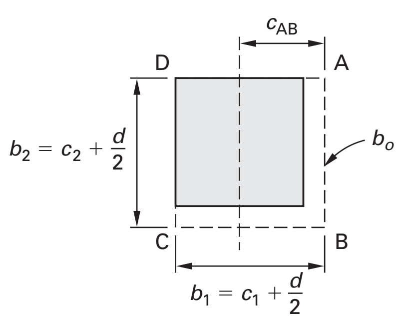
$$b_o = 2(c_1 + d) + 2(c_2 + d) = 2(c_1 + c_2 + 2d)$$
$$A_c = b_o d$$
$$c_{\mathrm{AB}} = \frac{b_1}{2}$$
$$J_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12}\right) + 2\left(b_2 d\left(\frac{b_1}{2}\right)^2\right) = \frac{b_1 d^3 + b_1^3 d}{6} + \frac{b_2 d b_1^2}{2}$$

interior rectangular column



$$b_o = 2\left(c_1 + \frac{d}{2}\right) + c_2 + d = 2c_1 + c_2 + 2d$$
$$A_c = b_o d$$
$$c_{\mathrm{AB}} = \frac{2b_1\left(\frac{b_1}{2}\right)}{b_o} = \frac{b_1^2}{b_o}$$
$$J_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + d b_1\left(\frac{b_1}{2} - c_{\mathrm{AB}}\right)^2\right) + d b_2 c_{\mathrm{AB}}^2$$

exterior rectangular column



$$b_o = c_1 + \frac{d}{2} + c_2 + \frac{d}{2} = c_1 + c_2 + d$$
$$A_c = b_o d$$
$$c_{\mathrm{AB}} = \frac{b_1\left(\frac{b_1}{2}\right)}{b_o} = \frac{b_1^2}{2b_o}$$
$$J_c = \frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + d b_1\left(\frac{b_1}{2} - c_{\mathrm{AB}}\right)^2 + d b_2 c_{\mathrm{AB}}^2$$

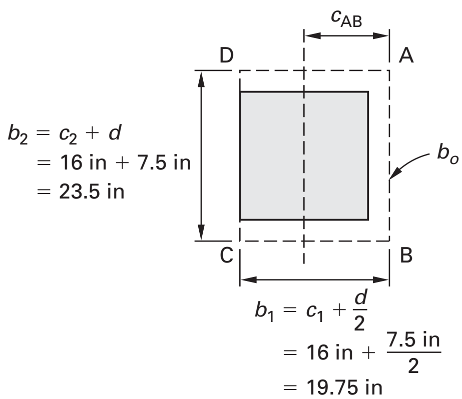
corner rectangular column

Figure 8.4 Section Properties for Eccentric Shear Stresses in Two-Way Slabs

Example 8.5
Shear Stresses at Exterior Column for a Flat Plate System

Check the stresses at an exterior column of the flat plate in Ex. 8.2 for a transfer of shear force of 60 kip and a transfer of moment of 35 ft-kip.

Solution:



The critical perimeter is located at $d/2$ from the face of 16 in by 16 in columns, as shown.

$$\begin{aligned} b_o &= 2\left(c_1 + \frac{d}{2}\right) + c_2 + d \\ &= (2)\left(16 \text{ in} + \frac{7.5 \text{ in}}{2}\right) + 16 \text{ in} + 7.5 \text{ in} \\ &= 63 \text{ in} \end{aligned}$$

The fraction of the unbalanced factored moment that transfers by flexure is

$$\begin{aligned} \gamma_f &= \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} \\ &= \frac{1}{1 + \frac{2}{3}\sqrt{\frac{19.75 \text{ in}}{23.5 \text{ in}}}} \\ &= 0.62 \end{aligned}$$

The remainder of the moment is

$$\gamma_v = 1 - \gamma_f = 1 - 0.62 = 0.38$$

This must transfer by eccentricity of shear. The centroid of the critical section from the right edge is

$$\begin{aligned} c_{\text{AB}} &= \frac{2b_1\left(\frac{b_1}{2}\right)}{b_o} \\ &= \frac{(19.75 \text{ in})^2}{63 \text{ in}} \\ &= 6.19 \text{ in} \end{aligned}$$

The torsional constant is

$$\begin{aligned} J_c &= 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + d b_1\left(\frac{b_1}{2} - c_{\text{AB}}\right)^2\right) + d b_2 c_{\text{AB}}^2 \\ &= 2\left(\begin{array}{l} \frac{(19.75 \text{ in})(7.5 \text{ in})^3}{12} + \frac{(7.5 \text{ in})(19.75 \text{ in})^3}{12} \\ \quad + (7.5 \text{ in})(19.75 \text{ in})\left(\frac{19.75 \text{ in}}{2} - 6.19 \text{ in}\right)^2 \end{array}\right) \\ &\quad + (7.5 \text{ in})(23.5 \text{ in})(6.19 \text{ in})^2 \\ &= 21{,}790 \text{ in}^4 \end{aligned}$$

The critical combined punching and eccentric shear stress is

$$\begin{aligned} v_u &= \frac{V_u}{b_o d} + \frac{\gamma_v M_u c_{\text{AB}}}{J_c} \\ &= \frac{60{,}000 \text{ lbf}}{(63 \text{ in})(7.5 \text{ in})} \\ &\quad + \frac{(0.38)(35{,}000 \text{ ft-lbf})\left(12 \frac{\text{in}}{\text{ft}}\right)(6.19 \text{ in})}{21{,}790 \text{ in}^4} \\ &= 172 \text{ psi} \end{aligned}$$

From Eqs. 8.9 and 8.11, the maximum shear stress that the joint can accommodate is

$$v_c = \frac{\phi V_c}{b_o d} \le \begin{cases} 4\phi\lambda\sqrt{f'_c} \\ \quad = (4)(0.75)(1.0)\sqrt{3500 \frac{\text{lbf}}{\text{in}^2}} \\ \quad = 177 \text{ psi} \quad \text{[controls]} \\ \phi\lambda\left(2 + \frac{4}{\beta}\right)\sqrt{f'_c} \\ \quad = (0.75)(1.0)\left(2 + \frac{4}{\frac{23 \text{ in}}{19.75 \text{ in}}}\right)\sqrt{3500 \frac{\text{lbf}}{\text{in}^2}} \\ \quad = 241 \text{ psi} \\ \phi\lambda\left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c} \\ \quad = (0.75)(1.0)\left(\frac{(30)(7.5 \text{ in})}{63 \text{ in}} + 2\right)\sqrt{3500 \frac{\text{lbf}}{\text{in}^2}} \\ \quad = 247 \text{ psi} \end{cases}$$

Thus, v_c is greater than v_u, and the shear stress at the exterior column satisfies the limiting stress.

9

Development of Reinforcement

In calculating the strength of reinforced concrete members, an implicit assumption is made that a complete bond exists between steel and concrete at the limit state. This requires that reinforcement must *develop* the design strength of the reinforcement.

1. Development of Reinforcement in Tension

ACI 318 permits three ways to develop bars in tension.

- straight embedment of the bar beyond the point of maximum stress
- extending the bar a sufficient distance beyond the point of maximum stress and providing a properly detailed 90° or 180° hook
- providing mechanical anchorage in the form of a properly welded cross bar or plate

Figure 9.1 illustrates each of the methods.

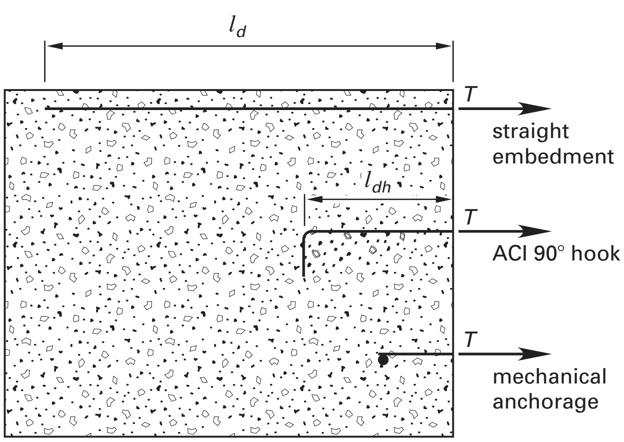


Figure 9.1 Representative Methods to Develop Bars in Tension

A. Straight Embedment

ACI Sec. 12.2 gives a general equation for the straight embedment length in tension.

$$l_d = \frac{3}{40}\left(\frac{f_y}{\lambda\sqrt{f'_c}}\right)\left(\frac{\psi_t\psi_e\psi_s}{\left(\frac{c_b+K_{\text{tr}}}{d_b}\right)}\right)d_b \qquad 9.1$$

ψ_t equals 1.0 for bottom bars and 1.3 for horizontal bars with 12 in or more of fresh concrete cast beneath. ψ_e is 1.0 for uncoated bars and 1.2 for epoxy-coated bars unless the cover is less than $3d_b$ or clear spacing is less than $6d_b$, in which case ψ_e is 1.5. ψ_s is 0.8 for no. 6 or smaller bars and 1.0 for bars larger than no. 6. λ is 1.0 for normal weight concrete and 0.75 for lightweight concrete. The parameters c_b and K_{tr} are dependent on the cover and transverse reinforcement surrounding the longitudinal bars.

For practical designs, minimum transverse reinforcement in the form of ties or stirrups is present, and the spacing and cover provided is sufficient to permit a simplification of the general equation to

$$l_d = \left(\frac{\psi_t\psi_e\psi_s f_y}{20\lambda\sqrt{f'_c}}\right)d_b \qquad 9.2$$

The code sets an upper bound on the value of $\sqrt{f'_c}$ of 100 psi and a lower bound on l_d of 12 in. ACI defines the development length of single bars in bundles as

- for two-bar bundles, the length of the individual bar
- for three-bar bundles, the length of the individual bar increased by 20%
- for four-bar bundles, the length of the individual bar increased by 33%

Example 9.1
Development Lengths for Grade 60 Bars

Generate a table giving the development lengths of no. 3 through no. 11 grade 60 rebars (yield strength of 60,000 psi), assuming normal weight concrete ($\lambda = 1$) with a compressive strength of 3000 psi and uncoated bottom bars ($\psi_e = \psi_t = 1$).

Solution:
For bars no. 6 and smaller, ψ_s equals 1.0 and the development length equation (Eq. 9.2) is

$$\begin{aligned} l_d &= \left(\frac{\psi_t \psi_e \psi_s f_y}{25\lambda\sqrt{f'_c}}\right) d_b \\ &= \frac{(1)(1)(1)\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right) d_b}{(25)(1)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}} \\ &= 43.8 d_b \end{aligned}$$

For no. 7 and larger bars, ψ_s equals 1.25, and the same equation is

$$\begin{aligned} l_d &= \left(\frac{\psi_t \psi_e \psi_s f_y}{25\lambda\sqrt{f'_c}}\right) d_b \\ &= \frac{(1)(1)(1.25)\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right) d_b}{(25)(1)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}} \\ &= 54.8 d_b \end{aligned}$$

The development lengths for these conditions are tabulated as shown.

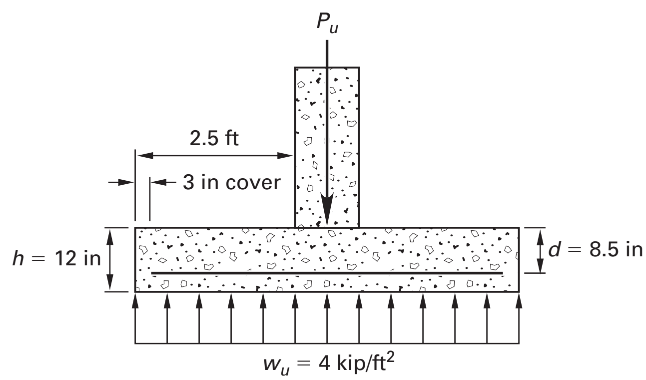
bar no.	d_b (in)	l_d (in)
3	0.375	16.0
4	0.500	21.9
5	0.625	27.4
6	0.750	32.9
7	0.875	48.0
8	1.000	54.8
9	1.128	61.8
10	1.270	70.6
11	1.410	77.3

Similar tables of development lengths are available (for example, Design Aid 11.2.9 in the *PCI Design Handbook*) and are more commonly used than the code equations.

ACI Sec. 12.2.5 permits a reduction in the development length when the area of steel furnished, $A_{s,\text{provided}}$, exceeds the area theoretically required at a section, $A_{s,\text{required}}$. The development length from the table or formula may be multiplied by the ratio $A_{s,\text{required}}/A_{s,\text{provided}}$ to give the allowed development length.

Example 9.2
Selecting Reinforcement to Ensure Development

A continuous wall footing is loaded to produce a uniformly distributed upward pressure of 4 kip/ft^2 under design factored loading. The width of footing is 6.0 ft and the concrete wall above is 12 in wide. The concrete is normal weight with a compressive strength of 3000 psi; reinforcement steel is grade 60. Determine the required area of flexural steel per foot of wall length and select appropriate reinforcement.



Solution:
Maximum bending moment occurs at the face of support.

$$\begin{aligned} M_u &= \frac{w_u a^2}{2} = \frac{\left(4\ \frac{\text{kip}}{\text{ft}^2}\right)(2.5\ \text{ft})^2}{2} \\ &= 12.5\ \text{ft-kip/ft} \end{aligned}$$

Calculate the flexural steel required using Eq. 3.14.

$$\begin{aligned} \phi M_n &= M_u \\ &= \phi \rho b d^2 f_y \left(1 - 0.59\ \rho\left(\frac{f_y}{f'_c}\right)\right) \end{aligned}$$

$$\begin{aligned} \left(12.5\ \frac{\text{ft-kip}}{\text{ft}}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right) &= 0.9\ \rho\left(12\ \frac{\text{in}}{\text{ft}}\right)(8.5\ \text{in})^2\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right) \\ &\quad \times \left(1 - 0.59\ \rho\left(\frac{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}{3{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right)\right) \end{aligned}$$

$$\rho = 0.00337$$

$$\begin{aligned} A_s &= \rho b d = (0.00337)\left(12\ \frac{\text{in}}{\text{ft}}\right)(8.5\ \text{in}) \\ &= 0.34\ \text{in}^2/\text{ft} \end{aligned}$$

The minimum steel required (from ACI 7.12.2) is

$$\begin{aligned} A_{s,\min} &= 0.0018bh \\ &= (0.0018)\left(12\ \frac{\text{in}}{\text{ft}}\right)(12\ \text{in}) \\ &= 0.26\ \text{in}^2/\text{ft} \end{aligned}$$

The flexural steel controls. For the given dimensions, the bottom reinforcement must develop over the distance from the face of the wall to the end of footing, less the required 3 in cover; thus, available embedment length is 30 in minus 3 in, or 27 in. Comparing the available embedment against the tabulated development lengths in Ex. 9.1, it is apparent that the largest bar that will fully develop is a no. 5 having a straight development length of 27.4 in. Therefore, use no. 5 bars at 11 in centers to furnish 0.34 in^2/ft.

B. Development of Standard Hooks in Tension

ACI Sec. 12.5 specifies the development length of standard hooks in tension. To qualify as a standard 90° or 180° hook, the diameter of bend and extension beyond the tangent point must satisfy the limits shown in Fig. R12.5 of ACI 318. The extension from the point of maximum stress to the outside edge of the hook is the development length, as shown in Fig. 9.2.

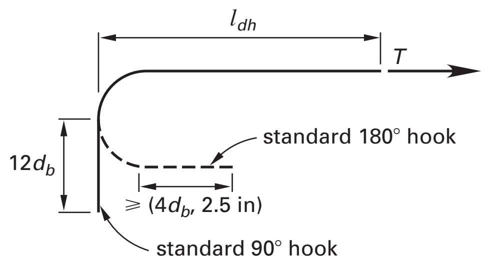


Figure 9.2 Development Length for Standard Hooks in Tension

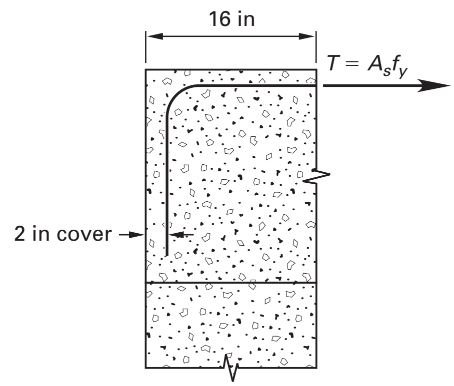
ACI Sec. 12.5.2 gives the development length for the standard hook in tension as

$$l_{dh} \geq \frac{0.02\psi_e f_y d_b}{\lambda\sqrt{f_c'}} \qquad 9.3$$

ψ_e is equal to 1.2 for epoxy-coated bars and 1.0 for other cases. λ is equal to 1.0 for normal weight concrete and 0.75 for lightweight concrete. If there is excess reinforcement, ACI Sec. 12.5.3 permits reducing l_{dh} by the factor $A_{s,\text{required}}/A_{s,\text{provided}}$. In the common case of a top bar terminating at an exterior column, a factor of 0.7 may be applied to the development length, provided that the side cover is at least 2.5 in and the end cover over the hook extension is at least 2 in. When there exists special enclosure of the hook by ties placed perpendicular or parallel to the development length, ACI Sec. 12.5.3 permits a reduction factor of 0.8. Lower bound values are specified in ACI Sec. 12.5.1 as $8d_b$ and 6 in.

Example 9.3
Development of Hooked Bar in Tension

Flexural steel in a girder terminates into an exterior column with a dimension of 16 in parallel to the flexural steel. Concrete is normal weight with a compressive strength of 4000 psi. Reinforcements are uncoated grade 60 rebars. Side cover is furnished by spandrel beams on each side with 2 in of clear cover beyond the hook extension. Calculate the largest diameter bar that can develop fully at the face of the column.



Solution:

The maximum extension of the hook beyond the critical location is 14 in. For a standard 90° hook fully developed in normal weight concrete with adequate side and end cover, the development length is calculated using criteria from ACI Sec. 12.5.

$$l_{dh} \geq \begin{cases} 0.7\left(\dfrac{0.02\psi_e f_y d_b}{\lambda\sqrt{f_c'}}\right) \\ \quad = (0.7)\left(\dfrac{(0.02)(1)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)d_b}{1\sqrt{4000\ \dfrac{\text{lbf}}{\text{in}^2}}}\right) \\ \quad = 13.3d_b \quad \text{[controls]} \\ 8d_b \\ 6\ \text{in} \end{cases}$$

The largest acceptable bar diameter is

$$\begin{aligned} 13.3d_b &\leq 14\ \text{in} \\ d_b &\leq 1.05\ \text{in} \end{aligned}$$

Thus, the largest acceptable bar size is no. 8.

2. Development of Reinforcement in Compression

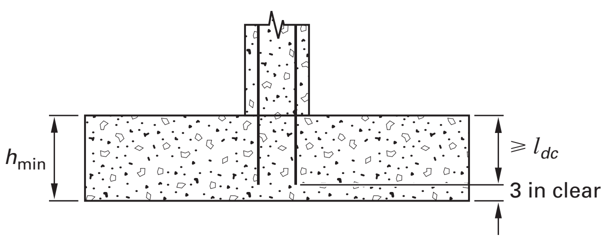
Hooks are ineffective in compression and only straight embedment counts. ACI Sec. 12.3 gives the compression development length for a rebar as

$$l_{dc} \geq \begin{cases} \dfrac{0.02 f_y d_b}{\lambda\sqrt{f'_c}} \\ 0.0003 f_y d_b \end{cases} \qquad 9.4$$

Where there is excess reinforcement, ACI Sec. 12.3.3 permits reducing l_{dc} by the factor $A_{s,\text{required}}/A_{s,\text{provided}}$. When the development length is confined by a spiral with a minimum 1/4 in diameter and 4 in pitch, or by no. 4 ties spaced less than or equal to 4 in, a factor of 0.75 may be applied to l_{dc}. A lower bound of 8 in applies.

Example 9.4
Development of Rebar in Compression

No. 8 dowels extend from a footing into a column above, as shown. Footing concrete is normal weight with compressive strength of 3000 psi. Reinforcements are grade 60 rebars. 3 in of clear cover is required. What is the smallest footing thickness that will permit full development of the dowels in compression?



Solution:
For the no. 8 dowel under the given conditions, from Eq. 9.4,

$$l_{dc} \geq \begin{cases} \dfrac{0.02 f_y d_b}{\lambda\sqrt{f'_c}} \\ \quad = \dfrac{(0.02)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)(1.00\ \text{in})}{1\sqrt{3000\ \dfrac{\text{lbf}}{\text{in}^2}}} \\ \quad = 22\ \text{in} \quad \text{[controls]} \\ 0.0003 f_y d_b \\ \quad = \left(0.0003\ \dfrac{\text{in}^2}{\text{lbf}}\right)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)(1.00\ \text{in}) \\ \quad = 18\ \text{in} \end{cases}$$

Thus, to fully develop and maintain the specified 3 in clearance at bottom, the footing must have a thickness of at least 25 in (22 in plus 3 in).

3. Development of Flexural Reinforcement

Critical locations are points of maximum stress or points at which adjacent reinforcement terminates. ACI Secs. 12.10 through 12.12 give requirements for extending flexural reinforcement beyond critical locations.

Generally, bars must extend a distance of at least $12d_b$ or the effective length, d, beyond the points where they are theoretically no longer needed. The continuing reinforcement must extend at least the development length beyond the location where the adjacent reinforcement can stop. Figure R12.10.2 summarizes the provisions graphically for a continuous beam or girder.

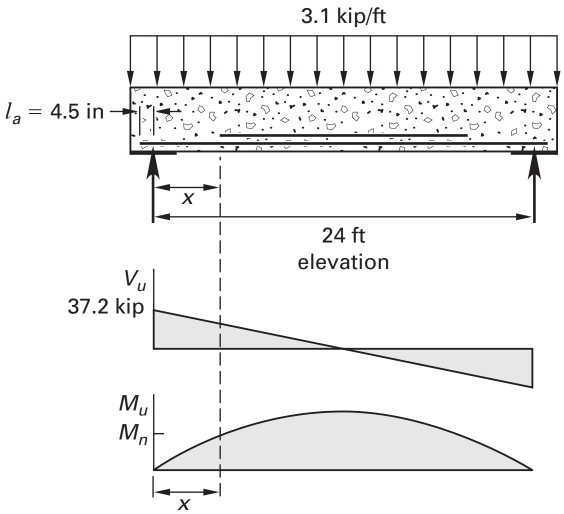
In the case of negative reinforcement ACI Sec. 12.12 requires that at least one-third the total negative reinforcement required over the support must extend past the point of inflection to a distance not less than $12d_b$, d, or $l_n/16$. For positive reinforcement, ACI Sec. 12.11 requires that at least one-third of the positive reinforcement in simple spans and one-fourth in continuous spans must extend at least 6 in beyond the face of supports. (ACI Sec. 7.13 sets more restrictive provisions for certain members to ensure structural integrity). ACI Sec. 12.11.3 imposes an additional requirement on the maximum bar size that can extend past the face of support in simple spans, or beyond the point of inflection for continuous spans. The requirement is that the bar diameter must be small enough to develop over a length of

$$l_d \leq \frac{M_n}{V_u} + l_a \qquad 9.5$$

M_n is the nominal moment strength of the continuing steel, V_u is the factored shear at the centerline of a simple support or at the point of inflection in a continuous span, and l_a is either the straight embedment length beyond the center of simple support or the required extensions beyond the point of inflection in a continuous span (that is, the greater of $12d_b$ or 12 in). For a simple span supported from below, ACI permits a 30% increase in the term M_n/V_u.

Example 9.5
Bar Cutoff in a Simple Span Beam

A simply supported beam supports a factored uniform load of 3.1 kip/ft on a 24 ft span. Bearing pads 12 in wide permit an extension of continuing bottom bars 4.5 inches beyond the support centerline. Bottom reinforcement consisting of two no. 8 bars and two no. 7 bars provides adequate flexural strength at midspan. The beam is rectangular, with b equal to 16 in and d equal to 20 in. The compressive strength of the concrete is 3000 psi, and the yield strength of the steel is 60,000 psi. Determine the location where the two no. 8 bars can terminate and verify that the two continuing no. 7 bars are adequate. Assume the shear strength at the cutoff point satisfies ACI Sec. 12.10.5.1.



Solution:

Calculate the nominal strength of the section with only two no. 7 bars.

$$a = \frac{A_s f_y}{0.85 f'_c b}$$
$$= \frac{(1.20 \text{ in}^2)\left(60 \ \frac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(3 \ \frac{\text{kip}}{\text{in}^2}\right)(16 \text{ in})}$$
$$= 1.76 \text{ in}$$
$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$
$$= (1.20 \text{ in}^2)\left(60 \ \frac{\text{kip}}{\text{in}^2}\right)\left(20 \text{ in} - \frac{1.76 \text{ in}}{2}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$
$$= 115 \text{ ft-kip}$$

The theoretical cutoff point for the two no. 8 bars is where the design strength with two no. 7 bars equals the factored bending moment.

$$M_u = \phi M_n = \frac{w_u L x}{2} - \frac{w_u x^2}{2}$$
$$(0.9)(115 \text{ ft-kip}) = \frac{\left(3.1 \ \frac{\text{kip}}{\text{ft}}\right)(24 \text{ ft})x}{2} - \frac{\left(3.1 \ \frac{\text{kip}}{\text{ft}}\right)x^2}{2}$$
$$\left(1.55 \ \frac{\text{kip}}{\text{ft}}\right)x^2 - (37.2 \text{ kip})x + (103.5 \text{ ft-kip}) = 0 \text{ ft-kip}$$

Solving the quadratic equation gives the two solutions

$$x = 3.21 \text{ ft}$$
$$x = 20.8 \text{ ft}$$

Theoretically, the two no. 8 bars could terminate at 3.2 ft from each support; however, ACI Sec. 12.10.3 requires an extension beyond this point equal to

$$\text{extension} \geq \begin{cases} d = 20 \text{ in} \\ 12 d_b = (12)(1 \text{ in}) = 12 \text{ in} \end{cases}$$

Thus, the bars must extend 20 in or 1.7 ft beyond the theoretical cutoff point, to a distance of 1.5 ft from support centerlines. Furthermore, the continuing bars must fully develop beyond the theoretical cutoff point. Referring to the tabulated development lengths in Ex. 9.1, the development length for a no. 7 bar under these conditions is 48.0 in. Checking this against the available embedment length,

$$x + l_a = (3.2 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right) + 4.5 \text{ in}$$
$$= 42.9 \text{ in} < 48 \text{ in} \quad \text{[unsatisfactory]}$$

ACI Sec. 12.11.3 also requires (from Eq. 9.5)

$$l_d \leq \frac{M_n}{V_u} + l_a$$

The factored shear force is

$$V_u = A_y - w_u x$$
$$= 37.2 \text{ kip} - \left(3.1 \ \frac{\text{kip}}{\text{ft}}\right)(3.2 \text{ ft})$$
$$= 27.3 \text{ kip}$$

Therefore, applying the 30% increase to M_n/V_u as permitted by ACI Sec. 12.11.3 gives

$$48.0 \text{ in} \leq (1.3)\left(\frac{1377 \text{ in-kip}}{27.3 \text{ kip}}\right) + 4.5 \text{ in}$$
$$\leq 70 \text{ in}$$

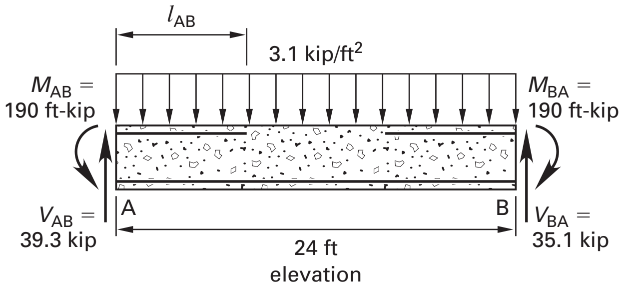
Thus, two no. 7 bars extending into support are adequate but require hooks or mechanical anchorage to develop fully beyond the cutoff points.

Example 9.6
Bar Cutoff in the Negative Moment Region of a Continuous Beam

The negative moment region at the left end of a continuous beam is governed by a loading pattern that results in end moments and reactions as shown. Top steel consists of four no. 7 bars over support A. Assume top bars must fully develop at A. Concrete is normal weight, and the following values apply.

$$f'_c = 3000 \text{ psi}$$
$$f_y = 60{,}000 \text{ psi}$$
$$b_w = 16 \text{ in}$$
$$d = 20 \text{ in}$$

Compute the distance from the left end at which all top steel can terminate.



Solution:
The development length for a no. 7 bottom bar is found in the table in Ex. 9.1 and is 48.0 in. A factor of 1.3 applies to this length for a top bar.

$$\begin{aligned} l_d &= (1.3)(48.0 \text{ in}) = 62 \text{ in} \\ &\geq l_{AB} \end{aligned}$$

Thus, the available embedment length is sufficient to develop the top bars at point A. At least one-third of the top steel must extend beyond the point of inflection (PI). Let x be the distance from A to PI.

$$M^- = M_{AB} + V_{AB}x - \frac{wx^2}{2} = 0$$

$$= -190 \text{ ft-kip} + (39.3 \text{ kip})x - \frac{\left(3.1 \ \frac{\text{kip}}{\text{ft}}\right) x^2}{2}$$

$$\left(-1.55 \ \frac{\text{kip}}{\text{ft}}\right) x^2 + (39.3 \text{ kip})x - 190 \text{ ft-kip}$$
$$= 0 \text{ ft-kip}$$

Using the quadratic formula gives

$$x = 6.50 \text{ ft}$$
$$x = 18.9 \text{ ft}$$

The point of inflection is 6.5 ft from the left end. The bars must extend beyond the PI at least

$$\text{extension} \geq \begin{cases} 12d_b = (12)(0.875 \text{ in}) = 10.5 \text{ in} \\ d = 20 \text{ in} \quad [\text{controls}] \\ \dfrac{l_n}{16} = \dfrac{(24 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{16} = 18 \text{ in} \end{cases}$$

Thus,

$$l_{AB} = x + \text{extension} = 6.5 \text{ ft} + (20 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = 8.2 \text{ ft}$$

4. Development of Web Reinforcement

ACI Sec. 12.13 gives requirements for the development of web reinforcement required to resist shear and torsion. Essentially these rules involve details that require standard bends around the longitudinal steel, which anchors the bars for full development at the member's midheight. Section 12.13 also permits bending of longitudinal bars to serve as shear reinforcement. This type of shear reinforcement (called a truss bar) is common in older construction but is rare in modern construction because of the high labor cost involved in fabricating the bent bars. Because these are largely detailing matters, the criteria are not summarized and illustrated in this book.

5. Mechanical Anchorage

ACI Sec. 12.6 permits mechanical devices to anchor reinforcement if the devices can fully develop the bar strength without damage to the concrete. Such devices may be used in combination with straight embedment length between the point of maximum stress and anchor.

A popular type of mechanical anchor is a bar welded perpendicularly to the bar that is to be developed. The anchor bar must have a diameter at least as large as that of the bar to be developed. Special welding procedures are required, and the ASTM A706 rebar is usually specified instead of ASTM A615 when bars are to be welded.

10

Prestressed Concrete

There are two general methods to overcome the relatively small tensile strength of concrete.

- Provide reinforcement that will resist tension after the concrete cracks.
- Prestress the regions that are subject to tensile stress under applied loads by deliberately inducing compression stresses.

Chapters 1 through 9 of this book have treated the first approach. This chapter summarizes the fundamentals of the second approach.

1. Prestressing Methods

Pretensioning and post-tensioning are the two general methods of prestressing concrete.

For pretensioned concrete, high-strength steel strands are stretched and temporarily anchored before casting the concrete. Figure 10.1 shows the operation schematically. When the concrete is sufficiently hardened, the strands are cut. The initial tension in the stretched strands transfers into the end regions of the member and induces stresses. If the strand force acts eccentrically to the center of gravity of the section, which is the usual case, it induces additional stresses to counteract stresses due to both axial compression and bending.

Pretensioning is an attractive approach for several reasons.

- Fabrication is normally done in plants, which permits better quality control of materials and workmanship.
- Components are standardized, which permits efficient use and reuse of formwork. (Chapter 3 of the *PCI Design Handbook*, 7th ed., shows typical cross sections that are widely used for pretensioned members.)
- Concrete will bond to the steel strands as it hardens. This provides corrosion resistance and ensures strain compatibility between the steel and the concrete at critical locations.

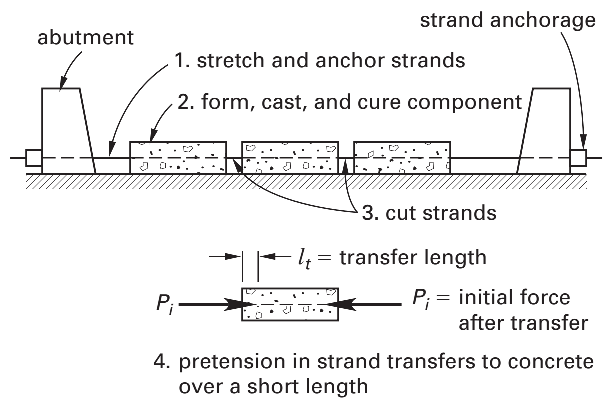


Figure 10.1 Schematic of Pretensioning Operation

An entire group of strands is a *tendon*. The dashed line representing the center of gravity of all prestressed steel at a section shows the tendon profile. This line is the *centroid of prestress*, and is typically labeled c.g.s. (center of gravity of steel). The distance from the *centroid of concrete* (typically labeled c.g.c., for centroid of gross concrete section) to the centroid of prestress is the *eccentricity of prestress* at a section.

Post-tensioning is the practice of stressing the tendon in a member after the concrete hardens. Strands enclosed in flexible sheaths or conduits are positioned in a desired profile, and the concrete is cast and cured to achieve a desired strength. The strands are then stretched and anchored using an appropriate anchorage system.

After anchorage, grout is sometimes pumped into the conduits to fill the inner space and encase the strands. After hardening, the grout provides corrosion protection and provides bond between the strands and the surrounding concrete. Alternatively, especially for single strands, grouting is omitted, leaving an unbonded strand, and the strand is coated with compounds that provide corrosion protection. The Post-Tensioning Institute's *PTI Post-Tensioning Manual* shows a variety of systems for post-tensioning.

Post-tensioning has several advantages over pretensioning.

- Reactive forces are applied directly to the hardened concrete, which means that the operation can be done without special end abutments.
- Strands can have curved profiles that better match the demands of the external loadings.
- Continuous systems are relatively easy and economical to construct.

Among the relative disadvantages of post-tensioning are these.

- Friction forces develop between the strands and conduits that cause loss of prestress.
- Strands are either unbonded or require grouting to achieve bond.
- Because post-tensioning is a field operation, quality control is usually not as good as for plant-cast members.

2. Materials

The concrete used in prestressed members is similar to that used for reinforced concrete members, as summarized in Ch. 1. Steel used for prestressing, however, is significantly different from the mild steel used in conventionally reinforced members. Appendix E of ACI 318 gives the cross-sectional properties of common prestressing materials.

While there are many types of steels used for prestressing, the most widely used in modern construction is a seven-wire strand that consists of six round or nearly round wires helically wrapped around a center wire. For modern construction, grade 270 strand is most common, and is available in diameters ranging from 3/8 in to 0.6 in.

Figure 10.2 shows qualitatively the stress-strain relationships for a grade 60 rebar and a grade 270 seven-wire strand. The grade 270 strand has over five times the tensile strength of the rebar, but the cold drawing of the wire significantly reduces the strand's ductility compared to that of the rebar. Furthermore, the arrangement of the wires causes an apparent deviation in the modulus of elasticity compared to that of a solid steel bar. Most important from a design standpoint, the grade 270 strand does not have the well-defined yield plateau of a rebar.

The stress-strain relationship for prestressing strand is nonlinear over much of the range that is important for assessing member strength. Reasonable approximations of the average stress-strain relationship for grade 270 strands are (according to the *PCI Design Handbook*, Design Aid 15.3.3)

$$f_{ps} = 28{,}800\varepsilon_{ps} \quad [\text{if } \varepsilon_{ps} \le 0.0085] \qquad 10.1$$

$$f_{ps} = 270 - \frac{0.04}{\varepsilon_{ps} - 0.007} \quad [\text{if } \varepsilon_{ps} > 0.0085] \qquad 10.2$$

f_{ps} is the strand stress in ksi, ε_{ps} is strand strain, and the constant 28,800 is the strand's modulus of elasticity. Similar expressions are available for other types of prestressing steels.

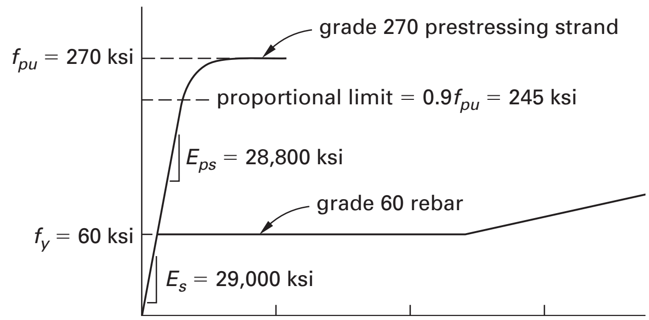


Figure 10.2 Comparative Stress-Strain Curves for Grade 60 Rebar and Grade 270 Low-Relaxation Strand

3. Changes in Prestress Force with Time

One complicating factor associated with prestressed concrete is that the force in the prestressing tendon changes with time. The change is mostly in the direction of diminishing force, but the application of external loads may cause tensile strains in the tendons that increase the force slightly. Losses occur at a fast rate initially but gradually diminish to achieve a steady state after several years. The stress that remains at the steady state is called the effective prestress, f_{pe}. It is convenient to describe the changes for pretensioned and post-tensioned members separately.

A. Pretensioned Members

Pretensioned members typically have strands that are several hundred feet in length during the pretensioning operation. ACI Sec. 18.5 permits the strands to be jacked to a stress not exceeding $0.94f_{py}$ or $0.8f_{pu}$. For grade 270 strand, f_{py} is equal to $0.9f_{pu}$, giving a maximum jacking stress of $0.8f_{pu}$ or 216 ksi. This stress is temporary and some of it is lost as the strand seats into its anchorage. After seating, the maximum stress is limited to $0.82f_{py}$ or $0.74f_{pu}$, which for grade 270 gives a stress of 200 ksi.

Steel stressed to such high stress is subject to relaxation, which is a time-dependent loss of stress under essentially constant strain. This stress loss occurs initially at a fast rate and gradually diminishes to a negligible rate. The grade 270 strands currently used are classified as low-relaxation, which limits the maximum relaxation loss to about 5 ksi over time.

When stress transfers into the member, reductions in tensile strains in the bonded tendons match the compressive strains, resulting in a loss of prestress due to elastic shortening. Behavior at this stage is linearly elastic and the loss of prestress due to elastic shortening is determined using basic equations of strength of materials. As subsequent external loads are applied to the member (for example, superimposed dead and live loads), the strain in the tendon may increase resulting in an elastic gain of stress.

Over the first few years of service, subsequent losses of prestress occur due to creep, shrinkage, and relaxation of steel. These time-dependent losses are interdependent and are difficult to estimate with precision. Recommended methods for estimating the losses are available (see the *PCI Design Handbook*, Sec. 5.7), but detailed estimates are seldom needed in design. Common practice is to take "lump sum" losses that are usually estimated as 10% of the anchorage stress immediately after transfer. (For example, the stress for a grade 270 low-relaxation strand immediately after transfer is estimated as $(0.9)(202\ \text{kip/in}^2) = 182$ ksi.)

Additional loss due to creep, shrinkage, and relaxation occurs over time, which gives a steady-state effective prestress in the range of 155 ksi to 160 ksi. The magnitude of the effective prestress affects the computed stresses and deflections in a member under service loads, but has practically no effect on the strength of a prestressed member.

B. Post-tensioned Members

The losses of prestress in post-tensioned members due to creep, shrinkage, and relaxation are similar to those for pretensioned members. However, there are significant differences between the two. In most cases, the strands in post-tensioned members are shorter than in pretensioned members. Consequently, the loss of prestress that occurs when the strand anchors is greater in a post-tensioned member. Since stress transfers directly to a post-tensioned member as strands are stressed, there is no elastic shortening loss when a particular strand is stressed.

In many cases, post-tensioned strands are stressed sequentially and the first strand to be stressed shortens and loses stress as stress is applied to each subsequent strand. Thus, if there are n strands, the tendon will experience an average loss equal to $1/(n-1)$ times the loss that occurs for the last strand stressed.

Most important, however, is that post-tensioned strands have curvature that creates friction loss as stressing proceeds. The friction loss is the sum of loss associated with accidental wobble over the projected length of strand plus the losses due to deliberate curvature. ACI Sec. 18.6 gives the requirements to compute these losses.

Fortunately, as noted for pretensioned members, a precise estimate of prestress loss is usually unnecessary and lump sum losses apply to most practical designs. Examples in the rest of this chapter follow this approach.

4. Serviceability of Prestressed Members

ACI 318 limits the behavior of prestressed members at all critical service load stages. Typically, this involves checking stresses and deflections immediately after the transfer of prestress, when the concrete is relatively weak and the prestress force is largest, and again under the service loads when all losses have occurred (the concrete has reached its design strength and the prestress is at its effective value).

A distinction is made among three classes of members.

- Class U—uncracked. The computed tensile stress in the precompressed zone is less than the modulus of rupture, $7.5\sqrt{f'_c}$.
- Class T—transition. The computed tensile stress in the precompressed zone is greater than the modulus of rupture but less than $12\sqrt{f'_c}$.
- Class C—cracked. The computed tensile stress in the precompressed zone is greater than $12\sqrt{f'_c}$.

For Class C members, the cracked transformed cross section applies; deflections are based on an effective moment of inertia in accordance with ACI Sec. 9.5.4, and crack widths are checked per ACI Sec. 18.4.4.1. In practice, there are few applications for Class C members and they will not be considered further in this book.

In analyzing Class U and T members for serviceability, ACI permits the assumption that an uncracked section will show linear elastic behavior. In a strict sense, a transformed area is needed to account for the differences in elastic properties of steel and concrete. However, a long history of satisfactory performance based on analysis of the gross concrete section has justified using this simpler approach. The basic assumptions for Class U and T analysis are as follows.

- Linear elastic behavior based on the gross concrete cross section applies.
- Strains vary linearly through the cross section.
- Changes in steel stress due to bending are small and may be neglected.
- The angle, α, between the centroid of gross concrete section (c.g.c.) and the tendon (c.g.s.) is small enough that

$$\sin\alpha \approx \tan\alpha$$
$$\cos\alpha = 1.0$$

Subject to these assumptions, the normal stress at a given section can be computed as

$$f = \frac{P}{A} \pm \frac{My}{I} \qquad 10.3$$

P is the prestress force at the stage of investigation. A is gross concrete area. M is the total moment taken about the c.g.c. and includes all actions due to prestress and external loads. I is the moment of inertia about the c.g.c. y is the distance from c.g.c. to a point in the cross section.

In computing extreme fiber stresses, it is usual to replace the stress due to bending, My/I, with the equivalent section modulus, $S = I/c$. c is the distance from the section centroid to the extreme fiber.

Any statically equivalent method may be used to determine the moment, M. A convenient approach is to treat the prestress as an external load acting on the concrete section and to calculate its effect as for any external load. Figure 10.3 shows the equivalent prestress loads for several typical cases. Isolating the tendon as a free-body diagram and solving for the forces required to maintain equilibrium gives the equivalent prestress loads. These loads apply as equal and opposite forces and couples on the centroid of concrete. For example, a free-body diagram of the depressed tendon in Fig. 10.3 requires a concentrated force, N, applied at midspan to maintain its shape when the tendon is pulled by the prestress force, P. Vertical equilibrium then requires

$$\sum F_v = 2P\sin\alpha - N = 0 \quad \text{10.4}$$

$$\sin\alpha = \tan\alpha = \frac{e_e + e_c}{0.5L} \quad \text{10.5}$$

$$N = \frac{4PL}{e_e + e_c} \quad \text{10.6}$$

Similar expressions can be derived for other tendon profiles (see the *PCI Design Handbook*, Design Aid 15.1.4).

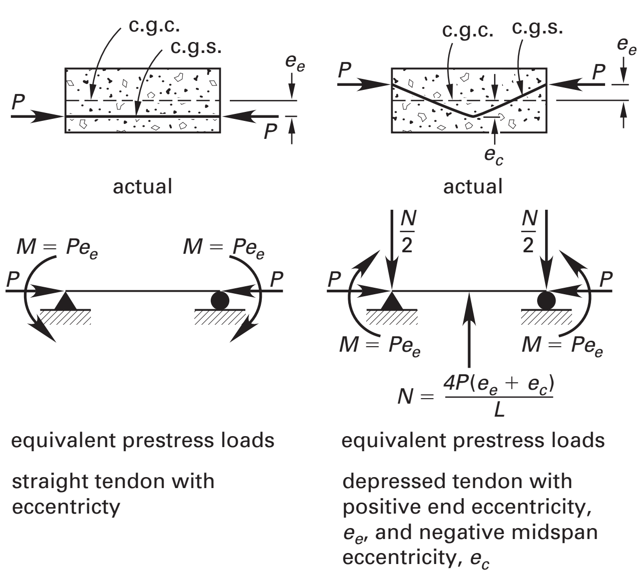


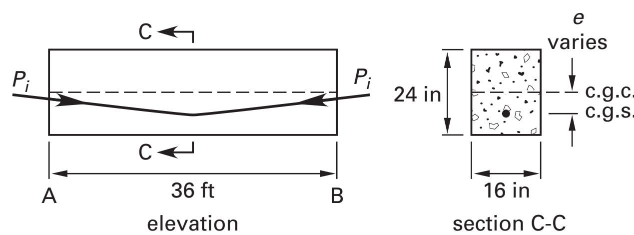
Figure 10.3 Equivalent Prestress Loads for Two Tendon Profiles

Example 10.1
Stress Calculations for a Pretensioned Beam

A pretensioned rectangular beam has the tendon profile shown below. The tendon consists of six ½ in diameter grade 270 strands with an eccentricity 2 in below c.g.c. at the ends and 10 in below c.g.c. at midspan. The cross section has these properties.

$$A = 384 \text{ in}^2$$
$$w = 400 \text{ lbf/ft}$$
$$I = 18{,}432 \text{ in}^2$$
$$S_{\text{top}} = S_{\text{bottom}} = 1536 \text{ in}^3$$

The strand stress is 180 ksi immediately after the transfer of prestress with a transfer length of 30 in. The beam cambers upward at transfer, and it spans, simply supported, over a length of 36 ft. Calculate the stresses at top and bottom fiber at the end of the transfer length immediately after transfer.



Solution:
The cross-sectional area of one ½ in nominal diameter grade 270 strand is given in ACI App. E as 0.153 in^2. Thus, for six strands, the initial prestressing force is

$$P_i = nA_b f_{pi} = (6)(0.153 \text{ in}^2)\left(180 \ \frac{\text{kip}}{\text{in}^2}\right)$$
$$= 165 \text{ kip}$$

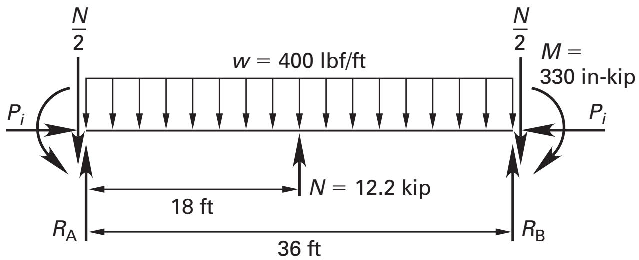
The upward prestress force acting on the concrete at midspan is

$$N = \frac{4P_i(e_c - e_e)}{L}$$
$$= \frac{(4)(165 \text{ kip})(10 \text{ in} - 2 \text{ in})}{(36 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}$$
$$= 12.2 \text{ kip}$$

End moments act to cause tension in the top of the beam. These end moments have magnitude

$$M = P_i e_e = (165 \text{ kip})(2 \text{ in}) = 330 \text{ in-kip}$$

The forces and couples acting on the member's c.g.c. immediately after transfer are as shown.



Because the effects of prestress are self-equilibrating, only the weight of the member contributes to the external reactions at points A and B.

$$
\begin{aligned}
R_{\mathrm{A}} = R_{\mathrm{B}} &= \frac{wL}{2} \\
&= \frac{\left(0.4\ \frac{\mathrm{kip}}{\mathrm{ft}}\right)(36\ \mathrm{ft})}{2} \\
&= 7.2\ \mathrm{kip}
\end{aligned}
$$

The total moment at the end of the transfer length (that is, at 2.5 ft from the end of the beam) is

$$
\begin{aligned}
M &= M_g + M_{ps} \\
&= \left(R_{\mathrm{A}} l_t - \frac{w l_t^2}{2}\right) - P e_e - \frac{N l_t}{2} \\
&= \left((7.2\ \mathrm{kip})(2.5\ \mathrm{ft}) - \frac{\left(0.4\ \frac{\mathrm{kip}}{\mathrm{ft}}\right)(2.5\ \mathrm{ft})^2}{2}\right) \\
&\quad - (165\ \mathrm{kip})(2\ \mathrm{in})\left(\frac{1\ \mathrm{ft}}{12\ \mathrm{in}}\right) - \frac{(12.2\ \mathrm{kip})(2.5\ \mathrm{ft})}{2} \\
&= -26\ \mathrm{ft\text{-}kip}
\end{aligned}
$$

The negative sign indicates a net moment causing tension stress in the top fiber. Following the convention that tensile stresses are positive and compression is negative, the stresses at the extreme fibers are

$$
\begin{aligned}
f_{\mathrm{top}} &= \frac{-P}{A} + \frac{M}{S_{\mathrm{top}}} \\
&= \frac{-165\ \mathrm{kip}}{384\ \mathrm{in}^2} + \frac{(26\ \mathrm{ft\text{-}kip})\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)}{1536\ \mathrm{in}^3} \\
&= -0.227\ \mathrm{ksi}
\end{aligned}
$$

$$
\begin{aligned}
f_{\mathrm{bottom}} &= \frac{-P}{A} - \frac{M}{S_{\mathrm{bottom}}} \\
&= \frac{-165\ \mathrm{kip}}{384\ \mathrm{in}^2} - \frac{(26\ \mathrm{ft\text{-}kip})\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)}{1536\ \mathrm{in}^3} \\
&= -0.633\ \mathrm{ksi}
\end{aligned}
$$

The negative signs indicate net compression stress at both top and bottom at the end of transfer length. ACI Sec. 18.4.1 limits the magnitude of normal stresses at the ends of simply supported members immediately after prestress to $0.6f'_{ci}$ compression and $6\sqrt{f'_{ci}}$ tension. When tension stress is excessive, which is a common condition, mild steel reinforcement must resist the total calculated tension force in the cross section. For this purpose, the mild steel is elastic and restricted to a maximum stress of 36 ksi.

Alternatively, the tendon profile may be adjusted or the bond between steel and concrete deliberately broken (usually by encasing lengths of strands in plastic sheathing) to alleviate the stresses at the ends.

Example 10.2
Deflections in a Pretensioned Beam

Calculate the midspan deflection in the beam of Ex. 9.2 immediately after transfer, given that the modulus of elasticity at that stage is 3600 ksi.

Solution:

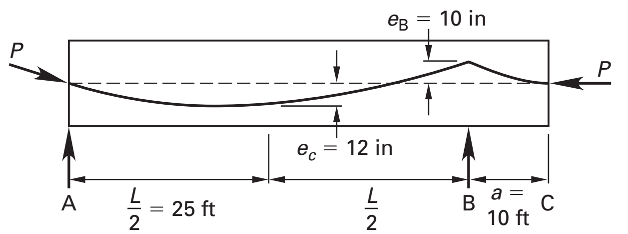
Superposition of the deflection caused by each load acting separately gives the total deflection. The end moment, $P_i e_e$, causes upward deflection; the concentrated midspan force, N, causes upward deflection; the weight of the beam, 0.4 kip/ft, causes downward deflection. Thus, the net midspan deflection is

$$
\begin{aligned}
\delta_i &= \frac{ML^2}{8E_cI} + \frac{NL^3}{48E_cI} - \frac{5wL^4}{384E_cI} \\
&= \frac{L^2}{E_cI}\left(\frac{M}{8} + \frac{NL}{48} - \frac{5wL^2}{384}\right) \\
&= \left(\frac{(36\ \mathrm{ft})^2\left(144\ \frac{\mathrm{in}^2}{\mathrm{ft}^2}\right)}{\left(3600\ \frac{\mathrm{kip}}{\mathrm{in}^2}\right)(18{,}400\ \mathrm{in}^4)}\right) \\
&\quad \times \left(\begin{array}{l} \dfrac{330\ \mathrm{in\text{-}kip}}{8} + \dfrac{(12.2\ \mathrm{kip})(36\ \mathrm{ft})\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)}{48} \\ \quad - \dfrac{(5)\left(0.4\ \frac{\mathrm{kip}}{\mathrm{ft}}\right)(36\ \mathrm{ft})^2\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)}{384} \end{array}\right) \\
&= 0.197\ \mathrm{in}\quad [\mathrm{upward}]
\end{aligned}
$$

The positive result indicates that a net upward deflection develops. This deflection is called *camber*. Camber increases with time due to the creep of the concrete. The prestress force diminishes with time due to creep, shrinkage, and relaxation, so the deflection components that are due to the end moment and force N will increase at a different rate than the deflection due to the constant beam weight. The *PCI Design Handbook* (Table 5.8.2) gives factors to apply to each component to obtain an estimate of the long-term deflections.

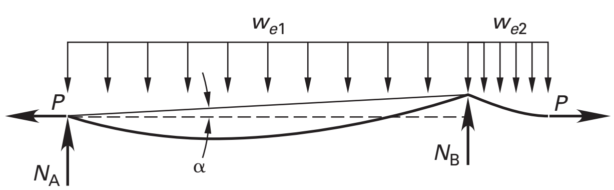
Example 10.3
Equivalent Loads for a Post-tensioned Beam

A tendon in the idealized parabolic profile shown is post-tensioned to an effective prestress of 400 kip. Compute the equivalent loads acting on the centroid of concrete.



Solution:

For the purpose of computing equivalent loads, the prestress force is assumed to be constant over its length. (In reality, it varies because of friction.) The idealized tendon is assumed to follow perfect parabolic curves, which results in a discontinuity in slope over support B. Practical considerations require a gradual change in tendon curvature, but the idealization is reasonable for the purpose of analysis (similar to treating the supports as knife-edge supports rather than as their true finite lengths). A free-body diagram of the strand requires transverse loads as shown.



The equivalent load for a parabolic strand over a full span is a uniformly distributed load, $8Ps/L^2$, where s denotes the sag at midspan. In the case of a strand that is higher at one end than the other, the sag is

$$\begin{aligned} s &= e_c + \frac{e_B}{2} = 12 \text{ in} + \frac{10 \text{ in}}{2} \\ &= (17 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \\ &= 1.42 \text{ ft} \end{aligned}$$

This gives an equivalent load of

$$\begin{aligned} w_{e1} &= \frac{8Ps}{L^2} = \frac{(8)(400 \text{ kip})(1.42 \text{ ft})}{(50 \text{ ft})^2} \\ &= 1.82 \text{ kip/ft} \end{aligned}$$

Similarly, for the overhang,

$$\begin{aligned} w_{e2} &= \frac{2Ps}{a^2} = \frac{(2)(400 \text{ kip})(0.83 \text{ ft})}{(10 \text{ ft})^2} \\ &= 6.64 \text{ kip/ft} \end{aligned}$$

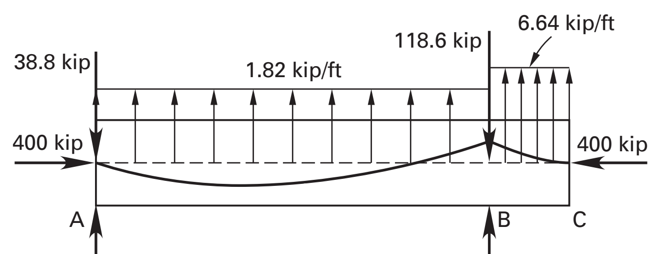
Equilibrium of the strands gives the transverse forces at A and B.

$$\sum M_A = -\frac{w_{e1}L^2}{2} - w_{e2}a\left(L + \frac{a}{2}\right) + N_B L = 0 \text{ ft-kip}$$

$$\begin{aligned} N_B &= \frac{\frac{w_{e1}L^2}{2} + w_{e2}a\left(L + \frac{a}{2}\right)}{L} \\ &= \left(\frac{\frac{\left(1.82 \frac{\text{kip}}{\text{ft}}\right)(50 \text{ ft})^2}{2} + \left(6.64 \frac{\text{kip}}{\text{ft}}\right)(10 \text{ ft})\left(50 \text{ ft} + \frac{10 \text{ ft}}{2}\right)}{50 \text{ ft}}\right) \\ &= 118.5 \text{ kip} \quad \text{[upward]} \end{aligned}$$

$$\sum F_v = -w_{e1}L - w_{e2}a + 118.5 \text{ kip} + N_A = 0 \text{ kip}$$

$$\begin{aligned} N_A &= w_{e1}L + w_{e2}a - 118.5 \text{ kip} \\ &= \left(1.82 \frac{\text{kip}}{\text{ft}}\right)(50 \text{ ft}) \\ &\quad + \left(6.64 \frac{\text{kip}}{\text{ft}}\right)(10 \text{ ft}) - 118.6 \text{ kip} \\ &= 38.8 \text{ kip} \quad \text{[upward]} \end{aligned}$$



equivalent prestress loads

The equivalent loads are self-equilibrating, and no external reactions develop due to the prestress in the statically determinate beam. This is generally not the case for statically indeterminate beams, as will be shown later in this chapter.

5. Flexural Strength of Prestressed Members

ACI Sec. 18.2 requires that prestressed beams meet all applicable limits on serviceability and strength. These limits require that, at every point of the beam, the design moment capacity must equal or exceed the moment due to factored loads, and that ductility limits must be met. This section summarizes the applicable criteria related to flexural strength.

A. General Analysis by Strain Compatibility

The stress-strain relationship for prestressing steel is nonlinear beyond the yield point, which makes computing the tendon stress difficult. Furthermore, many beams contain both prestressed and non-prestressed reinforcement, and these are generally positioned at different levels. Figure 10.4 shows the terminology for a rectangular beam with both prestressed and mild steel reinforcement.

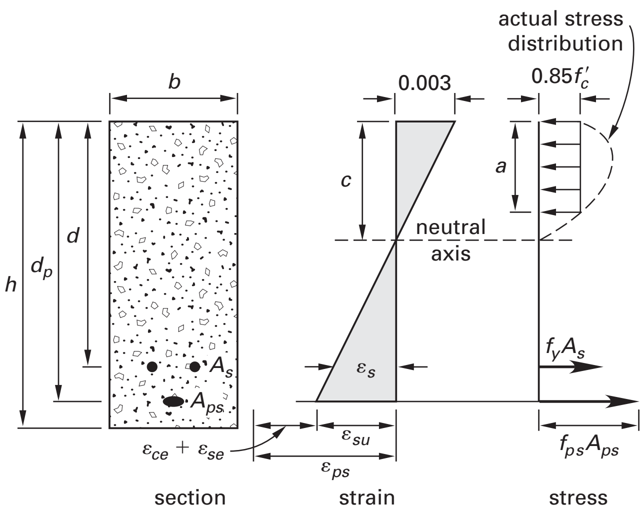


Figure 10.4 Notation for Moment Strength of a Rectangular Beam with both Mild Steel and Prestressing Steel

The same assumptions of ACI Sec. 10.2 that govern the strength of conventionally reinforced beams apply to prestressed members as well; the mild steel, however, will not necessarily yield at failure. A general strain compatibility analysis involves trial and error. Because most prestressed beams are proportioned to achieve high strains at failure, it is reasonable to base a first trial on the assumption that f_{ps} is equal to f_{pu}. Then three steps are repeated until a solution is found.

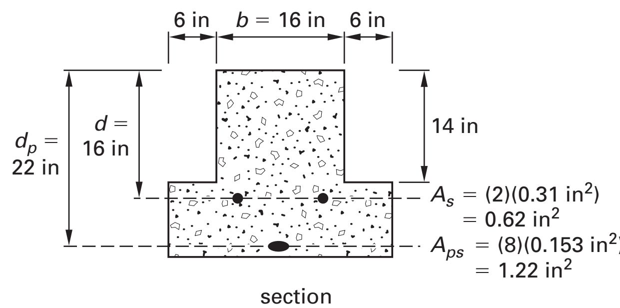
- Solve for the resulting depth to the neutral axis.
- Use strain compatibility to find the strains in the reinforcement.
- Use the stress-strain relationships to evaluate the stress in the reinforcement.

Iterate until the computed stresses agree with the trial values.

With the stresses in reinforcement determined, calculate the corresponding tensile forces and sum moments to find M_n. The effective strain in prestressing steel under no moment is equal to the effective strain plus the precompression strain in the concrete. However, these strains need not be determined exactly. For an effective prestress of at least $0.5f_{pu}$, 0.005 is a reasonable estimate for the initial strain in prestressing steel.

Example 10.4
Flexural Strength by Strain Compatibility Analysis

The inverted T-beam shown contains eight ½ in diameter grade 270 strands at an effective depth of 22 in, plus two no. 5 grade 60 rebars at an effective depth of 16 in. The rebars have a strain at first yield, ε_y, of 0.00207. Concrete is normal weight with a compressive strength of 5000 psi. Compute the nominal moment strength, M_n, by means of strain compatibility analysis.



Solution:

As an initial trial, assume the mild steel yields (that is, $f_s = f_y = 60$ ksi) and the stress in the prestressing steel is at ultimate ($f_{ps} = 270$ ksi). Then,

$$a = \frac{f_{ps}A_{ps} + f_sA_s}{0.85f'_cb}$$

$$= \frac{\left(270\ \frac{\text{kip}}{\text{in}^2}\right)(1.22\ \text{in}^2) + \left(60\ \frac{\text{kip}}{\text{in}^2}\right)(0.62\ \text{in}^2)}{(0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)(16\ \text{in})}$$

$$= 5.39\ \text{in}$$

$$c = \frac{a}{\beta_1} = \frac{5.39\ \text{in}}{0.8}$$

$$= 6.74\ \text{in}$$

Using similar triangles, the strains in the mild steel and the prestressing are

$$\frac{\varepsilon_s}{d-c} = \frac{0.003}{c}$$

$$\varepsilon_s = \frac{(0.003)(d-c)}{c}$$

$$= \frac{(0.003)(16\ \text{in} - 6.74\ \text{in})}{6.74\ \text{in}}$$

$$= 0.0041$$

$$> \varepsilon_y = 0.00207$$

Thus, the strain in the mild steel exceeds the yield strain; therefore, the stress in the mild steel is the yield stress, f_y.

$$\frac{\varepsilon_{su}}{d_p - c} = \frac{0.003}{c}$$

$$\begin{aligned}\varepsilon_{su} &= \frac{(0.003)(d_p - c)}{c} \\ &= \frac{(0.003)(22 \text{ in} - 6.74 \text{ in})}{6.74 \text{ in}} \\ &= 0.0068 \\ \varepsilon_{ps} &= (\varepsilon_{ce} + \varepsilon_{se}) + \varepsilon_{su} = 0.005 + 0.0068 \\ &= 0.0118 \\ &> 0.0086\end{aligned}$$

Thus, the stress is

$$\begin{aligned}f_{ps} &= 270 - \frac{0.04}{\varepsilon_{ps} - 0.007} \\ &= 270 \ \frac{\text{kip}}{\text{in}^2} - \frac{0.04 \ \frac{\text{kip}}{\text{in}^2}}{0.0118 - 0.007} \\ &= 262 \text{ ksi}\end{aligned}$$

The initial trial values are close to the calculated values. Strains in mild steel are well above yield, so it is reasonable to use $f_s = 60$ ksi for a second trial. The slightly lower stress in f_{ps} will give a smaller depth to neutral axis, and hence a slightly larger computed value for f_{ps}. For the second trial, assume f_{ps} is equal to 262 ksi and f_s equal to 60 ksi. The revised values are

$$\begin{aligned}a &= \frac{f_{ps}A_{ps} + f_sA_s}{0.85 f'_c b} \\ &= \frac{\left(262 \ \frac{\text{kip}}{\text{in}^2}\right)(1.22 \text{ in}^2) + \left(60 \ \frac{\text{kip}}{\text{in}^2}\right)(0.62 \text{ in}^2)}{(0.85)\left(5 \ \frac{\text{kip}}{\text{in}^2}\right)(16 \text{ in})} \\ &= 5.25 \text{ in}\end{aligned}$$

$$\begin{aligned}c &= \frac{a}{\beta_1} = \frac{5.25 \text{ in}}{0.8} \\ &= 6.56 \text{ in}\end{aligned}$$

$$\frac{\varepsilon_s}{d - c} = \frac{0.003}{c}$$

$$\begin{aligned}\varepsilon_s &= \frac{(0.003)(d - c)}{c} \\ &= \frac{(0.003)(16 \text{ in} - 6.56 \text{ in})}{6.56 \text{ in}} \\ &= 0.0043 \\ &> \varepsilon_y = 0.00207\end{aligned}$$

$$\frac{\varepsilon_{su}}{d_p - c} = \frac{0.003}{c}$$

$$\begin{aligned}\varepsilon_{su} &= \frac{(0.003)(d_p - c)}{c} \\ &= \frac{(0.003)(22 \text{ in} - 6.56 \text{ in})}{6.56 \text{ in}} \\ &= 0.0071 \\ \varepsilon_{ps} &= (\varepsilon_{ce} + \varepsilon_{se}) + \varepsilon_{su} = 0.005 + 0.0071 \\ &= 0.0121 \\ &> 0.0086 \\ f_{ps} &= 270 - \frac{0.04}{\varepsilon_{ps} - 0.007} \\ &= 270 \ \frac{\text{kip}}{\text{in}^2} - \frac{0.04 \ \frac{\text{kip}}{\text{in}^2}}{0.0121 - 0.007} \\ &= 262 \text{ ksi}\end{aligned}$$

Thus, the process converges. The neutral axis is well above the ledger, so the compression region is rectangular with 16 in width as assumed. The nominal strength is

$$\begin{aligned}M_n &= f_{ps}A_{ps}\left(d_p - \frac{a}{2}\right) + f_sA_s\left(d - \frac{a}{2}\right) \\ &= \left(262 \ \frac{\text{kip}}{\text{in}^2}\right)(1.22 \text{ in}^2)\left(22 \text{ in} - \frac{5.25 \text{ in}}{2}\right) \\ &\quad + \left(60 \ \frac{\text{kip}}{\text{in}^2}\right)(0.62 \text{ in}^2)\left(16 \text{ in} - \frac{5.25 \text{ in}}{2}\right) \\ &= (6690 \text{ in-kip})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \\ &= 558 \text{ ft-kip}\end{aligned}$$

B. Ductility Considerations

Ductility criteria with respect to achieving a desired strain in the reinforcement are the same for prestressed members as for conventionally reinforced beams. That is, to qualify as tension controlled, the strain increment in the extreme tension reinforcement from decompression to flexural failure must equal or exceed 0.005. Strains in the range from 0.004 to 0.005 are also permitted, provided that the ϕ-factor is taken as 0.65 for a strain of 0.004, increasing linearly to 0.9 at a strain of 0.005.

Requirements on minimum reinforcement take a different form for prestressed beams than they do for conventionally reinforced beams. Instead of imposing a minimum steel requirement, ACI Sec. 18.8.2 requires that the cracking moment of the member must be at least 20% larger than the calculated nominal moment strength. That is,

$$\phi M_n \geq 1.2 M_{\text{cr}} \qquad 10.7$$

An exception to this requirement is permitted if the member's shear and flexural strength are at least twice the demands at every section.

Example 10.5
Ductility Requirements for a Prestressed Beam

For the beam analyzed in Ex. 10.4, the area of the gross section is 504 in^2, the centroid of the gross section is 10.3 in from the bottom, the moment of inertia about the centroid is 23,900 in^4, and the concrete is normal weight with a compressive strength of 5000 psi. The effective prestress is 150 ksi. Determine the design flexural strength, and determine whether the member satisfies the $1.2M_{\text{cr}}$ requirement.

Solution:
The increment of strain in the prestressing steel was computed in Ex. 10.4 and found to be

$$\varepsilon_{su} = 0.0071$$

This exceeds the limit of 0.005 for a tension-controlled failure; therefore, a ϕ-factor of 0.9 applies.

The cracking moment is the external bending moment that overcomes the precompression stress on the bottom fiber and produces a tensile stress equal to the modulus of rupture. The modulus of rupture is

$$\begin{aligned} f_r &= 7.5\sqrt{f'_c} \\ &= 7.5\sqrt{5000\ \frac{\text{lbf}}{\text{in}^2}}\left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) \\ &= 0.530\ \text{ksi} \end{aligned}$$

Calculate the precompression stress on the bottom fiber for an effective prestress.

$$\begin{aligned} P_e &= f_{se}A_{ps} = \left(150\ \frac{\text{kip}}{\text{in}}\right)(1.22\ \text{in}^2) \\ &= 183\ \text{kip} \\ f_b &= \frac{-P_e}{A} - \frac{P_e e c_b}{I} \\ &= \frac{-183\ \text{kip}}{504\ \text{in}^2} - \frac{(183\ \text{kip})(8.3\ \text{in})(10.3\ \text{in})}{23{,}900\ \text{in}^4} \\ &= -1.02\ \text{ksi} \end{aligned}$$

Find the cracking moment.

$$\begin{aligned} \frac{M_{\text{cr}}c_b}{I} + f_b - f_r &= 0 \\ M_{\text{cr}} &= \left(\frac{I}{c_b}\right)(f_r - f_b) \\ &= \left(\frac{23{,}900\ \text{in}^2}{10.3\ \text{in}}\right)\left(1.02\ \frac{\text{kip}}{\text{in}^2} + 0.53\ \frac{\text{kip}}{\text{in}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &= 300\ \text{ft-kip} \end{aligned}$$

Check the requirement in Eq. 10.7.

$$\begin{aligned} \phi M_n &\geq 1.2M_{\text{cr}} \\ (0.9)(558\ \text{ft-kip}) &\geq (1.2)(300\ \text{ft-kip}) \\ 502\ \text{ft-kip} &\geq 360\ \text{ft-kip} \end{aligned}$$

The requirement is satisfied. The design flexural strength is 502 ft-kip.

C. Approximate Analysis Using ACI Equations

For unbonded tendons, and for many practical cases involving bonded tendons, the tendon stress at ultimate can be approximated, in lieu of the general strain compatibility analysis, with the use of equations in ACI Sec. 18.7.

For a *bonded member* without compression steel, having an effective prestress of at least $0.5f_{pu}$, the approximate stress at nominal moment strength is calculated from ACI Eq. 18-1.

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left(\frac{\rho f_{pu}}{f'_c} + \frac{d}{d_p}(\omega - \omega')\right)\right) \quad \text{10.8}$$

γ_p is equal to 0.28 for grade 270 low-relaxation strand, 0.40 for grade 270 stress-relieved strand, and 0.55 for grade 250 stress-relieved strand. The tension steel index, ω, and compression steel index, ω', are

$$\omega = \frac{\rho f_y}{f'_c} \quad \text{10.9}$$

$$\omega' = \frac{\rho' f_y}{f'_c} \quad \text{10.10}$$

If the compression steel index is nonzero, then additional requirements apply.

$$\left(\frac{\rho f_{pu}}{f'_c} + \left(\frac{d}{d_p}\right)(\omega - \omega')\right) \geq 0.17 \quad \text{10.11}$$

$$d' \leq 0.15d_p \quad \text{10.12}$$

The stress at nominal moment strength in an *unbonded member* is given by either of two expressions. When the span-to-depth ratio is less than or equal to 35,

$$f_{ps} \leq \begin{cases} f_{se} + 10{,}000\ \frac{\text{lbf}}{\text{in}^2} + \frac{f'_c}{100\rho_p} \\ f_{py} \\ f_{se} + 60{,}000\ \frac{\text{lbf}}{\text{in}^2} \end{cases} \quad \text{10.13}$$

When the span-to-depth ratio exceeds 35,

$$f_{ps} \leq \begin{cases} f_{se} + 10{,}000\ \frac{\text{lbf}}{\text{in}^2} + \frac{f'_c}{300\rho_p} \\ f_{py} \\ f_{se} + 30{,}000\ \frac{\text{lbf}}{\text{in}^2} \end{cases} \quad \text{10.14}$$

Example 10.6
Nominal Moment Strength Using ACI Approximate Equations

Recompute the strand stress at nominal moment strength for the beam in Ex. 10.4 using the ACI approximate equations to compute tendon stress.

Solution:
The beam does not contain compression steel, so the compression steel index, ω', is zero. From Eq. 10.9, the tension steel index for the mild steel is

$$\begin{aligned}\omega &= \frac{\rho f_y}{f'_c} = \frac{A_s f_y}{bdf'_c} \\ &= \frac{(0.62\ \text{in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)}{(16\ \text{in})(16\ \text{in})\left(5\ \frac{\text{kip}}{\text{in}^2}\right)} \\ &= 0.029\end{aligned}$$

The strand stress is

$$\begin{aligned}f_{ps} &= f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left(\frac{A_{ps}f_{pu}}{bdf'_c} + \frac{d}{d_p}(\omega - \omega')\right)\right) \\ &= \left(270\ \frac{\text{kip}}{\text{in}^2}\right) \\ &\quad \times \left(1 - \left(\frac{0.28}{0.8}\right)\left(\begin{array}{l}\dfrac{(1.28\ \text{in}^2)\left(270\ \frac{\text{kip}}{\text{in}^2}\right)}{(16\ \text{in})(22\ \text{in})\left(5\ \frac{\text{kip}}{\text{in}^2}\right)} \\ \quad + \left(\dfrac{16\ \text{in}}{22\ \text{in}}\right)(0.029 - 0)\end{array}\right)\right) \\ &= 249\ \text{ksi}\end{aligned}$$

Thus, the stress calculated by the ACI equation is 5% lower than the value obtained by the general strain compatibility method, 262 ksi. Furthermore, ACI Sec. 18.7.3 prevents including the mild steel in the computation of M_n without doing a strain compatibility analysis.

6. Shear Strength of Prestressed Members

ACI Sec. 11.1 gives the shear strength requirement for a prestressed beam in the same form as for a conventionally reinforced member.

$$V_u \le \phi(V_c + V_s) \qquad \text{10.15}$$

V_s is calculated using the same truss model as for nonprestressed beams.

$$V_s = \frac{A_v f_y d}{s} \qquad \text{10.16}$$

The basic requirements for a prestressed beam are as follows.

- In the vicinity of supports, the factored shear is computed at a distance $h/2$ from the face of support.
- d is taken as the distance from compression edge to the centroid of prestressed and non-prestressed reinforcement, but no less than $0.8h$.
- In lieu of a more detailed analysis, the contribution of concrete to shear resistance is given by ACI Sec. 11.3.2.

$$V_c \le \begin{cases} \max\begin{cases}\left(0.6\lambda\sqrt{f'_c} + \dfrac{700V_u d_p}{M_u}\right)b_w d \\ 2\lambda\sqrt{f'_c}b_w d\end{cases} \\ 5\lambda\sqrt{f'_c}b_w d\end{cases} \qquad \text{10.17}$$

  M_u is the factored moment occurring simultaneously with V_u. The term $V_u d_p/M_u$ must be taken as less than or equal to 1.0.

- Stirrups are not required if

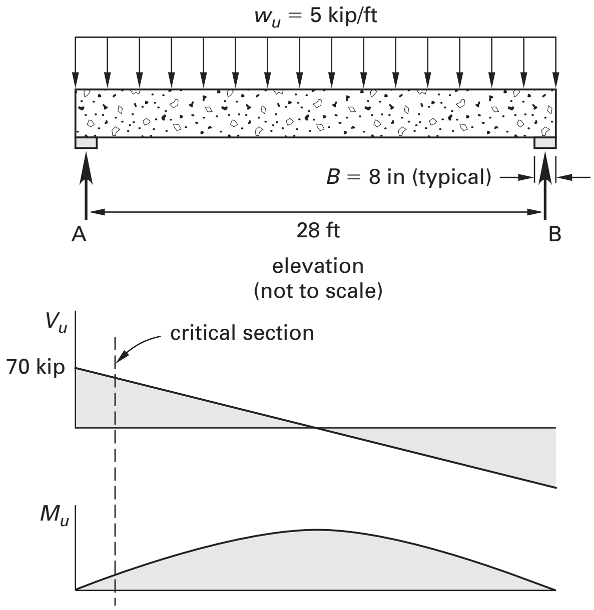
$$V_u \le 0.5\phi\sqrt{f'_c}b_w d \qquad \text{10.18}$$

  Otherwise stirrups must be provided at a spacing s, which is not to exceed

$$s \le \begin{cases}\min(0.75h,\ 24\ \text{in}) & [\text{if } V_s \le 4\sqrt{f'_c}b_w d] \\ \min(0.375h,\ 12\ \text{in}) & [\text{if } V_s > 4\sqrt{f'_c}b_w d] \\ \max\left(\dfrac{A_v f_{yt}}{0.75\sqrt{f'_c}b_w}, \dfrac{A_v f_{yt}}{50b_w}\right) \\ \dfrac{80A_v f_{yt} d}{A_{ps} f_{pu}}\sqrt{\dfrac{b_w}{d}}\end{cases}$$

Example 10.7
Shear Reinforcement in a Prestressed Beam

A simply supported rectangular beam spans 28 ft and bears on 8 in wide neoprene pads as shown. The given loads are factored gravity loads. The beam is 12 in wide by 28 in overall depth and is prestressed with 12 grade 270 ½ in diameter straight strands at 9 in eccentricity. Concrete is normal weight with a compressive strength of 5000 psi. Compute the required spacing of no. 3 U-stirrups with a yield strength of 60,000 psi at the point of maximum design shear.



Solution:

The most critical section for shear occurs at $h/2$ from the face of support.

$$x = \frac{h+B}{2} = \left(\frac{28\text{ in} + 8\text{ in}}{2}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) = 1.5\text{ ft}$$

The factored shear force at the critical section is

$$\begin{aligned} V_u &= \frac{w_u L}{2} - w_u x \\ &= \frac{\left(5\ \frac{\text{kip}}{\text{ft}}\right)(28\text{ ft})}{2} - \left(5\ \frac{\text{kip}}{\text{ft}}\right)(1.5\text{ ft}) \\ &= 62.5\text{ kip} \end{aligned}$$

The factored moment at the critical section is

$$\begin{aligned} M_u &= \frac{w_u L x}{2} - \frac{w_u x^2}{2} \\ &= \frac{\left(5\ \frac{\text{kip}}{\text{ft}}\right)(28\text{ ft})(1.5\text{ ft})}{2} - \frac{\left(5\ \frac{\text{kip}}{\text{ft}}\right)(1.5\text{ ft})^2}{2} \\ &= 99.4\text{ ft-kip} \end{aligned}$$

The effective depth for shear calculations is given in ACI 11.3.1.

$$d \geq \begin{cases} d_p = 23\text{ in} & \text{[controls]} \\ 0.8h = (0.8)(28\text{ in}) = 22.4\text{ in} \end{cases}$$

The shear resisted by concrete is given in ACI 11.3.2.

$$\frac{V_u d_p}{M_u} \leq \begin{cases} \dfrac{(62.5\text{ kip})(23\text{ in})\left(\dfrac{1\text{ ft}}{12\text{ in}}\right)}{99.4\text{ ft-kip}} = 1.21 \\ 1.0 \quad \text{[controls]} \end{cases}$$

$$V_c \leq \begin{cases} \left(0.6\lambda\sqrt{f'_c} + \dfrac{700 V_u d_p}{M_u}\right) b_w d \\ \quad = \left((0.6)(1)\sqrt{5000\ \dfrac{\text{lbf}}{\text{in}^2}} + \left(700\ \dfrac{\text{lbf}}{\text{in}^2}\right)(1.0)\right) \\ \qquad \times (12\text{ in})(23\text{ in}) \\ \quad = 205{,}000\text{ lbf} \\ 5\lambda\sqrt{f'_c} b_w d \\ \quad = (5)(1)\sqrt{5000\ \dfrac{\text{lbf}}{\text{in}^2}}(12\text{ in})(23\text{ in}) \\ \quad = 97{,}600\text{ lbf} \quad \text{[controls]} \end{cases}$$

Checking the criterion for stirrups (Eq. 10.18),

$$\begin{aligned} V_u &= 62.5\text{ kip} \\ 0.5\phi V_c &= (0.5)(0.75)(93\text{ kip}) \\ &= 34.9\text{ kip} \\ V_u &> 0.5\phi V_c \end{aligned}$$

Therefore, stirrups are required. From Eq. 10.15, the shear to be resisted by the stirrups is found using ACI Eq. 11-1.

$$\begin{aligned} V_u &\leq \phi(V_s + V_c) \\ V_s &= \frac{V_u}{\phi} - V_c \\ &= \frac{62.5\text{ kip}}{0.75} - 97.6\text{ kip} \\ &= -14.3\text{ kip} \\ \phi V_c &= (0.75)(97.6\text{ kip}) \\ &= 73.2\text{ kip} \\ V_s &< \phi V_c \end{aligned}$$

Provide minimum stirrups.

$$\begin{aligned} A_v &= 2A_b = (2)(0.11\text{ in}^2) \\ &= 0.22\text{ in}^2 \end{aligned}$$

$$s \le \begin{cases} \min(0.75h,\ 24\ \text{in}) \\ \quad = \min\big((0.75)(28\ \text{in}),\ 24\ \text{in}\big) \\ \quad = 21\ \text{in} \quad \text{[controls]} \\ \max\left(\dfrac{A_v f_{yt}}{0.75\sqrt{f'_c}b_w}, \dfrac{A_v f_{yt}}{50b_w}\right) \\ \quad = \max\left(\dfrac{(0.22\ \text{in}^2)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{0.75\sqrt{5000\ \dfrac{\text{lbf}}{\text{in}^2}}(12\ \text{in})}, \dfrac{(0.22\ \text{in}^2)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{\left(50\ \dfrac{\text{lbf}}{\text{in}^2}\right)(12\ \text{in})}\right) \\ \quad = 22\ \text{in} \\ \dfrac{80A_v f_{yt} d}{A_{ps} f_{pu}}\sqrt{\dfrac{b_w}{d}} \\ \quad = \dfrac{(80)(0.22\ \text{in}^2)\left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)(23\ \text{in})}{(12)(0.153\ \text{in}^2)\left(270\ \dfrac{\text{kip}}{\text{in}^2}\right)}\sqrt{\dfrac{12\ \text{in}}{23\ \text{in}}} \\ \quad = 35\ \text{in} \end{cases}$$

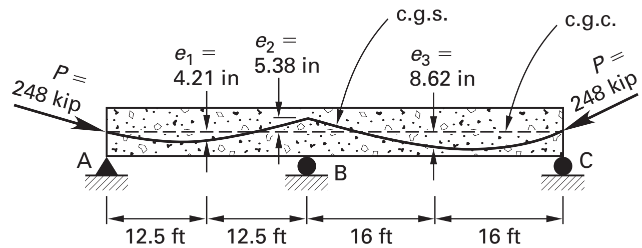
Use no. 3 stirrups at 21 in on centers.

7. Continuous Prestressed Concrete Beams

Continuous prestressed beams, especially those produced by post-tensioning, are popular and economical alternatives to simple span construction. An important difference in the behavior between simple and continuous spans is that continuous spans generally develop external reactions under the action of the prestress. This produces secondary moments in the members, which are significant at both the service and strength level. ACI Sec. 18.10.3 requires that the secondary moments combine with the factored loads using a load factor of 1.0. Moment redistribution in accordance with Sec. 18.10.4 may apply to the factored moments.

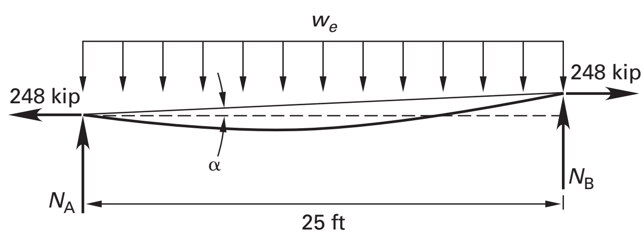
Example 10.8
Secondary Moments in a Prestressed Beam

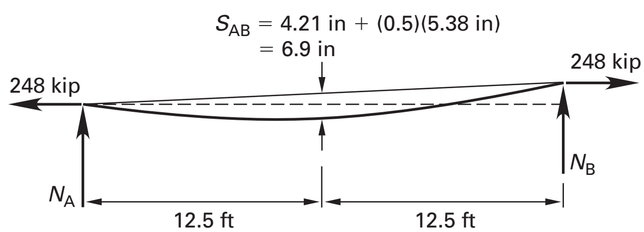
A two-span post-tensioned concrete beam has an effective prestress of 248 kip in the idealized profile shown. Flexural rigidity, EI, is constant over the entire length. Calculate reactions at the supports and determine the secondary moment over support B.



Solution:

Draw a free-body diagram of the tendon from A to B.





The equivalent prestress forces are

$$\begin{aligned} s_{\text{AB}} &= e_1 + 0.5e_2 \\ &= \big(4.21\ \text{in} + (0.5)(5.38\ \text{in})\big)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &= 0.575\ \text{ft} \\ w_{e,\text{AB}} &= \frac{8Ps_{\text{AB}}}{L^2} \\ &= \frac{(8)(248\ \text{kip})(0.575\ \text{ft})}{(25\ \text{ft})^2} \\ &= 1.83\ \text{kip/ft} \quad \text{[upward]} \end{aligned}$$

$$s_{BC} = e_3 + 0.5e_2 = \left(8.62 \text{ in} + (0.5)(5.38 \text{ in})\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = 0.943 \text{ ft}$$

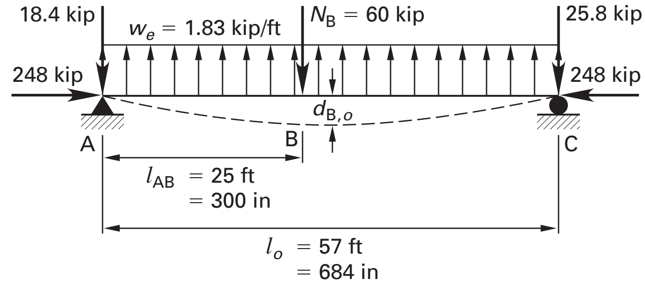
$$w_{e,BC} = \frac{8Ps_{BC}}{L^2} = \frac{(8)(248 \text{ kip})(0.943 \text{ ft})}{(32 \text{ ft})^2} = 1.83 \text{ kip/ft} \quad \text{[upward]}$$

$$N_A = \frac{w_{e,AB}l_{AB}}{2} - \frac{Pe_2}{l_{AB}} = \frac{\left(1.83 \frac{\text{kip}}{\text{ft}}\right)(25 \text{ ft})}{2} - \frac{(248 \text{ kip})(5.38 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)}{25 \text{ ft}} = 18.4 \text{ kip} \quad \text{[downward]}$$

$$N_B = \frac{w_{e,AB}l_{AB}}{2} + \frac{Pe_B}{l_{AB}} + \frac{w_{e,BC}l_{BC}}{2} + \frac{Pe_B}{l_{BC}}$$
$$= \frac{\left(1.83 \frac{\text{kip}}{\text{ft}}\right)(25 \text{ ft})}{2} + \frac{(248 \text{ kip})(5.38 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)}{25 \text{ ft}} + \frac{\left(1.83 \frac{\text{kip}}{\text{ft}}\right)(32 \text{ ft})}{2} + \frac{(248 \text{ kip})(5.38 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)}{32 \text{ ft}}$$
$$= 60.0 \text{ kip} \quad \text{[downward]}$$

$$N_C = \frac{w_{e,BC}l_{BC}}{2} - \frac{Pe_2}{l_{BC}} = \frac{\left(1.83 \frac{\text{kip}}{\text{ft}}\right)(32 \text{ ft})}{2} - \frac{(248 \text{ kip})(5.38 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)}{(32 \text{ ft})} = 25.8 \text{ kip} \quad \text{[downward]}$$

The structure is statically indeterminate to one degree. Choose the reaction at B as the unknown force and release it.



$$d_{B,o} = \frac{w_e l_{AB}}{24EI}(l_o^3 - 2l_{AB}l_o^2 + l_{AB}^3) - \frac{N_B l_{AB}^2 l_{BC}^2}{3EIl_o}$$
$$= \left(\frac{\left(1.83 \frac{\text{kip}}{\text{ft}}\right)(25 \text{ ft})}{24EI}\right) \times \left((57 \text{ ft})^3 - (2)(57 \text{ ft})(25 \text{ ft})^2 + (25 \text{ ft})^3\right) - \frac{(60.0 \text{ kip})(25 \text{ ft})^2(32 \text{ ft})^2}{3EI(57 \text{ ft})}$$
$$= \frac{22{,}430 \text{ ft}^3\text{-kip}}{EI} \quad \text{[upward]}$$

The flexibility coefficient is obtained by applying a unit force upward at the released point and computing the deflection caused by that force.

$$f_{BB} = \frac{l_{AB}^2 l_{BC}^2}{3EIl_o} = \frac{(25 \text{ ft})^2(32 \text{ ft})^2}{3EI(57 \text{ ft})} = \frac{3743 \text{ ft}^3}{EI} \quad \text{[upward]}$$

For consistent displacement at B,

$$R_B f_{BB} + d_{B,o} = 0 \text{ ft}$$
$$R_B = \frac{-d_{B,o}}{f_{BB}}$$
$$R_B = \frac{-22{,}430 \text{ kip-ft}^3}{3743 \text{ ft}^3} = -6.0 \text{ kip} = 6.0 \text{ kip} \quad \text{[downward]}$$

Equilibrium requires

$$\sum M_A = l_o R_C - R_B l_{AB} = 0 \text{ in-kip}$$
$$R_C = \frac{R_B l_{AB}}{l_o} = \frac{(6.0 \text{ kip})(25 \text{ ft})}{57 \text{ ft}} = 2.6 \text{ kip} \quad \text{[upward]}$$

Thus, the secondary moment induced by the prestress over support B is

$$\begin{aligned} M_{\mathrm{B,sec}} &= R_{\mathrm{C}} l_{\mathrm{BC}} \\ &= (2.6 \text{ kip})(32 \text{ ft}) \\ &= 83.2 \text{ ft-kip} \end{aligned}$$

This moment acts to cause compression on the top fiber.

11

Seismic Design of Reinforced Concrete Members

Concrete structures in regions of moderate or high seismic risk generally must meet the requirements of ACI Ch. 21. These requirements ensure there is sufficient toughness in the structure to dissipate the energy imparted by a design earthquake without collapsing. To function properly, members and connections must have enough ductility and strength to respond inelastically through many cycles of stress reversals.

Guiding principles for the design and detailing of ductile concrete members in regions of high seismic risk are as follows.

- Avoid brittle modes of failure. For example, limit the ratio of longitudinal steel so that yielding will occur before shear or bond failure.
- Recognize that material overstrength may be detrimental. For example, a yield strength larger than the specified minimum may result in shear or bond failure instead of the intended flexural yielding.
- Confine concrete to improve ductility and strength, by enclosing the core with spirals or closely spaced hoops and cross ties.
- Design for plastic hinge formation in beams, not in columns.
- Avoid discontinuities that create unreasonably large inelastic demands.
- Ensure that components that are not part of the lateral force resisting system retain their load-carrying capacity when subjected to the design displacement.

The following sections summarize the requirements for special frames, walls, and diaphragms in regions of high seismic risk. Similar but less restrictive criteria apply for intermediate structures located in regions of moderate seismic risk. Structures located in regions of low seismic risk are exempt from the provisions of ACI Ch. 21.

1. Flexural Members

ACI Sec. 21.5 defines a flexural member as one designed to resist earthquake forces by flexure, having a clear span at least four times its depth, and subjected to an axial compression load no larger than $0.1f'_cA_g$. Further restrictions require that the width, b_w, must exceed the smaller of $0.3h$ or 10 in and cannot exceed twice the width of the supporting member, or the width of the supporting member plus $3/4$ the dimension of the supporting member parallel to the beam span. Other requirements are as follows.

- The steel ratio at every section for both top and bottom longitudinal steel must be in the range from $200/f_y$ to 0.025.
- The positive moment strength at face of support must be at least 50% of the negative moment strength and the positive and negative strengths at every section along the span must be at least 25% of the larger negative moment strength at either support face.
- Transverse reinforcement in the form of closed hoops is required over a length equal to twice the member depth from the support face and on both sides of any interior point where plastic hinges might form under seismic loading. Spacing of the first hoop is no more than 2 in from the support face, and additional hoops are spaced at

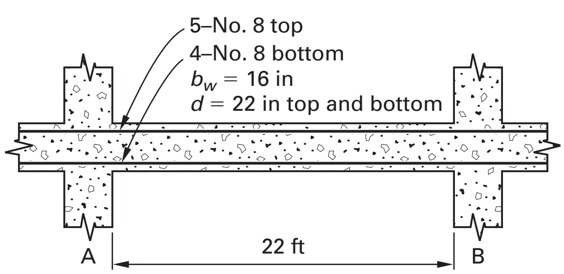
$$s \le \begin{cases} 0.25d \\ 6d_b \quad \text{[of the smallest longitudinal bar]} \\ 6 \text{ in} \end{cases} \qquad 11.1$$

  Hoops must brace the longitudinal bars to prevent their buckling in compression as well as to confine the concrete within.
- Shear reinforcement is determined based on factored gravity loads and the probable plastic moment capacities that could develop under seismic loading. For this purpose, probable moment strength is based on a yield stress of 1.25 times the

specified yield stress of the longitudinal reinforcement. For shear reinforcement, either hoops must be used, or, in regions where hoops are not required, stirrups must terminate with seismic hooks and be no farther apart than $d/2$. When the shear corresponding to probable moment strength is more than half the required shear strength and the factored axial compression force is less than $0.05f'_cA_g$, the contribution of concrete to shear strength is neglected (that is, V_c is zero).

Example 11.1
Shear Design for a Rectangular Beam in a Special Moment Frame

The rectangular reinforced concrete beam shown is part of a special moment frame in a region of high seismic risk. The concrete is normal weight with a specified compressive strength of 4000 psi, and the steel has a specified yield stress of 60,000 psi. The beam is subjected to a service dead load of 2 kip/ft and a live load due to office occupancy equal to 1.8 kip/ft (use a load factor of 0.5 on live load acting in combination with earthquake load). Treat the beam as singly reinforced for the purpose of computing nominal moment strength, and assume axial force is zero. Determine the hoop size and spacing in the end regions near the support faces.



elevation

Solution:

The probable moment strengths of the member are as follows.

With top steel yielding,

$$a_1 = \frac{A_{s1}(1.25f_y)}{0.85f'_cb_w} = \frac{(5)(0.79\text{ in}^2)(1.25)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(16\text{ in})} = 5.44\text{ in}$$

$$\begin{aligned} M_{\text{pr1}} &= A_{s1}(1.25f_y)\left(d - \frac{a_1}{2}\right) \\ &= (5)(0.79\text{ in}^2)(1.25)\left(60\ \frac{\text{kip}}{\text{in}^2}\right) \\ &\quad\times\left(22\text{ in} - \frac{5.44\text{ in}}{2}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) \\ &= 476\text{ ft-kip} \end{aligned}$$

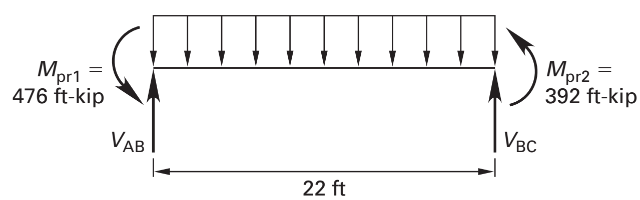
With bottom steel yielding,

$$a_2 = \frac{A_{s2}(1.25f_y)}{0.85f'_cb_w} = \frac{(4)(0.79\text{ in}^2)(1.25)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(16\text{ in})} = 4.36\text{ in}$$

$$\begin{aligned} M_{\text{pr2}} &= A_{s2}(1.25f_y)\left(d - \frac{a_2}{2}\right) \\ &= (4)(0.79\text{ in}^2)(1.25)\left(60\ \frac{\text{kip}}{\text{in}^2}\right) \\ &\quad\times\left(22\text{ in} - \frac{4.36\text{ in}}{2}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) \\ &= 392\text{ ft-kip} \end{aligned}$$

The gravity load acting in combination with earthquake load is

$$\begin{aligned} w_u &= 1.2w_d + 0.5w_l \\ &= (1.2)\left(2\ \frac{\text{kip}}{\text{ft}}\right) + (0.5)\left(1.8\ \frac{\text{kip}}{\text{ft}}\right) \\ &= 3.3\text{ kip/ft} \end{aligned}$$



Maximum shear occurs at the left end for the cycle shown above.

$$\begin{aligned} V_{\text{AB}} &= \frac{w_uL}{2} + \frac{M_{\text{pr1}} + M_{\text{pr2}}}{L} \\ &= \frac{\left(3.3\ \frac{\text{kip}}{\text{ft}}\right)(22\text{ ft})}{2} + \frac{476\text{ ft-kip} + 392\text{ ft-kip}}{22\text{ ft}} \\ &= 36.3\text{ kip} + 39.5\text{ kip} \\ &= 75.8\text{ kip} \end{aligned}$$

Because the contribution to the design shear due to probable moment strength is greater than 50%, neglect the contribution of concrete to shear strength. Assuming no. 3 overlapping hoops in the end region,

$$A_v = 4A_b = (4)(0.11\ \text{in}^2) = 0.44\ \text{in}^2$$

For shear strength, ACI Eq. 11-15 requires

$$\begin{aligned} s &= \frac{A_v f_y d}{V_s} \\ &= \frac{\phi A_v f_y d}{V_u} \\ &= \frac{(0.75)(0.44\ \text{in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)(22\ \text{in})}{75.8\ \text{kip}} \\ &= 5.7\ \text{in} \end{aligned}$$

The minimum stirrup spacing is given in Eq. 11.1.

$$s \le \begin{cases} 0.25d = (0.25)(22\ \text{in}) \\ \quad = 5.5\ \text{in} \quad [\text{controls}] \\ 6d = (6)(1\ \text{in}) \quad \left[\begin{array}{c}\text{of the smallest} \\ \text{longitudinal bar}\end{array}\right] \\ \quad = 6\ \text{in} \\ 6\ \text{in} \end{cases}$$

Use two no. 3 overlapping hoops at 5½ in on centers starting 2 in from support face and extending 50 in ($2h$) from face of each support.

2. Special Moment Frame Members Subjected to Combined Bending and Axial Force

ACI Sec. 21.6 requires that combined axial load and bending moment be considered for members that are part of the lateral force system in special moment frames and that have an axial compression force in excess of $0.1f'_cA_g$. For such members, the least cross-sectional dimension must be greater than or equal to 12 in, and the ratio of shorter side to longer side may not be more than 0.4. To ensure that plastic hinges form in the beams of special moment frames rather than in the columns, ACI Sec. 21.6.2 requires

$$\sum M_{nc} \ge \tfrac{6}{5}\sum M_{nb} \qquad 11.2$$

$\sum M_{nc}$ is the summation of the nominal moment strengths of the columns above and below a joint at the face of the joint, and $\sum M_{nb}$ is the nominal moment strength of beams at the left and right faces of the joint, as shown in Fig. 11.1. The factored axial load that results in the smallest moment strength for all relevant directions of the lateral force controls the moment strength of the column section.

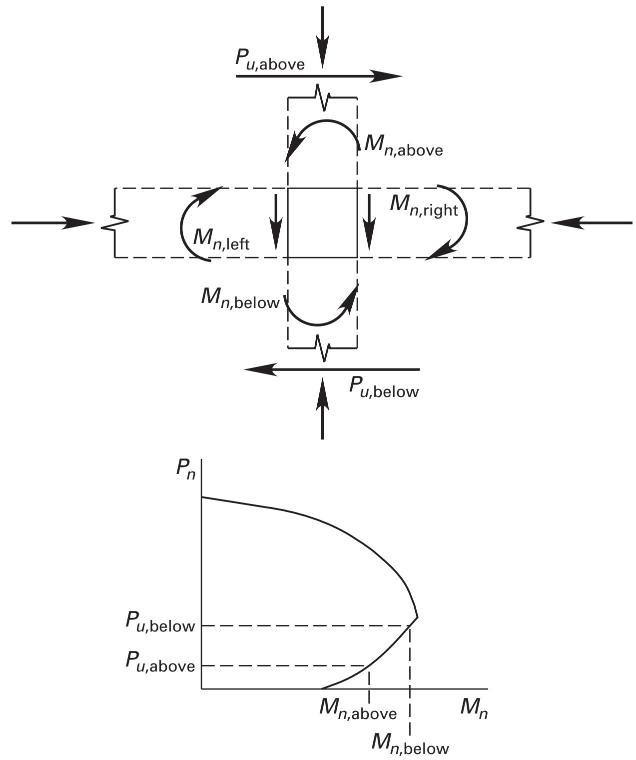


Figure 11.1 Definition of Moments Acting at Joint Faces of Special Moment Frame

ACI Sec. 21.6.3 limits the longitudinal reinforcement ratio.

$$0.01 \le \rho_g = \frac{A_{\text{st}}}{A_g} \le 0.06 \qquad 11.3$$

The following criteria govern transverse reinforcement.

- The volumetric ratio of spiral or circular hoop reinforcement shall not be less than

$$\rho_s \ge \begin{cases} \dfrac{0.12f'_c}{f_{yt}} \\ 0.45\left(\dfrac{A_g}{A_{ch}} - 1\right)\dfrac{f'_c}{f_{yt}} \end{cases} \qquad 11.4$$

- The total cross-sectional area of the rectangular hoop reinforcement shall not be less than

$$A_{sh} \ge \begin{cases} 0.3\left(\dfrac{sb_cf'_c}{f_{yt}}\right)\left(\dfrac{A_g}{A_{ch}} - 1\right) \\ 0.09\left(\dfrac{sb_cf'_c}{f_{yt}}\right) \end{cases} \qquad 11.5$$

b_c is the dimension of the hoop confining the concrete core, measured from center to center. Spacing of the transverse reinforcement must not exceed

$$s \leq \begin{cases} 0.25 \times \text{least member dimension} \\ 6d_b \quad \text{[of longitudinal steel]} \\ s_o \end{cases} \qquad 11.6$$

s_o is calculated from ACI Eq. 21-2, but may not be taken as less than 4 in or more than 6 in.

$$s_o = 4 + \frac{14 - h_x}{3} \quad [4 \text{ in} \leq s_o \leq 6 \text{ in}] \qquad 11.7$$

h_x is the maximum horizontal spacing of hoop legs or cross ties, which is limited to a maximum of 14 in. ACI Fig. R21.6.4.2 illustrates the hoop and cross ties arrangement for a rectangular column.

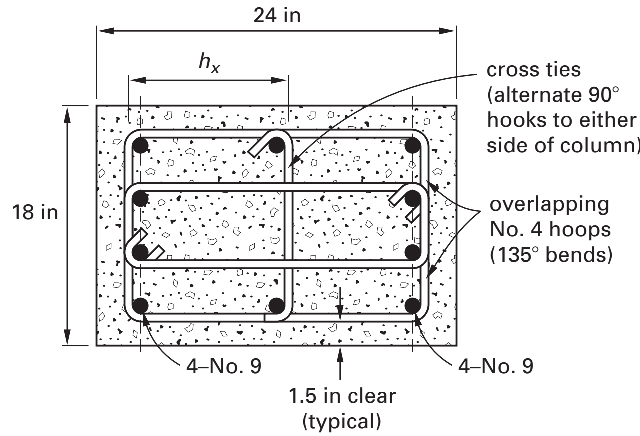
- ACI Sec. 21.6.4.1 requires that the transverse reinforcement must extend from support faces—and to both sides of other sections where plastic hinges might form—a minimum distance l_o, where

$$l_o \geq \begin{cases} \text{member depth at joint face or} \\ \quad \text{a location of likely} \\ \quad \text{plastic hinge} \\ \frac{1}{6} \times \text{member clear span} \\ 18 \text{ in} \end{cases} \qquad 11.8$$

- In regions where transverse reinforcement satisfying the above criteria is not required, spiral or hoop reinforcing is required at maximum spacing of six times the longitudinal bar diameter or 6 in.
- Columns that are not part of the lateral force resisting system must satisfy the deformation compatibility requirements of ACI Sec. 21.13. Columns supporting discontinuous stiff elements, such as shear walls, must have the transverse spirals or hoops throughout their length and the transverse reinforcement must continue into adjacent elements above and below the column for a distance equal to the tension development length of the longitudinal bars.

Example 11.2
Transverse Reinforcement for a Rectangular Column in a Special Moment Frame

The rectangular reinforced concrete column shown is part of a special moment frame in a region of high seismic risk. Concrete is normal weight with specified compressive strength of 5000 psi and steel has a specified yield stress of 60,000 psi. 10 no. 9 bars reinforce the cross section longitudinally in the pattern shown. Determine the required spacing of no. 4 hoops and cross ties in the vicinity of the joint face, and the extent of the hoops at that spacing for an unsupported column length of 10 ft.



section

Solution:

From Eq. 11.4, the total cross-sectional area of hoop legs must equal or exceed

$$A_{sh} \geq \begin{cases} 0.3\left(\dfrac{sb_cf'_c}{f_{yt}}\right)\left(\dfrac{A_g}{A_{ch}} - 1\right) \\ \quad = (0.3)\left(\dfrac{sb_c\left(5\ \dfrac{\text{kip}}{\text{in}^2}\right)}{60\ \dfrac{\text{kip}}{\text{in}^2}}\right) \\ \quad\quad \times \left(\dfrac{(18 \text{ in})(24 \text{ in})}{(15 \text{ in})(21 \text{ in})} - 1\right) \\ \quad = 0.0093sb_c \quad \text{[controls]} \\ 0.09\left(\dfrac{sb_cf'_c}{f_{yt}}\right) \\ \quad = (0.09)\left(\dfrac{sb_c\left(5\ \dfrac{\text{kip}}{\text{in}^2}\right)}{60\ \dfrac{\text{kip}}{\text{in}}^2}\right) \\ \quad = 0.0075sb_c \end{cases}$$

For the strong axis direction, with four no. 4 legs of the hoops confining the concrete,

$$\begin{aligned} b_c &= 24 \text{ in} - 3 \text{ in} - 0.5 \text{ in} \\ &= 20.5 \text{ in} \\ s &\leq \frac{A_{sh}}{0.0093b_c} \\ &= \frac{(4)(0.2 \text{ in}^2)}{(0.0093)(20.5 \text{ in})} \\ &= 4.2 \text{ in} \end{aligned}$$

In the weak direction, with two legs furnishing confinement,

$$\begin{aligned} b_c &= 18 \text{ in} - 3 \text{ in} - 0.5 \text{ in} \\ &= 14.5 \text{ in} \\ s &\le \frac{A_{sh}}{0.0093 b_c} \\ &= \frac{(3)(0.2 \text{ in}^2)}{(0.0093)(14.5 \text{ in})} \\ &= 4.4 \text{ in} \end{aligned}$$

Other spacing limits of ACI Sec. 21.6.4.2 require

$$\begin{aligned} h_x &= \frac{h}{2} - \text{cover} - d_h + \frac{d_{bl}}{2} \\ &= 12 \text{ in} - 1.5 \text{ in} - 0.5 \text{ in} + \frac{1.125 \text{ in}}{2} \\ &= 10.6 \text{ in} \end{aligned}$$

From Eq. 11.6,

$$\begin{aligned} s_o &= 4 \text{ in} + \left(\frac{14 \text{ in} - h_x}{3}\right) \\ &= 4 \text{ in} + \left(\frac{14 \text{ in} - 10.6 \text{ in}}{3}\right) \\ &= 5.1 \text{ in} \end{aligned}$$

From Eq. 11.5,

$$s \le \begin{cases} 0.25 \times \text{least member dimension} \\ \quad = (0.25)(18 \text{ in}) \\ \quad = 4.5 \text{ in} \quad \text{[controls]} \\ 6d_b = (6)(1.125 \text{ in}) \quad \text{[of longitudinal steel]} \\ \quad = 6.75 \text{ in} \\ s_o = 5.1 \text{ in} \end{cases}$$

The spacing of 4.2 in computed for the strong axis confinement controls (say, 4 in on centers). From Eq. 11.7, hoops are required at a distance from the face of the joint of

$$l_o \ge \begin{cases} \text{member depth at joint face} \\ \quad = 24 \text{ in} \quad \text{[controls]} \\ \frac{1}{6} \times \text{member clear span} \\ \quad = \left(\frac{1}{6}\right)(10 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right) \\ \quad = 20 \text{ in} \\ 18 \text{ in} \end{cases}$$

3. Joints in Special Moment Frame Members

ACI Sec. 21.7 gives requirements for the design of beam-column joints in special moment frames. Forces in reinforcement allow for overstrength using a tensile stress of $1.25f_y$. Because forces are conservatively estimated, the capacity reduction factor, ϕ, is specified to be 0.85 for shear in the joints rather than the traditional 0.75 for ordinary shear calculations.

ACI Sec. 21.7.2 and 21.7.5 specify the following requirements for longitudinal bar development (no. 3 through no. 11 bars).

- Bars that terminate must extend to the far face of the confined concrete in the core.
- For bars that continue through the joint, the width of joint parallel to the reinforcement must be greater than or equal to $20d_b$ of the largest longitudinal bar for normal weight concrete, and at least $26d_b$ for lightweight concrete.
- For 90° hooks in tension,

$$l_{dh} \ge \begin{cases} \psi\left(\frac{f_y d_b}{65\sqrt{f'_c}}\right) \\ 8\psi d_b \\ \psi(6 \text{ in}) \end{cases} \qquad 11.9$$

  The factor ψ is equal to 1.0 for normal weight concrete and 1.25 for lightweight concrete.
- For straight embedment in tension, l_d is 2.5 times the development length computed for a 90° hook if the depth of fresh concrete cast beneath the bar is less than 12 in, and is 3.25 times the 90° hook development length if the depth is 12 in or more.
- If any portion of the straight embedment length is outside the confined core, the required development length is

$$l_{dm} = 1.6l_d - 0.6l_{dcc} \qquad 11.10$$

  l_{dm} is the required development length when not fully within the core, and l_{dcc} is the length embedded in confined concrete.
- Development length for bars in compression is the same as for ordinary bars not subjected to stress reversals.

When there is confinement from framing members on four sides, and the width of every framing member is at least three-fourths the column width, the following provisions apply.

- Over a distance h of the shallowest framing member, at least one-half the hoop reinforcement required at the column end region must be provided.

- Nominal shear strength is

$$V_n = 20\sqrt{f'_c}A_j \qquad 11.11$$

When members framing in the perpendicular direction do not confine concrete within the joint faces, hoop reinforcement sized as for the column end regions must extend through the joint. When this is the case,

- For joints confined by members on three faces or on two opposite faces,

$$V_n = 15\sqrt{f'_c}A_j \qquad 11.12$$

- For other cases (for example, at corner columns),

$$V_n = 12\sqrt{f'_c}A_j \qquad 11.13$$

The term A_j is the effective area of the joint, which is the product of the overall depth of column times an effective width of

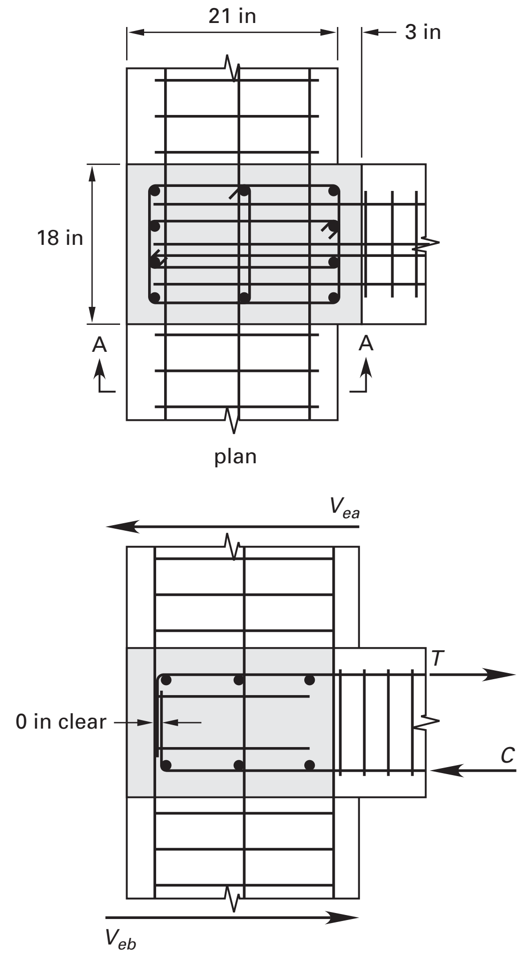
$$b_e \leq \begin{cases} b+h \\ b+2x \end{cases} \qquad 11.14$$

b is beam width, h is joint depth, and x is the shorter distance from beam edge to the edge of the column.

The criteria for nominal shear strength are for normal weight concrete. For lightweight concrete, a factor 0.75 applies to reduce the strength.

Example 11.3
Joint Reinforcement for a Rectangular Column in a Special Moment Frame

The column of Ex. 11.2 supports an 18 in wide by 20 in deep girder reinforced with four no. 8 bars top and bottom that properly develop with 90° hooks in the confined region. Beams with 21 in width confine the joint on two opposite sides as shown. Check the adequacy of the joint, ignoring the column shear caused by the lateral force. Beam is normal weight concrete with a compressive strength of 4000 psi.



section A-A
(bending moments and axial forces omitted for clarity)

Solution:
Because beams confine the joint on only three faces, no. 4 hoops and ties at 4 in centers are required through the joint, as well as over the length l_o above and below the joint, as computed in Ex. 11.2. The beam extends the full width of the column, so $b_e = b = 18$ in, and the depth of the joint in the direction considered is 24 in. From Eq. 11.11, for joints confined on three faces,

$$\begin{aligned} \phi V_n &= \phi 15\sqrt{f'_c}A_j \\ &= (0.85)(15)\sqrt{4000\ \frac{\text{lbf}}{\text{in}^2}}(18\text{ in})(24\text{ in})\left(\frac{1\text{ kip}}{1000\text{ lbf}}\right) \\ &= 348\text{ kip} \end{aligned}$$

Conservatively neglecting the shear in the column above, the shear in the joint is

$$\begin{aligned} V_u &= 1.25f_yA_s \\ &= (1.25)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)(4)(0.79\text{ in}^2) \\ &= 237\text{ kip} \\ &< \phi V_n \end{aligned}$$

The joint strength is adequate.

---

4. Special Reinforced Concrete Walls

ACI Sec. 21.9 applies to special concrete walls that are part of the lateral force resisting system of a building. ACI Sec. 21.9.2 prescribes percentages of longitudinal (that is, horizontal) and transverse (vertical) reinforcement for the wall.

$$\rho_t = \rho_l \geq 0.0025 \quad \text{[provided } V_u > A_{cv}\lambda\sqrt{f'_c}\text{]} \qquad 11.15$$

If V_u is less than or equal to $A_{cv}\lambda\sqrt{f'_c}$, the percentages of vertical and horizontal steel are calculated using the less restrictive criteria for ordinary walls in ACI Sec. 14.3.

If V_u is greater than $2A_{cv}\lambda\sqrt{f'_c}$, the wall reinforcement must be placed in two curtains. ACI Sec. 21.9.4.3 requires that if the wall height to length ratio, h_w/l_w, does not exceed 2.0 then the reinforcement ratio ρ_l must be at least ρ_t. In this ratio, h_w is the height of the entire wall that extends from base to roof, or, for wall segments pierced by openings, the height of an individual pier. ACI Sec. 21.9.2.3 contains criteria for developing and splicing the reinforcement. A_{cv} is the gross cross-sectional area computed as the product of web thickness times overall wall length, as shown in Fig. 11.2.

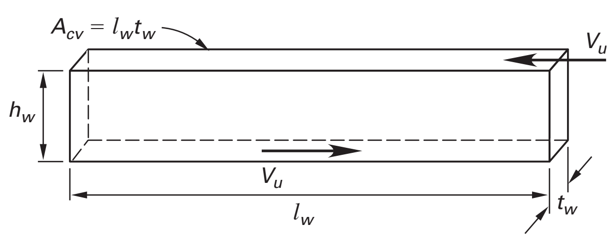


Figure 11.2 Notation for Special Wall Properties

A. Shear Strength

The design shear strength of the walls is

$$\phi V_n = \phi A_{cv}\left(\alpha_c\lambda\sqrt{f'_c} + \rho_t f_y\right) \qquad 11.16$$

In this formula,

$$\alpha_c = \begin{cases} 3.0 \quad \text{[for } \frac{h_w}{l_w} \leq 1.5\text{]} \\ 6.0 - 2.0\left(\frac{h_w}{l_w}\right) \quad \text{[for } 1.5 < \frac{h_w}{l_w} < 2.0\text{]} \\ 2.0 \quad \text{[for } \frac{h_w}{l_w} \geq 2.0\text{]} \end{cases} \qquad 11.17$$

The capacity reduction factor, ϕ, is 0.75 for ductile walls that are proportioned so the moment strength governs nominal strength. In some walls, however, it is impractical to provide sufficient shear reinforcement to prevent shear failure before flexural yielding. In these cases, ACI Sec. 9.3.4(a) imposes a lower ϕ-factor of 0.6 that applies when h_w/l_w is less than or equal to 1.5.

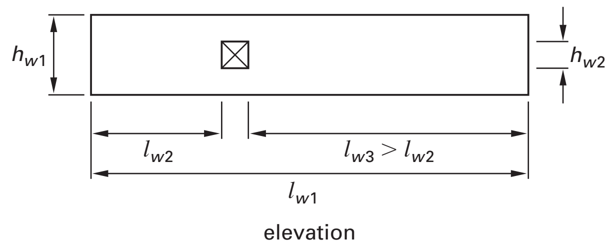


Figure 11.3 Special Wall with Opening

When one or more openings pierce a wall, the h_w/l_w ratio to be used in computing the nominal shear strength is the largest ratio for the entire wall or for the individual piers between openings. For example, for the wall shown in Fig. 11.3, with an offset opening near mid-height, shear strength is controlled by the larger h_w/l_w ratio, or $\max(h_{w1}/l_{w1}, h_{w2}/l_{w2})$. ACI Sec. 21.9.4.4 sets upper limits on wall segments.

$$V_n \leq \begin{cases} 8A_{cv}\sqrt{f'_c} \quad \text{[for the entire wall length]} \\ 10A_{cw}\sqrt{f'_c} \quad \text{[for an individual pier]} \end{cases} \qquad 11.18$$

A_{cv} is the combined area of wall segments $w1$ and $w2$. A_{cw} is the area of the individual segments $w1$ or $w2$.

B. Strength in Flexure and Axial Load

ACI Sec. 21.9.5 requires design of walls and wall segments for combined bending moment and axial load in accordance with provisions of ACI Sec. 10.2 and 10.3. Behavior is similar to a column subject to bending and axial load. For flanged regions, ACI Sec. 21.9.5.2 limits the effective flange width to the smaller of one-half the distance to adjacent webs and one-fourth the height of the wall segment.

ACI Sec. 21.9 permits either of two methods to determine whether special boundary members are required at extreme edges or around openings in the wall. ACI Sec. 21.9.6.3 provides the simpler of the two approaches, which requires boundary members if the computed index stress under factored gravity and earthquake forces exceeds $0.2f'_c$. Properties of the gross cross-sectional area and assumptions of linear elastic theory apply to calculate the index, which is not an actual stress but an easily calculated parameter for the purpose of this check. Boundary regions can terminate where the index is less than $0.15f'_c$. If boundary regions are required, ACI Sec. 21.9.6.4 requires computation of the depth from extreme compression edge to neutral axis, c, for the controlling factored load condition. The following limits apply on the extent of the boundary element:

- The region extends horizontally from extreme compression edge a distance of at least $c - 0.1l_w$ or $c/2$.
- For flanged sections, the boundary element includes the effective flange width and extends a minimum of 12 in into the web.
- Special confinement hoops are required, satisfying ACI Sec. 21.6.4.
- Special confinement hoops extend into the base support a distance equal to the development length of the largest diameter longitudinal bar in the element, except that when the base is a footing or a mat, the confinement extends only 12 in into the support.
- Horizontal reinforcement from the web must develop the yield stress, f_y, within the confined core of the boundary element and extend to within 6 in of the end of the wall.

Calculating the longitudinal steel in the boundary element precisely requires an interaction diagram that is tedious to construct. A reasonable estimate of the steel required can be made with an approximate method. Assume that

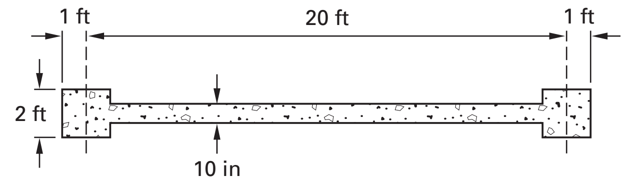
- the center of compression is at the centroid of the boundary element
- tension resistance from the vertical web steel is at the midlength of the wall
- tension resistance from the opposite side boundary element is at its centroid

Taking moments about the assumed center of compression gives the required area of tension steel in the opposite boundary element. The steel areas thus obtained can be used to construct an interaction diagram to validate the design.

Example 11.4
Longitudinal and Shear Reinforcement in a Special Shear Wall

The shear wall shown in cross section below is part of the lateral force resisting system in a high-rise building located in a region of high seismic risk. Concrete is normal weight with a compressive strength of 5000 psi; reinforcing steel has a specified yield stress of 60,000 psi. The overall wall height is 195 ft and the controlling factored load case at the base $(0.9D + 1.0E)$ subjects the wall to an axial compression force of 750 kip, an overturning moment of 28,000 ft-kip, and seismic shear of 450 kip. Determine

A. the vertical and longitudinal web reinforcement
B. the longitudinal steel required in the boundary members
C. the horizontal extent of boundary elements, if required



section at base

Solution:

A. Try minimum vertical and horizontal web reinforcement.

$$\begin{aligned} A_{cv} &= l_w t \\ &= (22\ ft)\left(12\ \frac{in}{ft}\right)(10\ in) \\ &= 2640\ in^2 \\ A_{sl} &= A_{st} = \rho_{min} t_w \\ &= (0.0025)\left(12\ \frac{in}{ft}\right)(10\ in) \\ &= 0.30\ \text{in}^2/ft \end{aligned}$$

Check the seismic shear.

$$\begin{aligned} limit &= A_{cv}\sqrt{f'_c} \\ &= 2640\ in^2\sqrt{5000\ \frac{lbf}{in^2}}\left(\frac{1\ kip}{1000\ lbf}\right) \\ &= 187\ kip \\ V_u &= 450\ kip > 2 \times limit \end{aligned}$$

Therefore, provide two curtains of web reinforcement furnishing 0.15 in^2/ft in each face; say no. 4 bars at 16 in on centers each way (satisfying the maximum 18 in spacing limit). The height-to-length ratio of the wall is

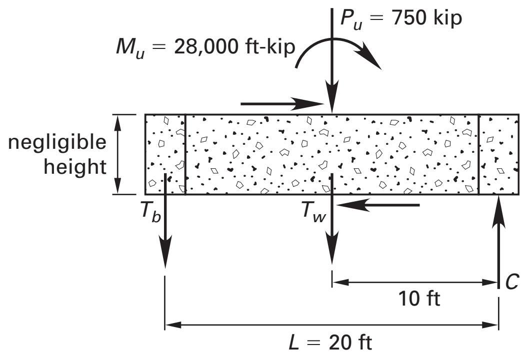
$$\begin{aligned} \frac{h_w}{l_w} &= \frac{195\ ft}{22\ ft} = 8.8 \\ &> 2.0 \end{aligned}$$

Therefore, from Eq. 11.16, α_c is 2.0.

$$\begin{aligned} \phi V_n &= \phi A_{cv}(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y) \\ &= (0.75)(2640\ in^2) \\ &\quad \times \left(\begin{array}{c} (2.0)(1.0)\sqrt{5000\ \frac{lbf}{in^2}} + (0.0025) \\ \times \left(60{,}000\ \frac{lbf}{in^2}\right) \end{array} \right) \\ &\quad \times \left(\frac{1\ kip}{1000\ lbf}\right) \\ &= 577\ kip \end{aligned}$$

Thus, V_u which is 450 kip, is less than ϕV_n, and 10 in web thickness with minimum reinforcement has adequate shear strength.

B. Calculate the longitudinal steel in each boundary element by the approximate method previously outlined. Given that the controlling case is $0.9D + 1.0E$, assume the longitudinal steel is sufficient for the gravity case $1.2D + 1.6L$.



Take moments about the line of action of C.

$$\begin{aligned} T_w &= A_{sw}f_y \\ &= \left(0.30\ \frac{in^2}{ft}\right)(18\ ft)\left(60\ \frac{kip}{in^2}\right) \\ &= 324\ kip \end{aligned}$$

$$\sum M_r = M_u - \frac{P_uL}{2} - \phi T_w\left(\frac{L}{2}\right) - \phi A_{st}f_yL = 0$$

$$\begin{aligned} A_{st} &= \frac{M_u - \frac{P_uL}{2} - \phi T_w\left(\frac{L}{2}\right)}{\phi f_y L} \\ &= \frac{\left(\begin{array}{c} 28{,}000\ \text{ft-kip} - \frac{(750\ kip)(20\ ft)}{2} \\ -\ (0.9)(324\ kip)\left(\frac{20\ ft}{2}\right) \end{array}\right)}{(0.9)\left(60\ \frac{kip}{in^2}\right)(20\ ft)} \\ &= 16.30\ in^2 \end{aligned}$$

Try 12 no. 10 bars in each boundary element.

$$A_{st} = (12)(1.27\ in^2) = 15.24\ in^2$$

A detailed check of the trial design by strain compatibility, omitted in this presentation, shows that the neutral axis depth falls within the compression side boundary element (that is, $c < 24\ in$) and that the 12 no. 10 bars are adequate.

C. Check boundary element criteria of ACI Sec. 21.9.6.3. The properties of the cross section for an elastic analysis are

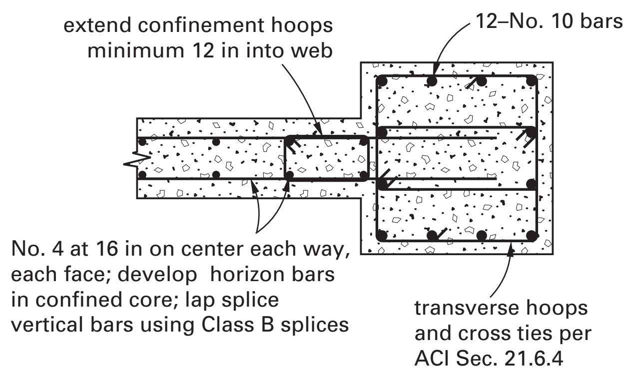
$$\begin{aligned} A_g &= bl_w - (b - t_w)(l_w - 2b) \\ &= (24\ in)(264\ in) - (24\ in - 10\ in) \\ &\quad \times (264\ in - (2)(24\ in)) \\ &= 3312\ in^2 \end{aligned}$$

$$\begin{aligned} I_g &= \frac{bl_w^3}{12} - \frac{(b - t_w)(l_w - 2b)^3}{12} \\ &= \frac{(24\ in)(264\ in)^3}{12} \\ &\quad - \frac{(24\ in - 10\ in)(264\ in - (2)(24\ in))^3}{12} \\ &= 25{,}000{,}000\ in^4 \end{aligned}$$

$$\begin{aligned} S_g &= \frac{2I_g}{l_w} \\ &= \frac{(2)(25{,}000{,}000\ in^4)}{264\ in} \\ &= 189{,}000\ in^3 \end{aligned}$$

$$\begin{aligned} f_c &= \frac{P_u}{A_g} + \frac{M_u}{S_g} \\ &= \frac{750\ kip}{3312\ in^2} + \frac{(28{,}000\ \text{ft-kip})\left(12\ \frac{in}{ft}\right)}{189{,}000\ in^3} \\ &= 2.0\ ksi \\ &> 0.2f_c' \end{aligned}$$

Special boundary elements are required. Because the depth of neutral axis falls in the flanged portion, ACI Sec. 21.9.6.4 controls, which requires extending the confinement steel to within 6 in of the end of the wall.



5. Reinforced Concrete Structural Diaphragms

Reinforced concrete slabs frequently serve as rigid diaphragms to help transfer lateral forces from wind or earthquake loads into parts of the lateral force resisting system. In regions of high seismic risk, the diaphragms must satisfy the design and detailing requirements of ACI Sec. 21.11. Figure 11.4 shows the basic elements of a rigid diaphragm transferring lateral loads acting in the north-south direction on a floor system.

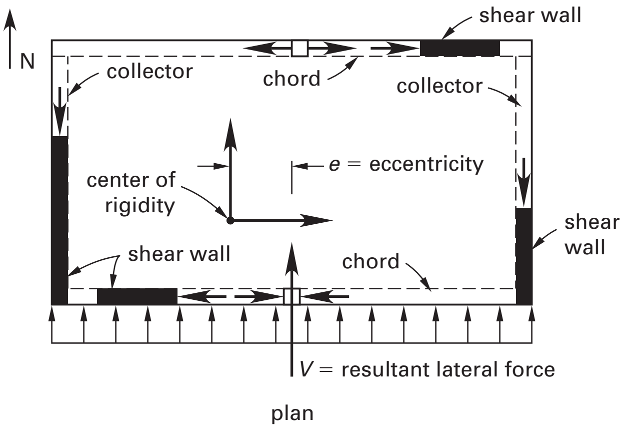


Figure 11.4 Components of a Typical Rigid Diaphragm

ACI Sec. 21.11.6 sets a minimum thickness of 2 in for diaphragms and requires a minimum reinforcement equivalent to that required for temperature and shrinkage in ACI Sec. 7.12. Elements of the diaphragm that have a calculated compressive stress exceeding $0.2f'_c$ at any point must have transverse confinement steel equivalent to the amount required for a compression member in ACI Sec. 21.9.6.4, and the transverse reinforcement must extend over the regions of the element for which compressive stresses exceed $0.15f'_c$. Stresses for this purpose are approximated using linear elastic theory based on the gross area of the element.

ACI Sec. 21.11.9 gives the shear strength of a diaphragm as

$$V_n = A_{cv}\left(2\lambda\sqrt{f'_c} + \rho_t f_y\right) \qquad 11.19$$

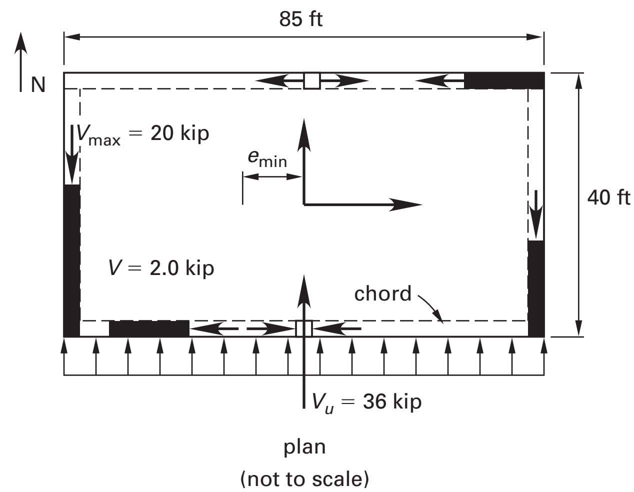
A_{cv} is the gross cross-sectional area resisting shear.

Some shear walls in low-rise buildings have height-to-length ratios such that it is not practical to provide sufficient shear reinforcement to ensure a flexural failure. For these cases, ACI Sec. 9.3.4(a) requires a ϕ-factor of 0.6, and the same factor applies to the diaphragm transferring shear to the walls. For other cases, a ϕ-factor of 0.75 applies. The maximum shear strength is limited to $8\sqrt{f'_c}A_{cv}$.

Shear is assumed to be uniformly distributed along the interfaces between the diaphragm and the shear walls and collectors. A simple model gives the chord forces by dividing the in-plane bending moment, M_u, by the distance between chords, and adding other axial forces that may be present.

Example 11.5
Shear and Chord Forces in a Rigid Diaphragm

The concrete rigid diaphragm shown is 6 in thick reinforced concrete containing at least minimum temperature and shrinkage steel in both directions. Concrete is normal weight with a compressive strength of 3000 psi; reinforcing steel has a yield stress of 60,000 psi. The diaphragm is laterally loaded with an earthquake force of 36 kip. Shear walls have rigidities such that, under code-imposed minimum eccentricity, the maximum shear in a north-south wall is 20 kip and the corresponding shears in east-west walls are 2 kip. Assume a high-rise building for which $\phi = 0.75$ applies to shear strength, and that the lateral load is the design load. Check shear in the diaphragm and forces in the diaphragm chords. Determine whether ACI 318 requires special confinement steel for the chords.



Solution:

For the 6 in diaphragm in the north-south direction, take $\rho_t = \rho_{min} = 0.0018$.

$$\begin{aligned}
A_{cv} &= Bh = (40\ ft)\left(12\ \frac{in}{ft}\right)(6\ in) \\
&= 2880\ in^2 \\
\phi V_n &= \phi A_{cv}\left(2\lambda\sqrt{f'_c} + \rho_t f_y\right) \\
&= (0.75)(2880\ in^2) \\
&\quad \times \left((2)(1)\sqrt{3000\ \frac{lbf}{in^2}} + (0.0018)\left(60{,}000\ \frac{lbf}{in^2}\right)\right) \\
&= 470{,}000\ lbf \\
V_u &= 20{,}000\ lbf \\
\phi V_n &> V_u
\end{aligned}$$

Shear strength is adequate.

For 36 kip distributed over the floor length and creating a maximum shear of 20 kip on the left shear wall, calculate the point of zero shear (that is, maximum moment) in the diaphragm.

$$\begin{aligned} x &= \left(\frac{V_{max}}{V_u}\right) L \\ &= \left(\frac{20\ kip}{36\ kip}\right)(85\ ft) \\ &= 47.2\ ft \\ M_u &= 0.5 V_{max} x \\ &= (0.5)(20\ kip)(47.2\ ft) \\ &= 472 \text{ ft-kip} \end{aligned}$$

The force in the chords due to the factored moment M_u is

$$\begin{aligned} C = T &= \frac{M_u}{B} = \frac{472 \text{ ft-kip}}{40\ ft} \\ &= 11.8\ kip \end{aligned}$$

The maximum chord force includes the forces along the collector in the east-west direction under the loading shown. Since each east-west wall resists 2 kip, the collector force at the point of maximum bending moment is approximately 1 kip; thus, the total chord force is approximately 12.8 kip.

Check whether the chords require special confinement steel. The index stress is

$$\begin{aligned} S_g &= \frac{hB^2}{6} \\ &= \frac{(6\ in)\left((40\ ft)\left(12\ \frac{in}{ft}\right)\right)^2}{6} \\ &= 230{,}000\ in^3 \\ f_c &= \frac{M_u}{S_g} \\ &= \frac{(472{,}000 \text{ ft-lbf})\left(12\ \frac{in}{ft}\right)}{230{,}000\ in^3} \\ &= 25\ psi \\ &< 0.2 f'_c \end{aligned}$$

Therefore, the chords do not require special confinement steel.

12

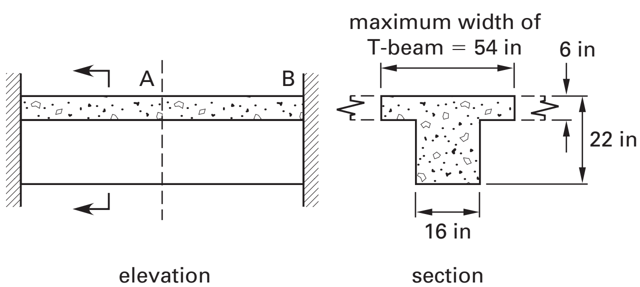
Practice Problems

Civil PE and SE Breadth Exam Problems

Problems 1 through 25 cover subjects likely to be found on both the structural depth section of the civil PE exam and the breadth sections of the SE exam.

Practice Problem 1

The beam shown is not part of a ductile moment-resisting frame. The service load moments at two locations are given. No redistribution of the negative moment is necessary.



The area of flexural steel at location is A is most nearly

(A) 1.0 in^2
(B) 2.2 in^2
(C) 3.3 in^2
(D) 4.4 in^2

Design Criteria

- concrete uses lightweight aggregates
- splitting tensile strength = 0.274 ksi
- $f'_c = 3$ ksi
- $f_y = 60$ ksi
- distance from face of concrete to center of gravity of steel is 2½ in
- for the dead load at A: M = 140 ft-kip (tension at bottom), V = 0 kip
- for the live load at A: M = 70 ft-kip, V = 9 kip
- for the dead load at B: M = −200 ft-kip (tension at top), V = 36 kip
- for the live load at B: M = −100 ft-kip, V = 18 kip

Solution

The effective depth of the beam is

$$\begin{aligned} d &= h - 2.5 \text{ in} = 22 \text{ in} - 2.5 \text{ in} \\ &= 19.5 \text{ in} \end{aligned}$$

The moment at A produces compression on the top, so

$$b = b_e = 54 \text{ in}$$

The factored bending moment is

$$M_u \geq \begin{cases} 1.2M_d + 1.6M_l \\ \quad = (1.2)(140 \text{ ft-kip}) + (1.6)(70 \text{ ft-kip}) \\ \quad = 280 \text{ ft-kip} \quad \text{[controls]} \\ 1.4M_d \\ \quad = (1.4)(140 \text{ ft-kip}) \\ \quad = 196 \text{ ft-kip} \end{cases}$$

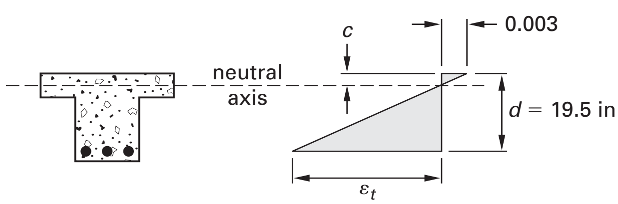
Assuming a tensioned-controlled flexural failure, that ϕ equals 0.9, and that the compression zone is within the flange, the area of the flexural steel can be calculated using

$$\begin{aligned} M_u &= \phi A_s f_y \left(d - \frac{A_s f_y}{(2)(0.85) f'_c b} \right) \\ (280 \text{ ft-kip}) \left(12 \ \frac{\text{in}}{\text{ft}} \right) &= 0.9 A_s \left(60 \ \frac{\text{kip}}{\text{in}^2} \right) \\ &\quad \times \left(19.5 \text{ in} - \frac{A_s \left(60 \ \frac{\text{kip}}{\text{in}^2} \right)}{(1.7) \left(3 \ \frac{\text{kip}}{\text{in}^2} \right) (54 \text{ in})} \right) \end{aligned}$$

Solving the quadratic equation gives $A_s = 3.31 \text{ in}^2$. The depth of the compression zone is

$$\begin{aligned} a &= \frac{A_s f_y}{0.85 f'_c b} \\ &= \frac{(3.31 \text{ in}^2) \left(60 \ \frac{\text{kip}}{\text{in}^2} \right)}{(0.85) \left(3 \ \frac{\text{kip}}{\text{in}^2} \right) (54 \text{ in})} \\ &= 1.44 \text{ in} \end{aligned}$$

The compression zone is within the 6 in flange thickness as assumed. Check the strain in the tension steel.



The depth to the neutral axis is

$$c = \frac{a}{\beta_1} = \frac{1.44 \text{ in}}{0.85} = 1.69 \text{ in}$$

From similar triangles,

$$\begin{aligned} \frac{\varepsilon_t}{d - c} &= \frac{0.003}{c} \\ \varepsilon_t &= \frac{0.003(d - c)}{c} \\ &= \frac{(0.003)(19.5 \text{ in} - 1.69 \text{ in})}{1.69 \text{ in}} \\ &= 0.032 \end{aligned}$$

Because the steel strain at design strength exceeds 0.005, the section is tension controlled and ϕ equals 0.9 as assumed. The minimum steel required is

$$\begin{aligned} A_{s,\min} &= \left(\frac{200}{f_y} \right) b_w d \\ &= \left(\frac{200 \ \frac{\text{lbf}}{\text{in}^2}}{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}} \right) (16 \text{ in})(19.5 \text{ in}) \\ &= 1.04 \text{ in}^2 \end{aligned}$$

Thus, the calculated $A_s = 3.31 \text{ in}^2$ controls.

The answer is (C).

Practice Problem 2

For the beam shown in Prob. 1, the service load shears at d-distance from the face of support B are as given in the following design criteria.

If no. 3 U-stirrups are used, the required minimum spacing of stirrups at location B is most nearly

(A) 3.7 in
(B) 10 in
(C) 16 in
(D) 20 in

Design Criteria

- concrete uses all-lightweight aggregates
- $f'_c = 3$ ksi
- $f_y = 60$ ksi
- distance from face of concrete to center of gravity of steel is 2½ in
- for the dead load at B, $V = 36$ kip
- for the live load at B, $V = 18$ kip

Solution

The factored shear is

$$V_u \geq \begin{cases} 1.2V_d + 1.6V_l \\ \quad = (1.2)(36 \text{ kip}) + (1.6)(18 \text{ kip}) \\ \quad = 72 \text{ kip} \quad \text{[controls]} \\ 1.4V_d \\ \quad = (1.4)(36 \text{ kip}) \\ \quad = 50.4 \text{ kip} \end{cases}$$

For no. 3 U-stirrups,

$$A_v = 2A_b = (2)(0.11\ \text{in}^2) = 0.22\ \text{in}^2$$

For lightweight concrete, when the splitting tensile strength is specified, the concrete's contribution to shear resistance is

$$\begin{aligned} V_c &= 2\lambda\sqrt{f'_c}b_w d \\ &= (2)(0.75)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}(16\ \text{in})(19.5\ \text{in}) \\ &= \frac{25{,}633\ \text{lbf}}{1000\ \frac{\text{lbf}}{\text{kip}}} \\ &= 25.6\ \text{kip} \\ V_s &= \frac{V_u}{\phi} - V_c \\ &= \frac{72\ \text{kip}}{0.75} - 25.6\ \text{kip} \\ &= 70.4\ \text{kip} \end{aligned}$$

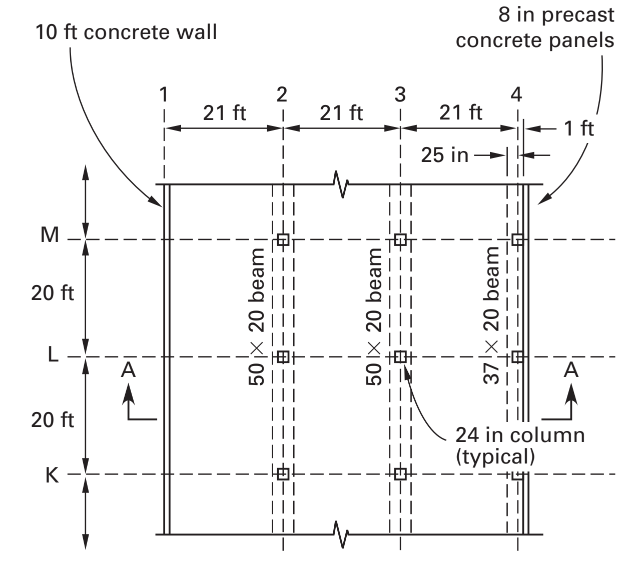
The spacing limits for the stirrups give

$$s \le \begin{cases} \dfrac{d}{2}\ \text{or}\ 24\ \text{in} = \dfrac{19.5\ \text{in}}{2} = 9.8\ \text{in} \\ \dfrac{A_v f_y}{50 b_w} = \dfrac{(0.22\ \text{in}^2)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{\left(50\ \dfrac{\text{lbf}}{\text{in}^2}\right)(16\ \text{in})} = 16.5\ \text{in} \\ \dfrac{A_v f_y d}{V_s} = \dfrac{(0.22\ \text{in}^2)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)(19.5\ \text{in})}{(70.4\ \text{kip})\left(1000\ \dfrac{\text{lbf}}{\text{kip}}\right)} \\ \quad = 3.7\ \text{in}\quad [\text{controls}] \end{cases}$$

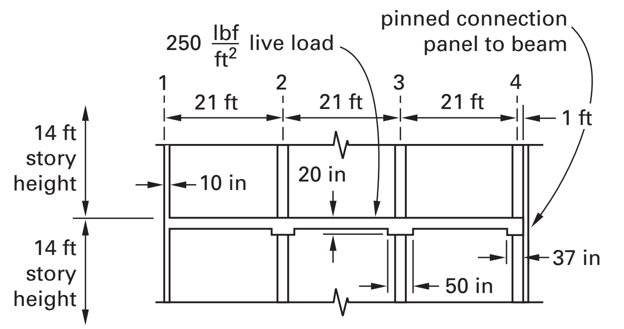
The answer is (A).

Practice Problem 3

A partial plan and a cross section of a typical floor for a multistory building are shown. The columns are 24 in square and the beams are as shown in section A-A. Assume that the beams and columns are adequate and that the fire rating does not control slab thickness.



partial floor plan



section A-A

For a one-way slab, the minimum slab thickness required to satisfy serviceability when deflections are not calculated is most nearly

(A) 6.9 in
(B) 7.4 in
(C) 8.0 in
(D) 8.5 in

Design Criteria

- $f'_c = 4$ ksi lightweight concrete with unit weight of $110\ \text{lbf/ft}^3$
- $f_y = 40$ ksi

Solution

Table 9.5(a) of ACI 318 gives the minimum slab thickness, h, in terms of the clear spans, which are the distances between supports from face to face. Because span 1–2 is integral with a 10 in wall at line 1, it is reasonable to consider that span as continuous at both ends. Span 2–3 is continuous at both ends. Span 3–4 is integral with a spandrel beam that offers relatively little restraint to rotation, and that span is therefore continuous at one end only. The clear spans are

$$\begin{aligned} l_{1-2} &= 21\ \text{ft} - \frac{10\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} - \frac{25\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 18.1\ \text{ft} \\ l_{2-3} &= 21\ \text{ft} - \frac{(2)(25\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 16.8\ \text{ft} \\ l_{3-4} &= 21\ \text{ft} - \frac{25\ \text{in} + (37\ \text{in} - 12\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 16.8\ \text{ft} \end{aligned}$$

The thickness is controlled either by the longer span with both ends continuous, 18.1 ft, or the 16.8 ft span that has only one end continuous. Because lightweight concrete is specified, according to the footnote to ACI Table 9.5(a), span limits are multiplied by the factor

$$\lambda_{lw} \geq \begin{cases} 1.65 - 0.005w_c \\ \quad = 1.65 - \left(0.005\ \frac{\text{ft}^3}{\text{lbf}}\right)\left(110\ \frac{\text{lbf}}{\text{ft}^3}\right) \\ \quad = 1.10 \quad \text{[controls]} \\ 1.09 \end{cases}$$

Because the steel is something other than grade 60, a factor is applied to account for the added stiffness that results by designing using $f_y = 40$ ksi.

$$\begin{aligned} \lambda_{40} &= 0.4 + \frac{f_y}{100{,}000} = 0.4 + \frac{40{,}000\ \frac{\text{lbf}}{\text{in}^2}}{100{,}000\ \frac{\text{lbf}}{\text{in}^2}} \\ &= 0.8 \end{aligned}$$

From ACI Table 9.5(a), the minimum thickness required is

$$h \geq \begin{cases} \dfrac{\lambda_{lw}\lambda_{40}l_{1-2}}{28} \\ \quad = \dfrac{(1.10)(0.8)(18.1\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{28} \\ \quad = 6.8\ \text{in} \\ \dfrac{\lambda_{lw}\lambda_{40}l_{3-4}}{24} \\ \quad = \dfrac{(1.10)(0.8)(16.8\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{24} \\ \quad = 7.4\ \text{in} \quad \text{[controls]} \end{cases}$$

The answer is (B).

Practice Problem 4

A partial plan and a cross section of a typical floor for a multistory building are shown in the illustration for Prob. 3. The columns are 24 in square and the beams are as shown in section A-A. Assume that the beams and columns are adequate. A solid one-way slab with an overall depth of 7½ in is used, which supports its self-weight plus a service live load of 150 lbf/ft^2 (nonreducible).

The steel area in in^2/ft in the slab over beam line 2 is most nearly

(A) 0.18 in^2/ft
(B) 0.32 in^2/ft
(C) 0.44 in^2/ft
(D) 0.53 in^2/ft

Design Criteria

- $f'_c = 4$ ksi
- $f_y = 40$ ksi reinforcing steel
- lightweight concrete with unit weight of 110 lbf/ft^3
- minimum cover for interior exposure

Solution

The flexural steel over beam line 3 is controlled by the average of adjacent clear spans. Because span 1–2 is integral with a 10 in wall at line 1, it is reasonable to consider that span as continuous at both ends. Span 2–3 is continuous at both ends. The clear spans are

$$\begin{aligned} l_{1-2} &= 21\ \text{ft} - \frac{10\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} - \frac{25\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 18.1\ \text{ft} \\ l_{2-3} &= 21\ \text{ft} - \frac{(2)(25\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 16.8\ \text{ft} \\ l_n &= \frac{l_{1-2} + l_{2-3}}{2} = \frac{18.1\ \text{ft} + 16.8\ \text{ft}}{2} \\ &= 17.5\ \text{ft} \end{aligned}$$

A 1 ft width of slab is used for design. For a 7.5 in slab of lightweight concrete,

$$\begin{aligned} w_d &= wbt \\ &= \left(110\ \frac{\text{lbf}}{\text{ft}^3}\right)(1\ \text{ft})\left(\frac{7.5\text{in}}{12\ \frac{\text{in}}{\text{ft}}}\right) \\ &= 69\ \text{lbf/ft} \end{aligned}$$

The factored load is

$$w_u \geq \begin{cases} 1.2w_d + 1.6w_l \\ \quad = (1.2)\left(69 \ \dfrac{\text{lbf}}{\text{ft}}\right) + (1.6)\left(150 \ \dfrac{\text{lbf}}{\text{ft}}\right) \\ \quad = 323 \text{ lbf/ft} \quad \text{[controls]} \\ 1.4w_d \\ \quad = (1.4)\left(69 \ \dfrac{\text{lbf}}{\text{ft}}\right) = 97 \text{ lbf/ft} \end{cases}$$

The controlling negative moment is at the exterior face of the first interior support for more than two spans.

$$\begin{aligned} M_u &= \frac{w_u l_n^2}{10} \\ &= \left(\frac{\left(323 \ \dfrac{\text{lbf}}{\text{ft}}\right)(17.5 \text{ ft})^2}{10}\right)\left(\frac{1 \text{ kip}}{1000 \text{ lbf}}\right) \\ &= 9.89 \text{ ft-kip} \end{aligned}$$

Minimum slab cover specified by ACI is 0.75 in; the estimated additional distance to the center of gravity of steel in a one-way slab is 0.25 in. Therefore,

$$\begin{aligned} d &= h - (0.75 \text{ in} + 0.25 \text{ in}) = 7.5 \text{ in} - 1.0 \text{ in} \\ &= 6.5 \text{ in} \end{aligned}$$

The required area of grade 40 steel to resist bending is

$$\begin{aligned} M_u &= \phi M_n = \phi A_s f_y \left(d - \frac{A_s f_y}{(2)(0.85) f_c' b}\right) \\ &(9.89 \text{ ft-kip})\left(12 \ \frac{\text{in}}{\text{ft}}\right) \\ &\quad = 0.9 A_s \left(40 \ \frac{\text{kip}}{\text{in}^2}\right) \\ &\quad\quad \times \left(6.5 \text{ in} - \frac{A_s \left(40 \ \dfrac{\text{kip}}{\text{in}^2}\right)}{(1.7)\left(4.0 \ \dfrac{\text{kip}}{\text{in}^2}\right)(12 \text{ in})}\right) \\ A_s &= 0.53 \text{ in}^2/\text{ft} \end{aligned}$$

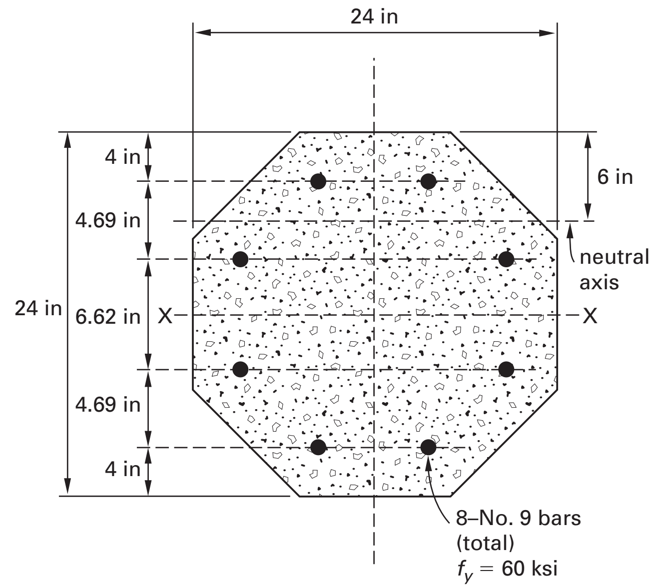
The minimum steel required for temperature and shrinkage reinforcement is

$$\begin{aligned} A_{s,\min} &= 0.002bh \\ &= (0.002)\left(12 \ \frac{\text{in}}{\text{ft}}\right)(7.5 \text{ in}) \\ &= 0.18 \text{ in}^2/\text{ft} \end{aligned}$$

The answer is (D).

Practice Problem 5

A reinforced concrete pile shown is subjected to bending about the X-X axis. The axial load is zero.



The neutral axis is 6 in below the top edge. The moment strength of the section is most nearly

(A) 280 ft-kip
(B) 320 ft-kip
(C) 420 ft-kip
(D) 660 ft-kip

Design Criteria

- $f_c' = 4$ ksi at 28 days hard rock concrete
- ASTM A615 grade 60 longitudinal reinforcing steel

Solution

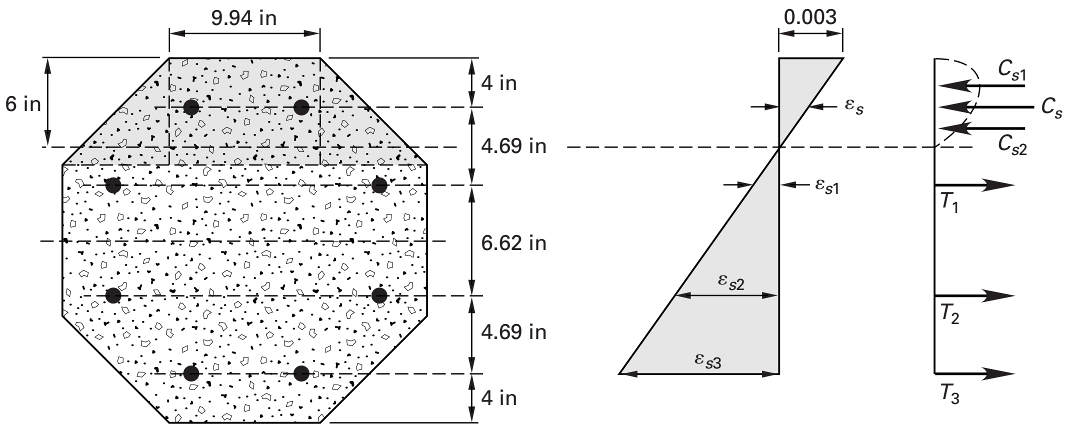
Break the compression region into two right triangular regions and a rectangular region as shown. The base length of the triangles is the same as the depth of the compression zone.

$$\begin{aligned} a &= \beta_1 c = (0.85)(6 \text{ in}) \\ &= 5.1 \text{ in} \end{aligned}$$

Let C_{c1} denote the resultant of the two triangular regions.

$$\begin{aligned} C_{c1} &= 0.85 f_c' A_c \\ &= (0.85)\left(4.0 \ \frac{\text{kip}}{\text{in}^2}\right)(2)\left(\frac{5.1 \text{ in}}{2}\right)(5.1 \text{ in}) \\ &= 88 \text{ kip} \end{aligned}$$

Illustration for Solution 5



The force C_{c1} acts at the centroid of the triangles, which from the top downward is

$$a_1 = \frac{2a}{3} = \frac{(2)(5.1 \text{ in})}{3} = 3.4 \text{ in}$$

The resultant compression in the remaining region is

$$\begin{aligned} C_{c2} &= 0.85 f'_c ba \\ &= (0.85)\left(4.0 \ \frac{\text{kip}}{\text{in}^2}\right)(9.94 \text{ in})(5.1 \text{ in}) \\ &= 172 \text{ kip} \end{aligned}$$

Compute the stresses in the reinforcing steel. For the grade 60 steel, the yield strain is 0.00207. Strains vary linearly, so, using similar triangles,

$$\begin{aligned} \frac{\varepsilon'_s}{6.0 \text{ in} - 4.0 \text{ in}} &= \frac{0.003}{6.0 \text{ in}} \\ \varepsilon'_s &= 0.001 \\ &< \varepsilon_y = 0.00207 \\ f'_s &= E_s \varepsilon' = \left(29{,}000 \ \frac{\text{kip}}{\text{in}^2}\right)(0.001) \\ &= 29 \text{ ksi} \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{\varepsilon_{s1}}{4.0 \text{ in} + 4.69 \text{ in} - 6.0 \text{ in}} &= \frac{0.003}{6.0 \text{ in}} \\ \varepsilon_{s1} &= 0.00135 \\ &< \varepsilon_y = 0.00207 \\ f_{s1} &= E_s \varepsilon_{s1} \\ &= \left(29{,}000 \ \frac{\text{kip}}{\text{in}^2}\right)(0.00135) \\ &= 39 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \frac{\varepsilon_{s2}}{4.0 \text{ in} + 4.69 \text{ in} + 6.62 \text{ in} - 6.0 \text{ in}} &= \frac{0.003}{6.0 \text{ in}} \\ \varepsilon_{s2} &= 0.00465 > \varepsilon_y \\ f_{s2} &= f_y \\ &= 60 \text{ ksi} \end{aligned}$$

Because ε_{s3} is greater than ε_{s2}, the stress in the lowest steel is also the yield stress. The remaining stress resultants are

$$\begin{aligned} C'_s &= (f'_s - 0.85 f'_c) A'_s \\ &= \left(29 \ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(4.0 \ \frac{\text{kip}}{\text{in}^2}\right)\right)(2.00 \text{ in}^2) \\ &= 51 \text{ kip} \end{aligned}$$

$$\begin{aligned} T_1 &= f_{s1} A_s = \left(39 \ \frac{\text{kip}}{\text{in}^2}\right)(2.00 \text{ in}^2) \\ &= 78 \text{ kip} \end{aligned}$$

$$\begin{aligned} T_2 = T_3 &= f_y A_s = \left(60 \ \frac{\text{kip}}{\text{in}^2}\right)(2.00 \text{ in}^2) \\ &= 120 \text{ kip} \end{aligned}$$

Take moments about the bottom tension force.

$$\begin{aligned}
M_n &= \sum (\text{force})(\text{lever arm}) \\
&= C_{c1}(d - a_1) + C_{c2}\left(d - \frac{a}{2}\right) + C_s'(d - d') \\
&\quad - T_1(d - 8.69\text{ in}) - T_2(d - 15.31\text{ in}) \\
&= (88\text{ kip})(20\text{ in} - 3.4\text{ in}) \\
&\quad + (172\text{ kip})\left(20\text{ in} - \frac{5.1\text{ in}}{2}\right) \\
&\quad + (51\text{ kip})(20\text{ in} - 4\text{ in}) \\
&\quad - (78\text{ kip})(20\text{ in} - 8.69\text{ in}) \\
&\quad - (120\text{ kip})(20\text{ in} - 15.31\text{ in}) \\
&= (3833\text{ in-kip})\left(\frac{1\text{ ft}}{12\text{ in}}\right) \\
&= 319\text{ ft-kip} \quad (320\text{ ft-kip})
\end{aligned}$$

The answer is (B).

Practice Problem 6

The existing 16 in by 36 in concrete beam shown supports a uniform service dead load of 1500 lbf per linear foot, including the weight of the beam.

Neglect any torsion consideration and use the simpler expression for shear resisted by concrete. Based on the shear capacity of the beam as indicated by the illustration and these criteria, the uniform service live load that can be supported is most nearly

(A) 4.8 kip/ft
(B) 6.2 kip/ft
(C) 8.6 kip/ft
(D) 11 kip/ft

Design Criteria

- $f_c' = 3$ ksi, normal weight concrete
- $f_y = 60$ ksi
- $d = 33.5$ in

Solution

The shear capacity of the concrete is

$$\begin{aligned}
V_c &= 2\lambda\sqrt{f_c'}b_w d \\
&= (2)(1)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}(16\text{ in})(33.5\text{ in})\left(\frac{1\text{ kip}}{1000\text{ lbf}}\right) \\
&= 58.7\text{ kip}
\end{aligned}$$

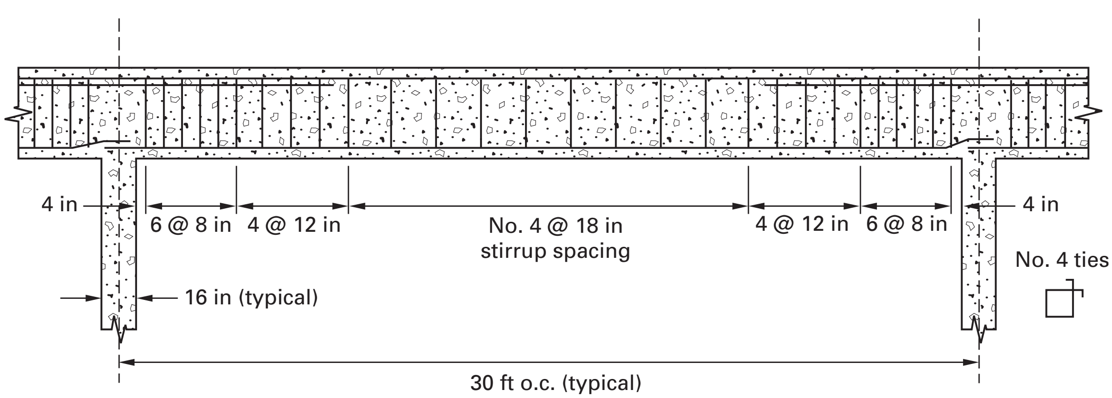
For the no. 4 stirrups, A_v equals 0.40 in^2. For the region where stirrups are spaced 8 in on centers,

$$\begin{aligned}
V_s &= \frac{A_v f_y d}{s} \\
&= \frac{(0.40\text{ in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)(33.5\text{ in})}{8\text{ in}} \\
&= 100.5\text{ kip}
\end{aligned}$$

Where the stirrups are spaced 12 in on centers,

$$\begin{aligned}
V_s &= \frac{A_v f_y d}{s} \\
&= \frac{(0.40\text{ in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)(33.5\text{ in})}{12\text{ in}} \\
&= 67.0\text{ kip}
\end{aligned}$$

Illustration for Practice Problem 6



For the 18 in stirrup spacing,

$$\begin{aligned} V_s &= \frac{A_v f_y d}{s} \\ &= \frac{(0.40 \text{ in}^2)\left(60 \ \frac{\text{kip}}{\text{in}^2}\right)(33.5 \text{ in})}{18 \text{ in}} \\ &= 44.7 \text{ kip} \end{aligned}$$

The clear span of the typical interior span is

$$l_n = 30 \text{ ft} - \frac{16 \text{ in}}{12 \ \frac{\text{in}}{\text{ft}}} = 28.7 \text{ ft}$$

The critical region near supports is at d-distance from face of supports where the factored shear is

$$\begin{aligned} V_u &= w_u \left(\frac{l_n}{2} - d\right) \\ &= w_u \left(\frac{28.7 \text{ ft}}{2} - \frac{33.5 \text{ in}}{12 \ \frac{\text{in}}{\text{ft}}}\right) \\ &= w_u (11.56 \text{ ft}) \end{aligned}$$

Therefore,

$$\begin{aligned} V_u &\le \phi(V_s + V_c) \\ w_u(11.56 \text{ ft}) &\le (0.75)(100.5 \text{ kip} + 58.7 \text{ kip}) \\ w_u &\le 10.3 \text{ kip/ft} \end{aligned}$$

Similarly, where the stirrups change to 12 in on centers at 4.33 ft from support face, the shear strength requirement gives

$$\begin{aligned} V_u &\le \phi(V_s + V_c) \\ w_u\left(\frac{28.7 \text{ ft}}{2} - 4.33 \text{ ft}\right) &\le (0.75)(67.0 \text{ kip} + 58.7 \text{ kip}) \\ w_u &\le 9.4 \text{ kip/ft} \end{aligned}$$

Lastly, where the stirrups change to the 18 in on centers spacing (this spacing slightly exceeds the code $d/2$ limit but is deemed acceptable),

$$\begin{aligned} V_u &\le \phi(V_s + V_c) \\ w_u\left(\frac{28.7 \text{ ft}}{2} - 8.33 \text{ ft}\right) &\le (0.75)(44.7 \text{ kip} + 58.7 \text{ kip}) \\ w_u &\le 12.9 \text{ kip/ft} \end{aligned}$$

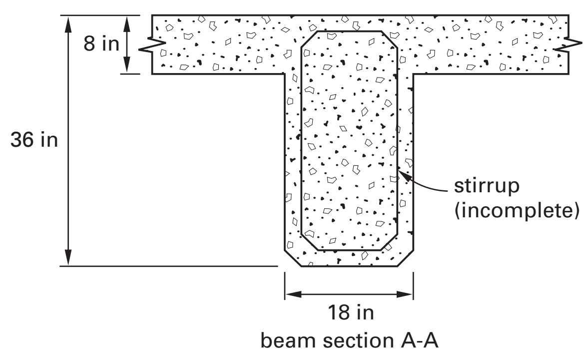
The critical location is at 4.33 ft where the factored load based on shear strength is 9.4 kip/ft. Solving for the corresponding service uniform live load gives

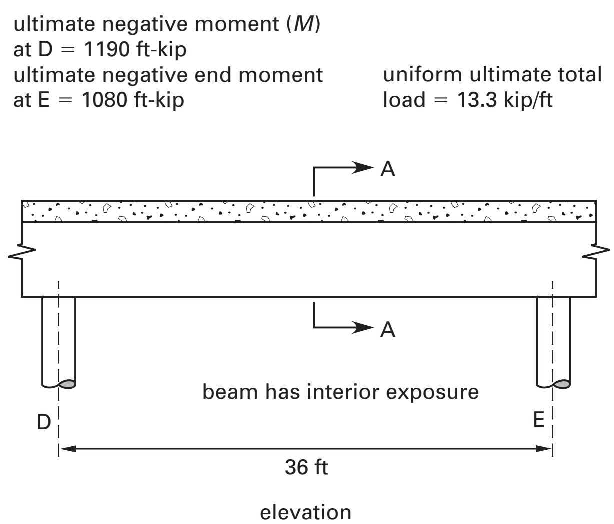
$$\begin{aligned} w_u &= 1.2 w_d + 1.6 w_l \\ 9.4 \ \frac{\text{kip}}{\text{ft}} &= (1.2)\left(1.5 \ \frac{\text{kip}}{\text{ft}}\right) + 1.6 w_l \\ w_l &= 4.8 \text{ kip/ft} \end{aligned}$$

The answer is (A).

Practice Problem 7

A typical interior span of a cast-in-place one-way slab and beam system is shown. The beam span is 36 ft. Beams are spaced 22 ft on center. The typical beam depth is 36 in with a web width of 18 in. Beam loading and end moment are given as ultimate loads and moments. All principal reinforcement is to be no. 8 bars. Neglect column width and any unbalanced loading.





The required area of positive steel at the critical location is 6.32 in^2. The distance from support D to the point where 50% of the bottom steel can terminate on the left side is most nearly

(A) 2.8 ft
(B) 6.4 ft
(C) 9.2 ft
(D) 12 ft

Design Criteria

- $f'_c = 4$ ksi, hard rock aggregate
- $f_y = 60$ ksi for principal reinforcement

Solution

The effective width of the T-beam for the positive region is

$$b_e \le \begin{cases} \dfrac{L}{4} = \dfrac{(36\text{ ft})\left(12\ \dfrac{\text{in}}{\text{ft}}\right)}{4} \\ \quad = 108\text{ in} \quad [\text{controls}] \\ b_w + 16h = 18\text{ in} + (16)(8\text{ in}) \\ \quad = 146\text{ in} \\ s_l + s_r = \left(\dfrac{22\text{ ft}}{2} + \dfrac{22\text{ ft}}{2}\right)\left(12\ \dfrac{\text{in}}{\text{ft}}\right) \\ \quad = 264\text{ in} \end{cases}$$

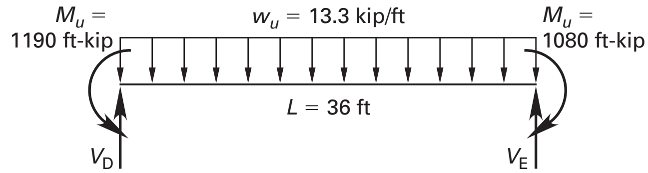
For the remaining 50% of the bottom steel,

$$A_s = (0.5)(6.32\text{ in}^2) = 3.16\text{ in}^2$$

$$a = \frac{A_s f_y}{0.85 f'_c b_e} = \frac{(3.16\text{ in}^2)\left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(4\ \dfrac{\text{kip}}{\text{in}^2}\right)(108\text{ in})} = 0.52\text{ in} < h = 8\text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{0.52\text{ in}}{0.85} = 0.61\text{ in} < \frac{3d}{8} \quad [\text{tension controlled, } \phi = 0.9]$$

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) = \frac{(0.9)(3.16\text{ in}^2)\left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)\left(33.5\text{ in} - \dfrac{0.52\text{ in}}{2}\right)}{12\ \dfrac{\text{in}}{\text{ft}}} = 473\text{ kip-ft}$$



$$\begin{aligned} \sum M_E &= 0\text{ ft-kip} \\ &= V_D L - \frac{w_u L^2}{2} - M_{u,D} - M_{u,E} \\ V_D &= \frac{w_u L}{2} + \frac{M_{u,D}}{L} + \frac{M_{u,E}}{L} \\ &= \frac{\left(13.3\ \dfrac{\text{kip}}{\text{ft}}\right)(36\text{ ft})}{2} + \frac{1190\text{ ft-kip}}{36\text{ ft}} - \frac{1080\text{ ft-kip}}{36\text{ ft}} \\ &= 242.5\text{ kip} \end{aligned}$$

The moment at a distance x from D is

$$\begin{aligned} M_u &= M_{u,D} + V_D x - \frac{w_u x^2}{2} \\ &= -1190\text{ ft-kip} + (242.5\text{ kip})x - \frac{\left(13.3\ \dfrac{\text{kip}}{\text{ft}}\right)x^2}{2} \end{aligned}$$

Equating the moment equation to the design moment and solving for x gives the distance to the theoretical cutoff point for 50% of the bottom steel.

$$\begin{aligned} \phi M_n &= M_u \\ 473\text{ ft-kip} &= -1190\text{ ft-kip} + (242.5\text{ kip})x - \left(13.3\ \frac{\text{kip}}{\text{ft}}\right)\frac{x^2}{2} \\ x &= 9.16\text{ ft} \end{aligned}$$

ACI requires that bars must extend beyond the point where they are theoretically no longer needed to a distance of

$$\text{extension} \ge \begin{cases} 12d_b = (12)(1\text{ in}) = 12\text{ in} \\ d = 33.5\text{ in} \quad [\text{controls}] \end{cases}$$

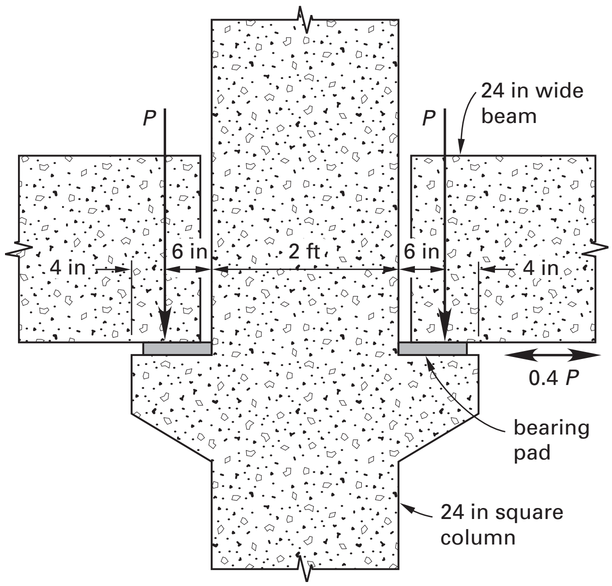
Therefore, the required cutoff point is

$$x' = 9.16\text{ ft} - \frac{33.5\text{ in}}{12\ \dfrac{\text{in}}{\text{ft}}} = 6.37\text{ ft} \quad (6.4\text{ ft})$$

The answer is (B).

Practice Problem 8

Concrete corbels support precast concrete beams as shown. Properly sized bearing pads transfer the vertical service load reactions to the corbel such that the resultant is 6 in from the face of the column.



The overall width and depth of the corbel are both 16 in, and main steel is positioned 1 in from the top. The minimum area of the main steel required by ACI 318 is most nearly

(A) 0.64 in^2

(B) 1.1 in^2

(C) 1.7 in^2

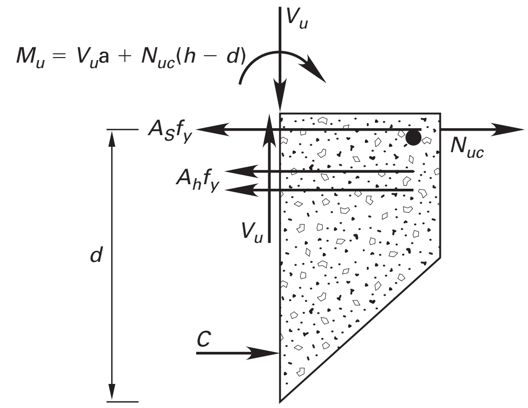
(D) 2.1 in^2

Design Criteria

- $f'_c = 4$ ksi, normal weight concrete
- $f_y = 60$ ksi
- $P_{\text{dead load}} = 48$ kip
- $P_{\text{live load}} = 32$ kip
- bearing pad transfers a horizontal force equal to 40% of vertical force
- corbels are cast monolithically with the column

Solution

The main steel must resist the horizontal force, the moment, and a portion of the vertical force through shear friction. A free-body diagram shows the statically equivalent loads acting at the interface between corbel and column.



The factored forces and moment are

$$V_u \geq \begin{cases} 1.4P_d = (1.4)(48 \text{ kip}) \\ \quad = 67.2 \text{ kip} \\ 1.2P_d + 1.6P_l = (1.2)(48 \text{ kip}) + (1.6)(32.0 \text{ kip}) \\ \quad = 108.8 \text{ kip} \quad [\text{controls}] \end{cases}$$

$$\begin{aligned} N_{uc} &= 0.4V_u = (0.4)(108.8 \text{ kip}) \\ &= 43.5 \text{ kip} \\ M_u &= V_u a + N_{uc}(h - d) \\ &= (108.8 \text{ kip})(6 \text{ in}) \\ &\quad + (43.5 \text{ kip})(16 \text{ in} - 15 \text{ in}) \\ &= 696 \text{ in-kip} \end{aligned}$$

Because the design of the corbel is primarily a problem in shear transfer, the resistance factor is taken as the factor for shear design, 0.75. To resist the moment, usual design practice is to estimate the internal lever arm as $0.9d$. Thus,

$$\begin{aligned} A_f &= \frac{M_u}{\phi f_y (0.9d)} \\ &= \frac{696 \text{ in-kip}}{(0.75)\left(60 \dfrac{\text{kip}}{\text{in}^2}\right)(0.9)(15 \text{ in})} \\ &= 1.15 \text{ in}^2 \end{aligned}$$

The additional area to resist the normal force is

$$\begin{aligned} A_n &= \frac{N_{uc}}{\phi f_y} = \frac{43.5 \text{ kip}}{(0.75)\left(60 \dfrac{\text{kip}}{\text{in}^2}\right)} \\ &= 0.97 \text{ in}^2 \end{aligned}$$

The total area of shear friction reinforcement is

$$\begin{aligned} A_{vf} &= \frac{V_u}{\phi \mu f_y} = \frac{108.8 \text{ kip}}{(0.75)(1.4)\left(60 \dfrac{\text{kip}}{\text{in}^2}\right)} \\ &= 1.73 \text{ in}^2 \end{aligned}$$

The minimum area of the main steel is limited to

$$\begin{aligned} A_{s,\min} &= 0.04\left(\frac{f'_c}{f_y}\right)bd \\ &= (0.04)\left(\frac{4\ \frac{\text{kip}}{\text{in}^2}}{60\ \frac{\text{kip}}{\text{in}^2}}\right)(16\text{ in})(15\text{ in}) \\ &= 0.64\text{ in}^2 \end{aligned}$$

The main steel required by ACI is

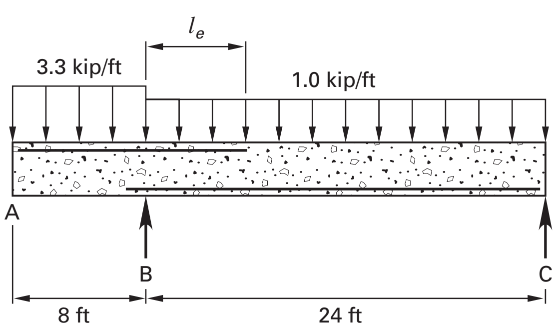
$$A_s \geq \begin{cases} A_f + A_n = 1.15\text{ in}^2 + 0.97\text{ in}^2 \\ \quad = 2.12\text{ in}^2 \quad \text{[controls]} \\ \dfrac{2A_{vf}}{3} + A_n = \dfrac{(2)(1.73\text{ in}^2)}{3} + 0.97\text{ in}^2 \\ \quad = 2.12\text{ in}^2 \\ A_{s,\min} = 0.64\text{ in}^2 \end{cases}$$

The value of 2.12 in^2 (2.1 in^2) controls.

The answer is (D).

Practice Problem 9

The beam shown has an effective depth of 16 in and width of 12 in. The cantilever must support a uniformly distributed factored load of 3.3 kip/ft, which produces a design moment of 106 ft-kip at the face of support. Flexural design results in the selection of two no. 8 bars, which fully develop by straight embedment into the back span BC.



The minimum length of embedment to the right of B for the given loading is most nearly

(A) 62 in
(B) 110 in
(C) 120 in
(D) 140 in

Design Criteria

- $f'_c = 4$ ksi, normal weight concrete
- $f_y = 60$ ksi
- bars enclosed by stirrups with 6 in clear spacing between bars

Solution

The bars must fully develop to the right of support B. For a no. 8 top bar in normal weight concrete, the development length is

$$\begin{aligned} l_d &= \left(\frac{f_y\psi_t\psi_e}{20\lambda\sqrt{f'_c}}\right)d_b \\ &= \left(\frac{\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)(1.3)(1.0)}{(20)(1)\sqrt{4000\ \frac{\text{lbf}}{\text{in}^2}}}\right)(1\text{ in}) \\ &= 62\text{ in} \end{aligned}$$

Because there are only two top bars, both must extend past the point of inflection a minimum distance in accordance with ACI Sec. 12.12.3.

$$\text{extension} \geq \begin{cases} d = 16\text{ in} \\ \dfrac{l_n}{16} = \dfrac{(24\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{16} \\ \quad = 18\text{ in} \quad \text{[controls]} \\ 12d_b = (12)(1\text{ in}) = 12\text{ in} \end{cases}$$

To locate the point of inflection, write an equation for the bending moment for span BC. The simplest way to write the expression is to position the origin at C with x positive to the left.

Taking moments about support B,

$$\begin{aligned} \sum M_\text{B} &= 0\text{ in-kip} \\ &= C_yL + \frac{w_{u,\text{AB}}a^2}{2} - \frac{w_{u,\text{BC}}L^2}{2} \\ C_y &= \frac{w_{u,\text{BC}}L^2}{2L} - \frac{w_{u,\text{AB}}a^2}{2L} \\ &= \frac{\left(1\ \frac{\text{kip}}{\text{ft}}\right)(24\text{ ft})^2}{(2)(24\text{ ft})} - \frac{\left(3.3\ \frac{\text{kip}}{\text{ft}}\right)(8\text{ ft})^2}{(2)(24\text{ ft})} \\ &= 7.6\text{ kip} \quad \text{[upward]} \end{aligned}$$

The moment about the point at distance x from C is

$$\sum M_x = 0 \text{ in-kip}$$

$$M_u - C_y x + \frac{w_{u,BC}x^2}{2} = 0 \text{ in-kip}$$

$$M_u = C_y x - w_{u,\text{BC}}\left(\frac{x^2}{2}\right)$$

$$= (7.6 \text{ kip})x - \left(1 \ \frac{\text{kip}}{\text{ft}}\right)\left(\frac{x^2}{2}\right)$$

Thus, the point of inflection is

$$x = \frac{7.6 \text{ kip}}{0.5 \ \frac{\text{kip}}{\text{ft}}} = 15.2 \text{ ft} \quad \text{[from C]}$$

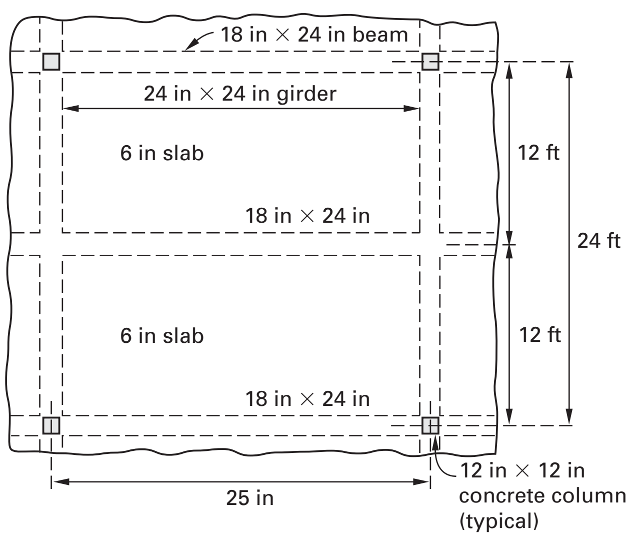
The required length of embedment is

$$l_e \geq \begin{cases} l_d = 62 \text{ in} \\ (L - x) + \text{extension} \\ \quad = (24 \text{ ft} - 15.2 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right) + 18 \text{ in} \\ \quad = 124 \text{ in} \quad \text{[controls]} \end{cases}$$

The answer is (C).

Practice Problem 10

An owner has an existing two-story office building to be converted to a warehouse. The maximum allowable live load that the second floor will safely support must be determined. A typical interior bay is shown.



Based on the flexural strength of the negative steel in the beams, the maximum uniformly distributed service live load capacity of the floor system is most nearly

(A) 75 lbf/ft^2

(B) 100 lbf/ft^2

(C) 150 lbf/ft^2

(D) 200 lbf/ft^2

Design Criteria

- $f'_c = 3.5$ ksi (from core samples), hard rock concrete
- $f_y = 40$ ksi
- beam reinforcement (1½ in clearance to stirrups):

  four no. 9 bottom

  four no. 10 top to supports

  eight no. 3 stirrups at 8 in on center each end (starting 4 in from support)
- all reinforcement adequately embedded to develop fully
- do not consider crack control and deflection criteria
- ACI approximate method applies to beams
- typical bay is applicable to all interior and exterior bays
- storage live load is not reducible
- treat all superimposed load as service live load

Solution

For the negative moment regions, the width of the compression zone is the web width, 18 in. Thus the equivalent depth of the compression zone is

$$a = \frac{A_s f_y}{0.85 f'_c b_w}$$

$$= \frac{(4)(1.27 \text{ in}^2)\left(40 \ \frac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(3.5 \ \frac{\text{kip}}{\text{in}^2}\right)(18 \text{ in})}$$

$$= 3.80 \text{ in}$$

The effective depth, d, can be determined from the given information

$$d = h - \text{cover} - d_{\text{stirrup}} - 0.5d_b$$

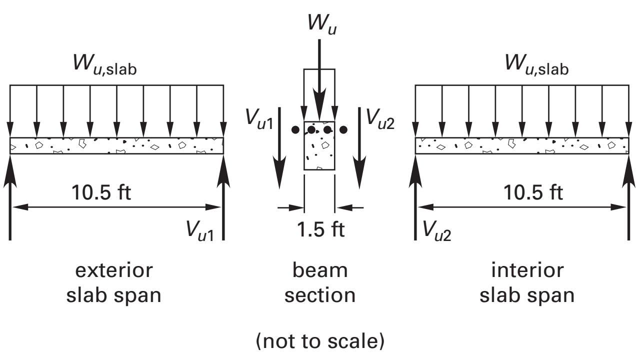
$$= 24 \text{ in} - 1.5 \text{ in} - 0.375 \text{ in} - (0.5)(1.25 \text{ in})$$

$$= 21.5 \text{ in}$$

The strain in the tension steel at flexural failure is

$$\begin{aligned}\varepsilon_t &= \frac{(0.003)\left(d - \frac{a}{\beta_1}\right)}{\frac{a}{\beta_1}} \\ &= \frac{(0.003)\left(21.5\text{ in} - \frac{3.80\text{ in}}{0.85}\right)}{\frac{3.80\text{ in}}{0.85}} \\ &= 0.011 > 0.005 \quad \text{[therefore tension controlled]} \\ \phi M_n &= \phi A_s f_y \left(d - \frac{a}{2}\right) \\ &= (0.9)(4)(1.27\text{ in}^2)\left(40\ \frac{\text{kip}}{\text{in}^2}\right) \\ &\quad \times \left(21.5\text{ in} - \frac{3.80\text{ in}}{2}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) \\ &= 299\text{ ft-kip}\end{aligned}$$

Using the ACI approximate method for one-way systems, the worst loading on the typical beam occurs at the first interior beam.



The 6 in normal weight slab weighs 75 lbf/ft^2, so the factored load on the slab is

$$w_{u,\text{slab}} = (1.2)\left(75\ \frac{\text{lbf}}{\text{ft}^2}\right) + 1.6w_l$$

w_l is the service live load. The 1.5 ft wide by 2 ft beam weighs 450 lbf/ft. Thus, the load on the controlling beam is

$$\begin{aligned}w_u &= V_{u1} + V_{u2} + 1.6bw_l + 1.2w_{\text{bm}} \\ &= (1.15)\left(90\ \frac{\text{lbf}}{\text{ft}} + 1.6w_l\right)\left(\frac{10.5\text{ ft}}{2}\right) \\ &\quad + \left(90\ \frac{\text{lbf}}{\text{ft}} + 1.6w_l\right)\left(\frac{10.5\text{ ft}}{2}\right) \\ &\quad + (1.6)(1.5\text{ ft})w_l + (1.2)\left(450\ \frac{\text{lbf}}{\text{ft}}\right) \\ &= 1556\ \frac{\text{lbf}}{\text{ft}} + (20.5\text{ ft})w_l\end{aligned}$$

The critical negative bending moment is at the exterior face of the first interior support.

$$M_u = \frac{w_u l_n^2}{10}$$

Thus, based on flexural strength,

$$\begin{aligned}M_u &= \phi M_n = \frac{w_u l_n^2}{10} \\ w_u &= \frac{10(\phi M_n)}{l_n^2} \\ &= \frac{(10)(299\text{ ft-kip})\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)}{(25\text{ ft} - 2\text{ ft})^2} \\ &= 5650\text{ lbf/ft}\end{aligned}$$

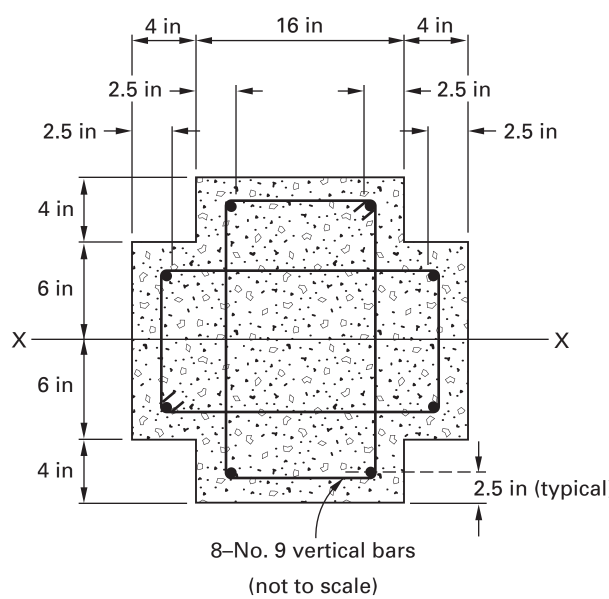
Substituting this value into the earlier equation,

$$\begin{aligned}w_u &= 1556\ \frac{\text{lbf}}{\text{ft}} + (20.5\text{ ft})w_l \\ 5650\ \frac{\text{lbf}}{\text{ft}} &= 1556\ \frac{\text{lbf}}{\text{ft}} + (20.5\text{ ft})w_l \\ w_l &= 200\text{ lbf/ft}^2\end{aligned}$$

The answer is (D).

Practice Problem 11

For the column cross section shown, bending is about the X-X axis. Do not apply a capacity reduction factor, ϕ.



The axial compression force on the column interaction diagram corresponding to balanced strain conditions for this section is most nearly

(A) 500 kip
(B) 700 kip
(C) 900 kip
(D) 1200 kip

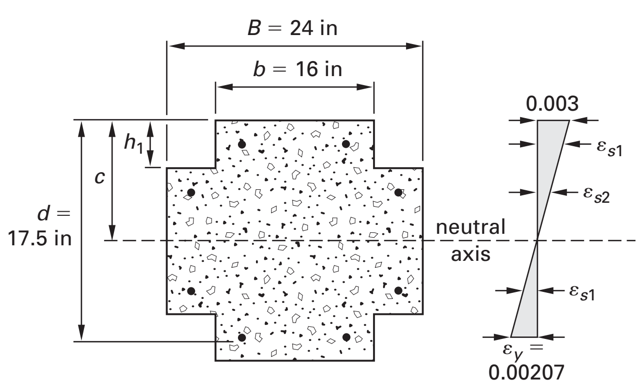
Design Criteria

- $f'_c = 5$ ksi
- $f_y = 60$ ksi

Solution

Balanced strain conditions require that the extreme tension steel reaches its yield strain exactly when the strain at the compression edge reaches ultimate, 0.003. For grade 60 reinforcing steel,

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60\ \frac{\text{kip}}{\text{in}^2}}{29{,}000\ \frac{\text{kip}}{\text{in}^2}} = 0.00207$$



Using similar triangles gives the distance from the extreme compression edge to the neutral axis.

$$\frac{c}{0.003} = \frac{d}{0.003 + \varepsilon_y}$$

$$c = \frac{0.003d}{0.003 + \varepsilon_y} = \frac{(0.003)(17.5\ \text{in})}{0.003 + 0.00207} = 10.36\ \text{in}$$

Similarly, the strains in the reinforcement at various depths are

$$\varepsilon'_{s1} = \frac{0.003(c - d'_1)}{c} = \frac{(0.003)(10.36\ \text{in} - 2.5\ \text{in})}{10.36\ \text{in}} = 0.00228$$

$$\varepsilon'_{s2} = \frac{0.003(c - d'_2)}{c} = \frac{(0.003)(10.36\ \text{in} - 6.5\ \text{in})}{10.36\ \text{in}} = 0.0011$$

$$\varepsilon_{s1} = \frac{0.003(d_1 - c)}{c} = \frac{(0.003)(13.5\ \text{in} - 10.36\ \text{in})}{10.36\ \text{in}} = 0.00091$$

Because the yield strain is 0.00207, the stress in the topmost reinforcement and in the lower tension steel is the yield stress, 60 ksi. The strains in the remaining steel layers are below the yield strain, which means that Hooke's law is valid for these layers.

$$f'_{s2} = E_s\varepsilon'_{s2} = \left(29{,}000\ \frac{\text{kip}}{\text{in}^2}\right)(0.0011) = 31.9\ \text{ksi}$$

$$f_{s1} = E_s\varepsilon_{s1} = \left(29{,}000\ \frac{\text{kip}}{\text{in}^2}\right)(0.00091) = 26.4\ \text{ksi}$$

The force components corresponding to the regions of the cross section are

$$C_c = 0.85f'_c\big((\beta_1 c)b + (\beta_1 c - h_1)(B - b)\big) = (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)\left(\begin{array}{l}(0.8)(10.36\ \text{in})(16\ \text{in}) \\ \quad + \big((0.8)(10.36\ \text{in}) - 4\ \text{in}\big) \\ \quad \times (24\ \text{in} - 16\ \text{in})\end{array}\right) = 709\ \text{kip}$$

$$C'_{s1} = (f'_{s1} - 0.85f'_c)A'_{s1} = \left(60\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)\right)(2.0\ \text{in}^2) = 112\ \text{kip}$$

$$C'_{s2} = (f'_{s2} - 0.85f'_c)A'_{s2} = \left(31.9\ \frac{\text{kip}}{\text{in}^2} - (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)\right)(2.0\ \text{in}^2) = 55\ \text{kip}$$

$$T_1 = f_{s1}A_{s1} = \left(26.4\ \frac{\text{kip}}{\text{in}^2}\right)(2.0\ \text{in}^2)$$
$$= 53\ \text{kip}$$
$$T_2 = f_yA_{s2} = \left(60\ \frac{\text{kip}}{\text{in}^2}\right)(2.0\ \text{in}^2)$$
$$= 120\ \text{kip}$$

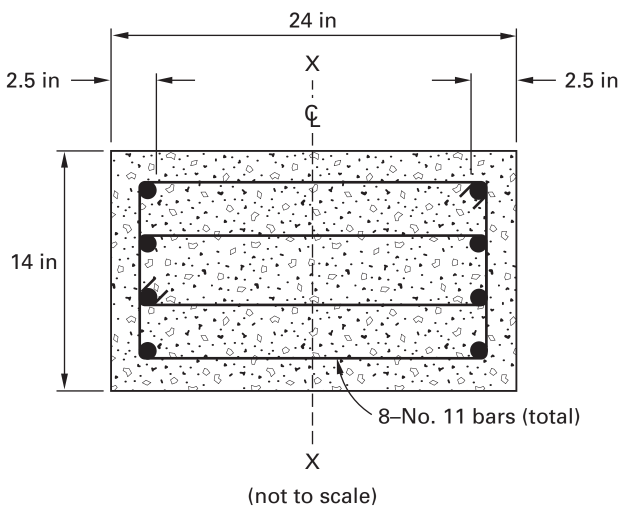
The axial load on the column at balanced strain conditions is the algebraic sum of the forces in the components.

$$\begin{aligned} P_b &= C_c + C'_{s1} + C'_{s2} - T_1 - T_2 \\ &= 709\ \text{kip} + 112\ \text{kip} + 55\ \text{kip} - 53\ \text{kip} - 120\ \text{kip} \\ &= 703\ \text{kip}\quad (700\ \text{kip}) \end{aligned}$$

The answer is (B).

Practice Problem 12

A reinforced concrete tied column that is part of a braced frame is shown in cross section.



Assuming a compression-controlled failure, the factored bending moment that controls the capacity of the column for the given loading is most nearly

(A) 140 ft-kip
(B) 190 ft-kip
(C) 220 ft-kip
(D) 510 ft-kip

Design Criteria

The column carries vertical loads (P) and moments (M) about the centerline X-X as follows. These are unfactored service load values.

- dead load only, $P_d = 450$ kip, $M_d = 100$ ft-kip
- live load only, $P_l = 200$ kip, $M_l = 40$ ft-kip
- moments are equal at top and bottom and bend column in reverse curvature
- effective unsupported length of column about X-X axis is 24 ft
- use approximate method of ACI 318 to account for slenderness effects
- $f'_c = 4$ ksi concrete
- $f_y = 60$ ksi reinforcing steel

Solution

The factored axial load on the column is

$$P_u \geq \begin{cases} 1.4P_d = (1.4)(450\ \text{kip}) = 630\ \text{kip} \\ 1.2P_d + 1.6P_l \\ \qquad = (1.2)(450\ \text{kip}) + (1.6)(200\ \text{kip}) \\ \qquad = 860\ \text{kip}\quad [\text{controls}] \end{cases}$$

For a compression-controlled failure, the larger value, 860 kip, controls. Therefore, compute the corresponding moment for a combination of dead and live load.

$$\begin{aligned} M_1 = M_2 &= 1.2M_d + 1.6M_l \\ &= (1.2)(100\ \text{ft-kip}) + (1.6)(40\ \text{ft-kip}) \\ &= 184\ \text{ft-kip} \end{aligned}$$

For a rectangular column, ACI Sec. 10.10.1.2 permits

$$\begin{aligned} r &= 0.3h = (0.3)(24\ \text{in}) \\ &= 7.2\ \text{in} \end{aligned}$$
$$\begin{aligned} \frac{kl_u}{r} &= \frac{(24\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{7.2\ \text{in}} \\ &= 40 \end{aligned}$$

For reverse curvature bending, the ratio M_1/M_2 is negative. From ACI Sec. 10.10.1, the limiting slenderness ratio for which slenderness effects can be neglected is

$$\frac{kl_u}{r} < \begin{cases} 34 - 12\left(\frac{M_1}{M_2}\right) = 34 - (12)(-1) = 46 \\ 40\quad [\text{controls}] \end{cases}$$

Since the actual slenderness ratio, 40, equals the limit of 40, calculate the effect of slenderness on the bending moments. For 4 ksi normal weight concrete,

$$\begin{aligned}E_c &= 33w_c^{1.5}\sqrt{f'_c}\\ &= \left(33\ \frac{\text{ft}^3}{\text{lbf}}\right)\left(145\ \frac{\text{lbf}}{\text{ft}^3}\right)^{1.5}\sqrt{4000\ \frac{\text{lbf}}{\text{in}^2}}\left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right)\\ &= 3640\ \text{ksi}\end{aligned}$$

$$\begin{aligned}I_g &= \frac{bh^3}{12} = \frac{(14\ \text{in})(24\ \text{in})^3}{12}\\ &= 16{,}128\ \text{in}^4\end{aligned}$$

To account for creep in the column, adjust the rigidity of the column by the dead-to-total load factor.

$$\begin{aligned}\beta_{dns} &= \frac{1.2P_d}{P_u} = \frac{(1.2)(450\ \text{kip})}{860\ \text{kip}}\\ &= 0.63\end{aligned}$$

$$\begin{aligned}EI &= \frac{0.4E_cI_g}{1+\beta_{dns}}\\ &= \frac{(0.4)\left(3640\ \frac{\text{kip}}{\text{in}^2}\right)(16{,}128\ \text{in}^4)}{1+0.63}\\ &= 14{,}400{,}000\ \text{kip-in}^2\end{aligned}$$

$$\begin{aligned}P_c &= \frac{\pi^2 EI}{kL^2}\\ &= \frac{\pi^2(14{,}400{,}000\ \text{kip-in}^2)}{(24\ \text{ft})^2\left(144\ \frac{\text{in}^2}{\text{ft}^2}\right)}\\ &= 1712\ \text{kip}\end{aligned}$$

$$C_m \geq \begin{cases} 0.6 + 0.4\left(\dfrac{M_1}{M_2}\right) \\ \qquad = 0.6 + (0.4)(-1.0) = 0.2 \\ 0.4 \quad [\text{controls}] \end{cases}$$

The amplifier for the design moment is

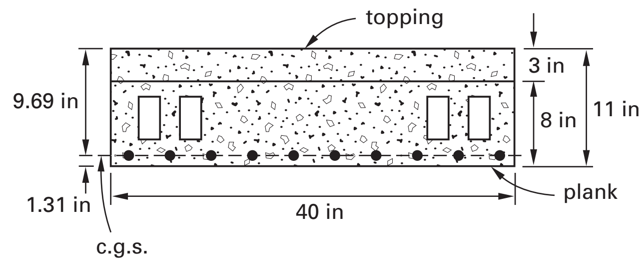
$$\begin{aligned}\delta &= \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}}\\ &= \frac{0.4}{1 - \dfrac{860\ \text{kip}}{(0.75)(1712\ \text{kip})}}\\ &= 1.21\end{aligned}$$

$$\begin{aligned}M_c &= \delta M_2\\ &= (1.21)(184\ \text{ft-kip})\\ &= 223\ \text{ft-kip} \quad (220\ \text{ft-kip})\end{aligned}$$

The answer is (C).

Practice Problem 13

A pretensioned cored slab is shown.



The nominal ultimate moment capacity of the section, M_n, is most nearly

(A) 160 ft-kip
(B) 200 ft-kip
(C) 240 ft-kip
(D) 280 ft-kip

Design Criteria

- f'_c of plank and topping = 4 ksi, normal weight concrete
- f_u = 270 ksi prestressing steel, low-relaxation strand
- effective prestress after all losses is 170 ksi
- reinforcement is 10 seven-wire strands with diameter of 3/8 in
- area of one 3/8 in diameter strand = 0.085 in^2
- use approximate equations of ACI 318

Solution

Assume the topping slab is fully composite with the precast hollow core section. The neutral axis of the section at ultimate should be well above the cored region; therefore, $b = 40$ in and $d_p = 9.69$ in.

$$\begin{aligned}\rho_p &= \frac{A_{ps}}{bd} = \frac{(10)(0.085\ \text{in}^2)}{(40\ \text{in})(9.69\ \text{in})}\\ &= 0.00219\end{aligned}$$

For 270 ksi low-relaxation strand, $f_{py}/f_{pu} = 0.9$; therefore, $\gamma_p = 0.28$. The strand stress at ultimate is

$$\begin{aligned} f_{ps} &= f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left(\frac{\rho_p f_{pu}}{f'_c}\right)\right) \\ &= \left(270\ \frac{\text{kip}}{\text{in}^2}\right) \\ &\quad \times \left(1 - \left(\frac{0.28}{0.85}\right)\left(\frac{(0.00219)\left(270\ \frac{\text{kip}}{\text{in}^2}\right)}{4\ \frac{\text{kip}}{\text{in}^2}}\right)\right) \\ &= 257\ \text{ksi} \end{aligned}$$

$$\begin{aligned} a &= \frac{A_{ps} f_{ps}}{0.85 f'_c b} \\ &= \frac{(10)(0.085\ \text{in}^2)\left(257\ \frac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(40\ \text{in})} \\ &= 1.61\ \text{in} \end{aligned}$$

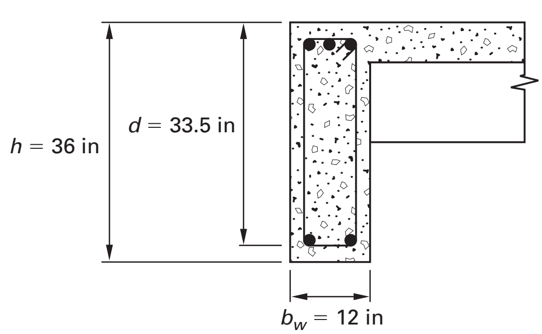
The compression zone is within the topping slab; therefore, the use of the full width of 40 in is justified.

$$\begin{aligned} M_n &= A_{ps} f_{ps}(d_p - \frac{a}{2}) \\ &= (10)(0.085\ \text{in}^2)\left(257\ \frac{\text{kip}}{\text{in}^2}\right)\left(9.69\ \text{in} - \frac{1.61\ \text{in}}{2}\right) \\ &\quad \times \left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &= 162\ \text{ft-kip} \quad (160\ \text{ft-kip}) \end{aligned}$$

The answer is (A).

Practice Problem 14

The spandrel beam shown has a 24 ft clear span and supports floor loads from a one-way ribbed slab. These loads create a factored uniformly distributed torsional moment along the entire clear span equal to 4 ft-kip/ft. Stirrups are no. 3 with 1.5 in cover. The overhanging portion of the slab is not detailed to resist torsion.



The factored torsional moment at the most critical location is most nearly

(A) 21 ft-kip
(B) 27 ft-kip
(C) 34 ft-kip
(D) 48 ft-kip

Design Criteria

- $f'_c = 3$ ksi
- $f_y = 60$ ksi

Solution

The critical section for torsion is located a distance d from the face of support. Thus, for a uniformly distributed torsional loading,

$$\begin{aligned} T_u &= t_u\left(\frac{l_n}{2} - d\right) \\ &= \left(4\ \frac{\text{ft-kip}}{\text{ft}}\right)\left(\frac{24\ \text{ft}}{2} - \frac{33.5\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}}\right) \\ &= 36.8\ \text{ft-kip} \end{aligned}$$

Properties of the section resisting torsion are

$$\begin{aligned} p_{cp} &= 2(b_w + h) = (2)(12\ \text{in} + 36\ \text{in}) \\ &= 96\ \text{in} \\ A_{cp} &= b_w h = (12\ \text{in})(36\ \text{in}) \\ &= 432\ \text{in}^2 \end{aligned}$$

The threshold torsion for non-prestressed members is

$$\begin{aligned} T_{ut} &= \phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) \\ &= 0.75\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\left(\frac{\left(432\ \text{in}^2\right)^2}{96\ \text{in}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &\quad \times \left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) \\ &= 6.7\ \text{ft-kip} \end{aligned}$$

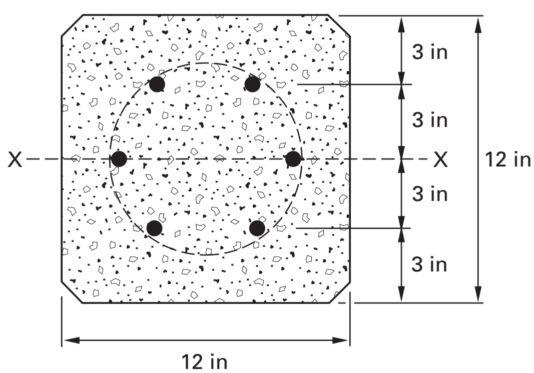
Because the factored torsional moment exceeds the threshold torsional moment, torsional shear stresses are significant. The spandrel beam is an example of compatibility torsion for which the code permits a redistribution of forces after torsional cracking. The maximum torsional moment that applies in this case is

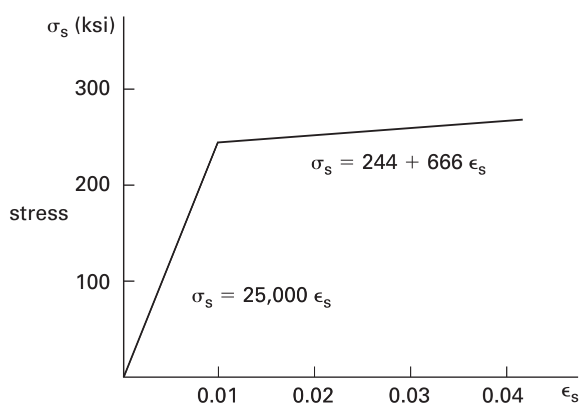
$$\begin{aligned} T_u &\le 4\phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) \\ &\le 4T_{ut} \\ &\le (4)(6.7 \text{ ft-kip}) \\ &\le 26.8 \text{ ft-kip} \quad (27 \text{ ft-kip}) \quad \text{[controls]} \end{aligned}$$

The answer is (B).

Practice Problem 15

A prestressed concrete pile is shown in cross section.





stress and strain relationship for 270 ksi stress-relieved strand

Due to the soil condition, the pile has to resist both axial load and bending moment during earthquake. The maximum allowable bending moment about the X-X axis computed by a strain-compatibility analysis is most nearly

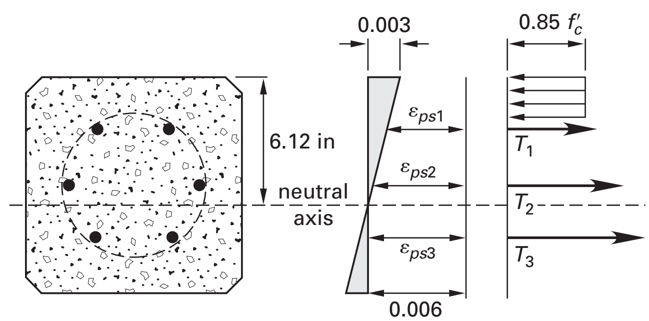
(A) 50 ft-kip
(B) 75 ft-kip
(C) 150 ft-kip
(D) 600 ft-kip

Design Criteria

- $f'_c = 5$ ksi @ 28 days concrete
- $f_{pu} = 270$ ksi prestressed strands
- $A_s = 0.115$ in^2 per strand
- $A_c = 144$ in^2 (neglect champfers)
- effective strain in strands at decompression is 0.006
- $P_u = 140$ kip
- use $\phi = 0.65$ for both axial and bending
- distance from extreme compression edge to neutral axis is 6.12 in

Solution

The depth to the neutral axis is 6.12 in, so the strain diagram at the design condition is as shown.



The effective prestrain is given as 0.006. From similar triangles, the final strains at the three levels are

$$\begin{aligned} \varepsilon_{ps1} &= 0.006 - \frac{(0.003)(6.12 \text{ in} - 3 \text{ in})}{6.12 \text{ in}} \\ &= 0.00447 \\ \varepsilon_{ps2} &= 0.006 - \frac{(0.003)(6.12 \text{ in} - 6 \text{ in})}{6.12 \text{ in}} \\ &= 0.00594 \\ \varepsilon_{ps3} &= 0.006 + \frac{(0.003)(9.00 \text{ in} - 6.12 \text{ in})}{6.12 \text{ in}} \\ &= 0.00741 \end{aligned}$$

Because the final strains are below the given yield strain of 0.01, the corresponding strand stresses are

$$\begin{aligned} f_{ps1} &= E_{ps}\varepsilon_{ps1} = \left(25{,}000\ \frac{\text{kip}}{\text{in}^2}\right)(0.00447) \\ &= 112 \text{ ksi} \\ f_{ps2} &= E_{ps}\varepsilon_{ps2} = \left(25{,}000\ \frac{\text{kip}}{\text{in}^2}\right)(0.00594) \\ &= 148.5 \text{ ksi} \\ f_{ps3} &= E_{ps}\varepsilon_{ps3} = \left(25{,}000\ \frac{\text{kip}}{\text{in}^2}\right)(0.00741) \\ &= 185 \text{ ksi} \end{aligned}$$

The concrete compression resultant is

$$\begin{aligned} C_c &= 0.85 f'_c b(\beta_1 c) \\ &= (0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)(12 \text{ in})(0.8)(6.12 \text{ in}) \\ &= 250 \text{ kip} \end{aligned}$$

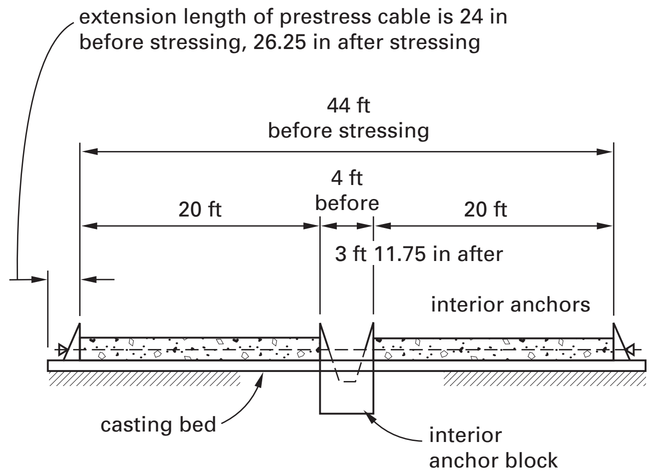
Taking the moments of the forces about the plastic centroid, and noting that the applied 140 kip and the force from the center strands act through the plastic centroid, calculate the moment capacity.

$$\begin{aligned} M_n &= f_{ps3}A_{ps3}x_3 - f_{ps1}A_{ps1}x_1 + C_c\left(\frac{h}{2} - \frac{\beta_1 c}{2}\right) \\ &= \left(185\ \frac{\text{kip}}{\text{in}^2}\right)(2)(0.115 \text{ in}^2)(3 \text{ in}) \\ &\quad - \left(112\ \frac{\text{kip}}{\text{in}^2}\right)(2)(0.115 \text{ in}^2)(3 \text{ in}) \\ &\quad + (250 \text{ kip})\left(6 \text{ in} - \frac{(0.8)(6.12 \text{ in})}{2}\right) \\ &= 938 \text{ in-kip} \\ \phi M_n &= (0.65)(938 \text{ in-kip})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \\ &= 50.8 \text{ ft-kip} \quad (50 \text{ ft-kip}) \end{aligned}$$

The answer is (A).

Practice Problem 16

Two concrete members are post-tensioned concentrically as shown. The prestress tendon is a single cable extending between exterior anchorages and pulled from one end. The interior anchorages move toward each other under the prestress load. After stressing, the distance between the interior anchorages has been reduced by 0.25 in. The total extension of the prestress cable at the exterior anchorage was 24 in before stressing and 26.25 in after stressing. This difference in length is due to the movement of interior supports, compressive strain in the concrete, and tensile strain in the prestress cable.



The force in the prestressing tendon after stressing is most nearly

(A) 200 kip
(B) 300 kip
(C) 400 kip
(D) 500 kip

Design Criteria

- no friction between concrete members and bed
- assume concrete performs elastically in this stress range
- $E_s = 30 \times 10^6$ psi
- $E_c = 4 \times 10^6$ psi
- $A_{\text{concrete}} = 144 \text{ in}^2$
- $A_{\text{steel}} = 3.0 \text{ in}^2$

Solution

The 0.25 in movement between the interior abutments is a rigid body movement that has the effect of reducing the original length of the prestress tendon. Thus,

$$\begin{aligned} l_s &= \big((2)(20 \text{ ft}) + 4 \text{ ft}\big)\left(12\ \frac{\text{in}}{\text{ft}}\right) - 0.25 \text{ in} \\ &= 527.75 \text{ in} \end{aligned}$$

The total elongation of the prestress tendon is similarly reduced by the rigid body displacement of the interior abutments.

$$\begin{aligned} \Delta_o &= l_f - l_i - \Delta_{\text{rigid}} \\ &= 26.25 \text{ in} - 24.00 \text{ in} - 0.25 \text{ in} \\ &= 2 \text{ in} \end{aligned}$$

Compatibility of deformation in the tendon and concrete requires

$$\Delta_o = \frac{Pl_s}{E_{ps}A_{ps}} + \frac{Pl_c}{E_cA_c}$$

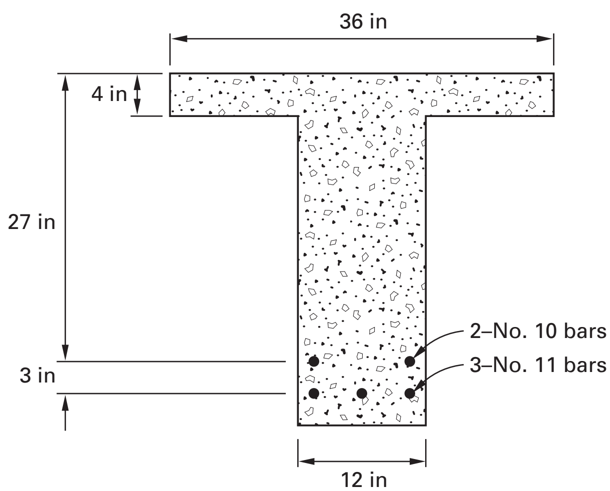
$$2.00 \text{ in} = \frac{P(527.75 \text{ in})}{\left(30 \times 10^3 \frac{\text{kip}}{\text{in}^2}\right)(3.00 \text{ in}^2)} + \frac{P(480 \text{ in})}{\left(4 \times 10^3 \frac{\text{kip}}{\text{in}^2}\right)(144.0 \text{ in}^2)}$$

$$P = 298.6 \text{ kip} \quad (300 \text{ kip})$$

The answer is (B).

Practice Problem 17

A precast concrete T-beam has flexural reinforcement placed in two layers as shown.



At flexural capacity, the strain in the extreme tension reinforcement is most nearly

(A) 0.005
(B) 0.012
(C) 0.022
(D) 0.034

Design Criteria

- $f'_c = 5$ ksi, normal weight aggregate
- $f_y = 60$ ksi

Solution

The total area of tension steel is

$$\begin{aligned} A_s &= 3A_{11} + 2A_{10} \\ &= (3)(1.56 \text{ in}^2) + (2)(1.27 \text{ in}^2) \\ &= 7.22 \text{ in}^2 \end{aligned}$$

The tension force in the steel at ultimate is

$$\begin{aligned} T &= f_yA_s = \left(60 \ \frac{\text{kip}}{\text{in}^2}\right)(7.22 \text{ in}^2) \\ &= 433 \text{ kip} \end{aligned}$$

Equilibrium of the internal forces gives the required area of the compression zone.

$$\begin{aligned} C &= T \\ 0.85f'_cA_c &= 433 \text{ kip} \\ A_c &= \frac{433 \text{ kip}}{(0.85)\left(5 \ \frac{\text{kip}}{\text{in}^2}\right)} \\ &= 102 \text{ in}^2 \end{aligned}$$

The area of equivalent compression zone is less than the flange area, 144 in^2. The compression zone is thus entirely within the flange, and the depth of compression zone is

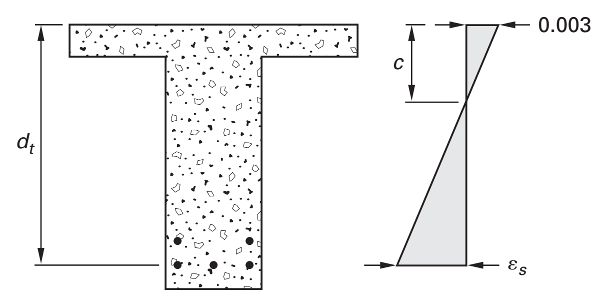
$$\begin{aligned} a &= \frac{A_c}{b_f} = \frac{102 \text{ in}^2}{36 \text{ in}} \\ &= 2.83 \text{ in} \end{aligned}$$

The depth of the neutral axis is

$$\begin{aligned} c &= \frac{a}{\beta_1} = \frac{2.83 \text{ in}}{0.8} \\ &= 3.54 \text{ in} \end{aligned}$$

From similar triangles, the strain in the extreme tension steel at flexural failure is

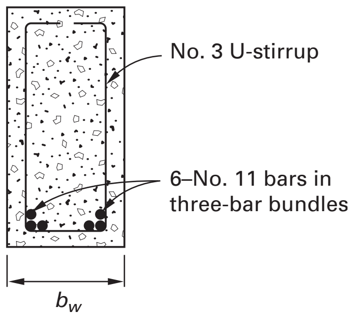
$$\begin{aligned} \frac{\varepsilon_s}{d_t - c} &= \frac{0.003}{c} \\ \varepsilon_s &= \frac{0.003\,(d_t - c)}{c} \\ &= \frac{(0.003)(30 \text{ in} - 3.54 \text{ in})}{3.54 \text{ in}} \\ &= 0.022 \end{aligned}$$



The answer is (C).

Practice Problem 18

Six no. 11 reinforcing bars are placed in a rectangular beam and bundled in groups of three in the arrangement shown.



Based on the spacing limits of ACI 318, the minimum width of the web, b_w, is most nearly

(A) 8 in
(B) 10 in
(C) 12 in
(D) 14 in

Design Criteria

- maximum aggregate size is 1 in
- interior exposure
- development of reinforcement is not a consideration

Solution

The clear spacing between rebars is a function of the nominal bar diameter, but for bundles of bars, ACI Sec. 7.6.6.5 requires that an equivalent bar diameter must be used. The equivalent bar diameter is that of a fictitious circle having the same area as the bars in the bundle, but need not be greater than 2 in. Thus,

$$\begin{aligned} A &= 3A_b = (3)(1.56 \text{ in}^2) \\ &= 4.68 \text{ in}^2 \\ \frac{\pi d_{be}^2}{4} &= A \\ d_{be} &= \sqrt{\frac{4A}{\pi}} \\ &= \sqrt{\frac{(4)(4.68 \text{ in}^2)}{\pi}} \\ &= 2.44 \text{ in} \end{aligned}$$

From ACI Sec. 7.6, the minimum clear spacing between bars is

$$s \geq \begin{cases} d_{be} = 2.44 \text{ in} \quad \text{[controls]} \\ \dfrac{4 \times \text{aggregate size}}{3} = \dfrac{(4)(1 \text{ in})}{3} = 1.33 \text{ in} \\ 1 \text{ in} \end{cases}$$

For interior exposure, the minimum clear cover from side of member to the stirrups is 1.5 in. The web must be wide enough to accommodate the cover and diameter of stirrups on both sides, the diameters of the bars in the bundles, and the clear distance between bundles. Thus,

$$\begin{aligned} b_w &\geq (2)(\text{cover} + \text{stirrup diameter}) + 4d_b + s \\ &\geq (2)(1.5 \text{ in} + 0.375 \text{ in}) + (4)(1.41 \text{ in}) + 2.44 \text{ in} \\ &\geq 11.8 \text{ in} \quad (12 \text{ in}) \end{aligned}$$

The answer is (C).

Practice Problem 19

A solid one-way slab has an overall thickness of 4.0 in and is reinforced for temperature and shrinkage with no. 4 grade 60 reinforcing bars. The minimum spacing of the no. 4 bars required by the ACI code is most nearly

(A) 12 in
(B) 18 in
(C) 20 in
(D) 28 in

Solution

The required area of temperature and shrinkage steel for a slab reinforced with grade 60 steel is

$$\begin{aligned} A_{s,\min} &= 0.0018bh \\ &= (0.0018)\left(12 \frac{\text{in}}{\text{ft}}\right)(4.0 \text{ in}) \\ &= 0.0864 \text{ in}^2/\text{ft} \end{aligned}$$

For a no. 4 bar, the area of one bar is 0.20 in^2. Therefore, the required spacing to furnish the temperature and shrinkage steel is

$$\begin{aligned} s &\leq \frac{A_b}{A_{s,\min}} \\ &\leq \left(\frac{0.20 \text{ in}^2}{0.0864 \dfrac{\text{in}^2}{\text{ft}}}\right)\left(12 \frac{\text{in}}{\text{ft}}\right) \\ &\leq 28 \text{ in} \end{aligned}$$

ACI sets a maximum spacing for bars in one-way slabs as

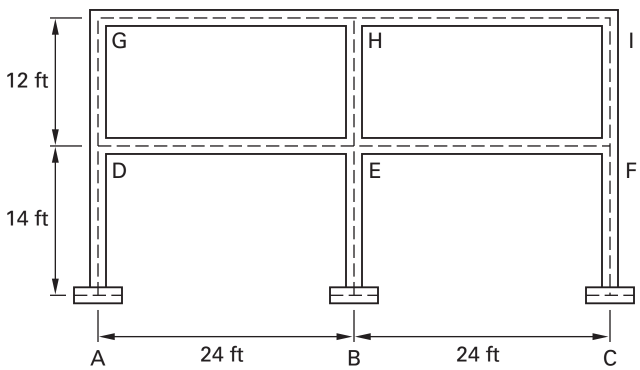
$$s \le \begin{cases} 5h = (5)(4.0\text{ in}) = 20\text{ in} \\ 18\text{ in} \quad \text{[controls]} \end{cases}$$

Thus, the spacing limit of 18 in controls.

The answer is (B).

Practice Problem 20

For the rigid frame shown, columns are 18 in by 18 in and girders are 24 in overall depth by 30 in wide. Centerline-to-centerline dimensions are given.



The effective length of member EH is most nearly

(A) 8 ft
(B) 12 ft
(C) 16 ft
(D) 20 ft

Design Criteria

- normal weight concrete with $f'_c = 4$ ksi in all members
- use alignment chart of ACI Fig. R10.10.1.1

Solution

For normal weight concrete, ACI Sec. 10.10.4.1 defines member rigidity as

$$\begin{aligned} EI_{\text{col}} &= E_c(0.7I_g) \\ &= 57{,}000\sqrt{f'_c}(0.7)\left(\frac{bh^3}{12}\right) \\ &= 57{,}000\sqrt{4000\ \frac{\text{lbf}}{\text{in}^2}}(0.7)\left(\frac{(18\text{ in})(18\text{ in})^3}{12}\right) \\ &= 22.1\times 10^9\text{ in-lbf} \end{aligned}$$

$$\begin{aligned} EI_{\text{bm}} &= E_c(0.35I_g) \\ &= 57{,}000\sqrt{f'_c}(0.35)\left(\frac{bh^3}{12}\right) \\ &= 57{,}000\sqrt{4000\ \frac{\text{lbf}}{\text{in}^2}}(0.35)\left(\frac{(30\text{ in})(24\text{ in})^3}{12}\right) \\ &= 43.6\times 10^9\text{ in-lbf} \end{aligned}$$

The stiffness parameters for the top (joint H) and bottom (joint E) ends of column EH are

$$\begin{aligned} \psi_{\text{E}} &= \frac{\sum\left(\frac{EI}{L}\right)_{\text{col}}}{\sum\left(\frac{EI}{L}\right)_{\text{bm}}} \\ &= \frac{\frac{22.1\times 10^9\text{ in-lbf}}{14\text{ ft}} + \frac{22.1\times 10^9\text{ in-lbf}}{12\text{ ft}}}{\frac{43.6\times 10^9\text{ in-lbf}}{24\text{ ft}} + \frac{43.6\times 10^9\text{ in-lbf}}{24\text{ ft}}} \\ &= 0.94 \end{aligned}$$

$$\begin{aligned} \psi_{\text{H}} &= \frac{\sum\left(\frac{EI}{L}\right)_{\text{col}}}{\sum\left(\frac{EI}{L}\right)_{\text{bm}}} \\ &= \frac{\frac{22.1\times 10^9\text{ in-lbf}}{12\text{ ft}}}{\frac{43.6\times 10^9\text{ in-lbf}}{24\text{ ft}} + \frac{43.6\times 10^9\text{ in-lbf}}{24\text{ ft}}} \\ &= 0.51 \end{aligned}$$

Use Fig. R10.10.1.1(b) to find the effective length of the column. Draw a straight line from the value of ψ_{E}, 0.94, on the left scale to the value of ψ_{H}, 0.51, on the right scale. This line intersects the center scale at 1.2, so this is the effective length, k.

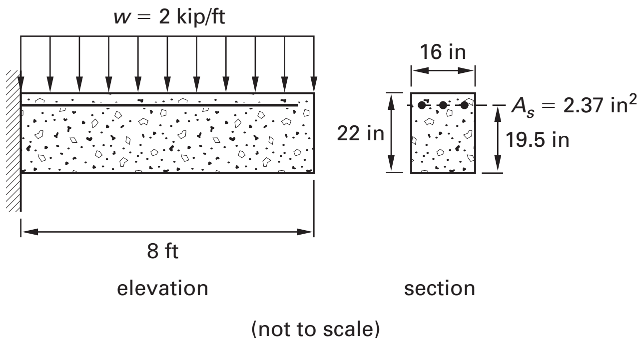
Thus, the effective length is the unsupported length times the effective length factor.

$$\begin{aligned} kl_u &= k(L - h_g) = (1.2)(12\text{ ft} - 2\text{ ft}) \\ &= 12\text{ ft} \end{aligned}$$

The answer is (B).

Practice Problem 21

The singly reinforced rectangular beam shown cantilevers 8 ft and supports a uniformly distributed service load of 2 kip/ft, which includes the weight of the beam.



The effective moment of inertia required to compute immediate deflections for this loading is most nearly

(A) 4000 in^4
(B) 5500 in^4
(C) 8300 in^4
(D) 14,000 in^4

Design Criteria

- $f'_c = 3$ ksi, normal weight concrete
- $f_r =$ modulus of rupture $= 410$ psi
- $n =$ modular ratio $= 10$
- reinforcing can fully develop at every point on beam

Solution

The maximum applied moment occurs at the face of support.

$$M_a = \frac{wL^2}{2} = \frac{\left(2\ \frac{\text{kip}}{\text{ft}}\right)(8\ \text{ft})^2}{2} = 64\ \text{ft-kip}$$

The moment of inertia of the uncracked section is

$$I_g = \frac{bh^3}{12} = \frac{(16\ \text{in})(22\ \text{in})^3}{12} = 14{,}197\ \text{in}^4$$

The cracking moment is

$$M_{\text{cr}} = \frac{f_r I_g}{y_t} = \left(\frac{\left(410\ \frac{\text{lbf}}{\text{in}^2}\right)(14{,}197\ \text{in}^4)}{11\ \text{in}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \times \left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) = 44\ \text{ft-kip}$$

For the cracked transformed section, the depth of neutral axis in a singly reinforced beam is kd.

$$\rho n = \frac{nA_s}{bd} = \frac{(10)(2.37\ \text{in}^2)}{(16\ \text{in})(19.5\ \text{in})} = 0.076$$

$$kd = d\left(\sqrt{(\rho n)^2 + 2\rho n} - \rho n\right) = (19.5\ \text{in})\left(\sqrt{0.076^2 + (2)(0.076)} - 0.076\right) = 6.26\ \text{in}$$

The moment of inertia of the cracked transformed section is

$$I_{\text{cr}} = \frac{b\,(kd)^3}{3} + nA_s\,(d - kd)^2 = \frac{(16\ \text{in})\,(6.26\ \text{in})^3}{3} + (10)(2.37\ \text{in}^2)\,(19.5\ \text{in} - 6.26\ \text{in})^2 = 5463\ \text{in}^4$$

Thus, the effective moment of inertia to be used to calculate immediate deflections is

$$I_e = \left(\frac{M_{\text{cr}}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{\text{cr}}}{M_a}\right)^3\right) I_{\text{cr}} = \left(\frac{44\ \text{ft-kip}}{64\ \text{ft-kip}}\right)^3 (14{,}197\ \text{in}^4) + \left(1 - \left(\frac{44\ \text{ft-kip}}{64\ \text{ft-kip}}\right)^3\right)(5463\ \text{in}^4) = 8300\ \text{in}^4$$

The answer is (C).

Practice Problem 22

A tied reinforced concrete column must support concentric axial compression forces of 600 kip dead load and 490 kip live load. Architectural considerations require a rectangular column with a width of 16 in.

If the longitudinal steel ratio is set to 4%, the smallest overall thickness required by ACI 318 (rounded to the nearest inch) is

(A) 16 in
(B) 22 in
(C) 24 in
(D) 28 in

Design Criteria

- $f'_c = 5$ ksi, normal weight aggregate
- $f_y = 60$ ksi

Solution

The factored design load is

$$\begin{aligned} P_u &= 1.2P_d + 1.6P_l \\ &= (1.2)(600 \text{ kip}) + (1.6)(490 \text{ kip}) \\ &= 1504 \text{ kip} \end{aligned}$$

ACI Sec. 10.3.6 gives the strength of a concentrically loaded tied column as

$$\phi P_{n,\max} = 0.8\phi(0.85f'_c(A_g - A_{\text{st}}) + f_y A_{\text{st}})$$

The factor 0.8 is required to account for accidental eccentricity. For a steel ratio of 4%, A_{st} is equal to $0.04A_g$, and the design condition can be expressed as

$$\phi P_{n,\max} = P_u$$

$$0.8\phi\big(0.85f'_c(A_g - 0.04A_g) + f_y(0.04A_g)\big) = P_u$$

$$(0.8)(0.65)\left(\begin{array}{c} (0.85)\left(5\ \dfrac{\text{kip}}{\text{in}^2}\right)(0.96A_g) \\ +\left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)(0.04A_g) \end{array}\right) = 1504 \text{ kip}$$

$$A_g = 446 \text{ in}^2$$

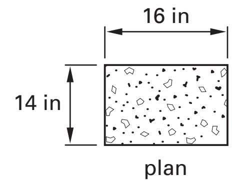
For a rectangular column with b equal to 16 in,

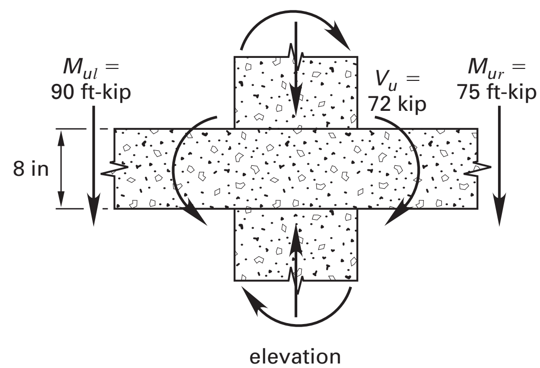
$$\begin{aligned} bh &= A_g \\ h &= \frac{A_g}{b} = \frac{446 \text{ in}^2}{16 \text{ in}} \\ &= 27.9 \text{ in} \quad (28 \text{ in}) \end{aligned}$$

The answer is (D).

Practice Problem 23

A two-way flat plate is subjected to factored gravity loads that produce a punching shear force of 72 kip and unbalanced moments with 90 ft-kip to the left of the column centerline and 75 ft-kip to the right of the column centerline, as shown.

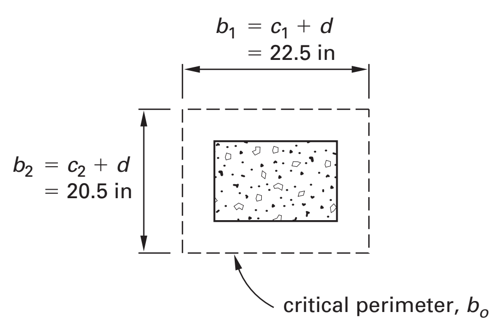




The slab has an overall thickness of 8 in and an average effective depth of 6.5 in. Columns are 14 in by 16 in in cross section, and are oriented such that bending is about the stronger axis. The maximum shear stress in the critical perimeter is most nearly

(A) 150 psi
(B) 200 psi
(C) 250 psi
(D) 300 psi

Solution



The critical perimeter resisting punching shear is

$$\begin{aligned} b_o &= 2\left((c_1 + d) + (c_2 + d)\right) \\ &= (2)\left(16 \text{ in} + 6.5 \text{ in} + 14 \text{ in} + 6.5 \text{ in}\right) \\ &= 86 \text{ in} \end{aligned}$$

ACI Sec. 13.5.3.2 requires that a fraction of the unbalanced moments transfer to the columns through flexure and the complement must transfer through eccentricity of shear, as given in ACI Sec. 11.11.7.1.

$$\begin{aligned}\gamma_f &= \frac{1}{1+\frac{2}{3}\sqrt{\frac{b_1}{b_2}}}\\ &= \frac{1}{1+\frac{2}{3}\sqrt{\frac{22.5\text{ in}}{20.5\text{ in}}}}\\ &= 0.59\end{aligned}$$

$$\begin{aligned}M_u &= (1-\gamma_f)(M_{ul}-M_{ur})\\ &= (1-0.59)(90\text{ ft-kip}-75\text{ ft-kip})\\ &= 6.15\text{ ft-kip}\end{aligned}$$

The properties of the critical section resisting the unbalanced moments are

$$\begin{aligned}A_c &= 2d(c_1+c_2+2d)\\ &= (2)(6.5\text{ in})\big(16.0\text{ in}+14.0\text{ in}+(2)(6.5\text{ in})\big)\\ &= 559\text{ in}^2\end{aligned}$$

$$\begin{aligned}J_c &= \frac{d\,(c_1+d)^3+(c_1+d)d^3}{6}+\frac{d(c_2+d)\,(c_1+d)^2}{2}\\ &= \frac{\left(\begin{array}{l}(6.5\text{ in})\,(16.0\text{ in}+6.5\text{ in})^3\\ \quad+(16.0\text{ in}+6.5\text{ in})(6.5\text{ in})^3\end{array}\right)}{6}\\ &\quad+\frac{\left(\begin{array}{l}(6.5\text{ in})(14.0\text{ in}+6.5\text{ in})\\ \quad\times(16.0\text{ in}+6.5\text{ in})^2\end{array}\right)}{2}\\ &= 47{,}100\text{ in}^4\end{aligned}$$

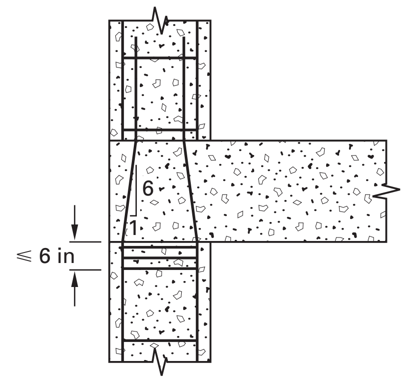
The critical stress is the sum of the stress due to direct punching shear plus that due to eccentricity of shear.

$$\begin{aligned}v_u &= \frac{V_u}{A_c}+\frac{M_u(c_1+d)}{2J_c}\\ &= \frac{(72\text{ kip})\left(1000\,\frac{\text{lbf}}{\text{kip}}\right)}{559\text{ in}^2}\\ &\quad+\frac{\left(\begin{array}{l}(6.15\text{ ft-kip})\left(12\,\frac{\text{in}}{\text{ft}}\right)\\ \quad\times\left(1000\,\frac{\text{lbf}}{\text{kip}}\right)(16.0\text{ in}+6.5\text{ in})\end{array}\right)}{(2)(47{,}100\text{ in}^4)}\\ &= 146\text{ psi}\end{aligned}$$

The answer is (A).

Practice Problem 24

A no. 10 longitudinal bar in an exterior tied column is to be offset bent at a 1-to-6 slope as shown, so that it can be lap spliced to the longitudinal bars in a column above.



elevation

The area of tie reinforcement required within 6 in of the lower bend is most nearly

(A) 0.22 in^2
(B) 0.35 in^2
(C) 0.55 in^2
(D) 0.75 in^2

Design Criteria

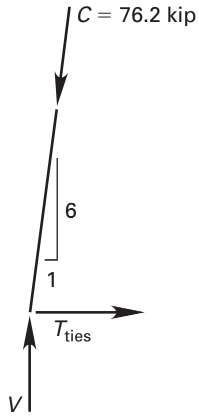
- longitudinal bars are fully stressed in compression under design loads
- $f_y = 60$ ksi

Solution

The design compressive force in the no. 10 bar is

$$\begin{aligned}C = f_yA_b &= \left(60\,\frac{\text{kip}}{\text{in}^2}\right)(1.27\text{ in}^2)\\ &= 76.2\text{ kip}\end{aligned}$$

At the bend, ACI Sec. 7.8.1.3 requires that ties, spirals, or restraint from horizontal members must resist 1.5 times the horizontal component of the force in the bar. Because the column is an exterior member, restraint from horizontal framing is ineffective and the horizontal force must be resisted by ties.



Horizontal equilibrium requires

$$\begin{aligned} \sum F_x &= 0 \text{ kip} \\ T_{\text{ties}} - C\sin\alpha &= 0 \text{ kip} \\ T_{\text{ties}} &= C\sin\alpha \\ T_{\text{ties}} &= (76.2 \text{ kip})\left(\frac{1 \text{ in}}{\sqrt{(1 \text{ in})^2 + (6 \text{ in})^2}}\right) \\ &= 12.5 \text{ kip} \end{aligned}$$

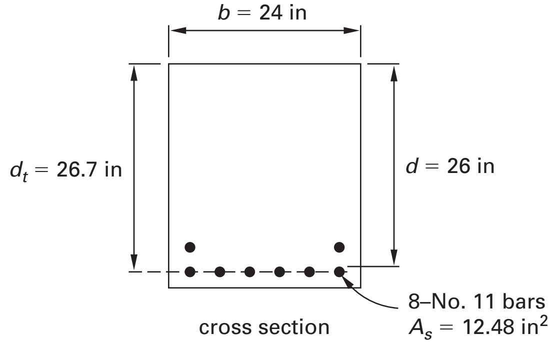
The area of ties required is

$$\begin{aligned} \phi A_s f_y &= 1.5T_{\text{ties}} \\ A_s &= \frac{(1.5)(12.5 \text{ kip})}{(0.9)\left(60 \dfrac{\text{kip}}{\text{in}^2}\right)} \\ &= 0.35 \text{ in}^2 \end{aligned}$$

The answer is (B).

Practice Problem 25

The cross section of a reinforced concrete beam is shown. Main steel consists of eight no. 11 bars with six in the bottom layer and two in an upper layer arranged as shown.



The capacity reduction factor, ϕ, that applies to the nominal moment strength of the section is most nearly

(A) 0.65
(B) 0.75
(C) 0.85
(D) 0.90

Design Criteria

- $f'_c = 4$ ksi
- $f_y = 60$ ksi

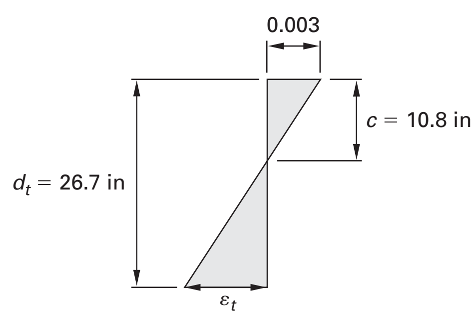
Solution

Assuming that flexural steel yields, the depth of the equivalent compression zone is

$$\begin{aligned} a &= \frac{A_s f_y}{0.85 f'_c b} = \frac{(12.48 \text{ in}^2)\left(60{,}000 \dfrac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(4000 \dfrac{\text{lbf}}{\text{in}^2}\right)(24 \text{ in})} \\ &= 9.18 \text{ in} \end{aligned}$$

The depth of the neutral axis is

$$\begin{aligned} c &= \frac{a}{\beta_1} = \frac{9.18 \text{ in}}{0.85} \\ &= 10.8 \text{ in} \end{aligned}$$



The strain in the extreme tension steel is found using similar triangles.

$$\begin{aligned} \frac{\varepsilon_t}{d_t - c} &= \frac{0.003}{c} \\ \varepsilon_t &= \frac{0.003(d_t - c)}{c} \\ &= \frac{(0.003)(26.7 \text{ in} - 10.8 \text{ in})}{10.8 \text{ in}} \\ &= 0.0044 \end{aligned}$$

Because ε_t exceeds the yield strain for grade 60 rebar ($\varepsilon_y = 0.00207$), the steel yields as assumed. The strain also exceeds the lower bound tensile strain permitted by ACI 318 for a flexural member, 0.004. However, the value of ε_t falls in the transition zone between a compression-controlled failure (taken in ACI 318 as a tensile strain of 0.002) and tension-controlled failure (0.005). In this zone, the capacity reduction factor varies linearly between 0.65 for a compression-controlled failure and 0.9 for a tension-controlled failure. Interpolating,

$$\frac{0.0044 - 0.002}{0.005 - 0.002} = \frac{\phi - 0.65}{0.90 - 0.65}$$

$$\begin{aligned} \phi &= 0.65 + \left(\frac{0.0044 - 0.002}{0.005 - 0.002}\right) \\ &\quad \times (0.90 - 0.65) \\ &= 0.85 \end{aligned}$$

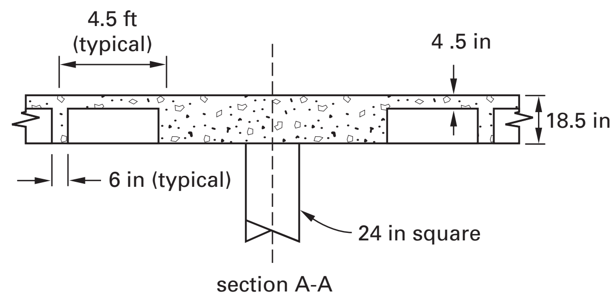
The answer is (C).

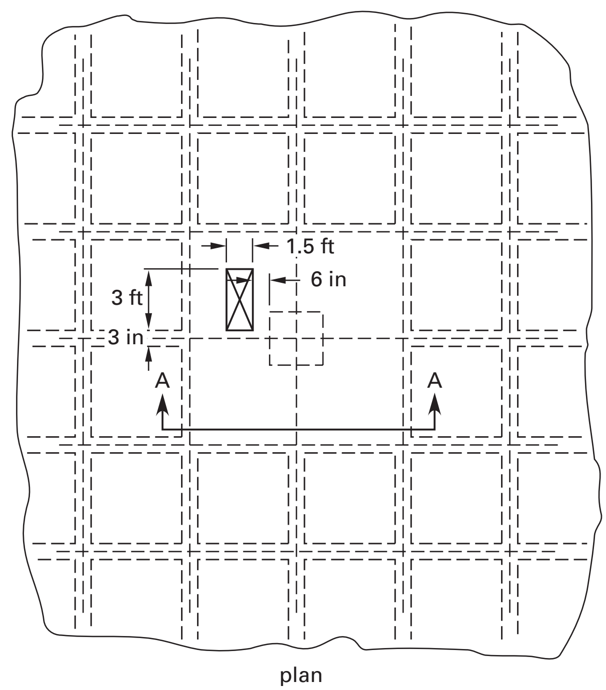
SE Breadth Exam Problems

Problems 26 through 35 cover subjects likely to be found on the breadth sections of the SE exam.

Practice Problem 26

A typical panel for an interior column of a concrete waffle floor slab is shown. The columns are 24 in square and are spaced 36 ft on center each way. Use tributary area for loads. Do not provide for unbalanced shears.





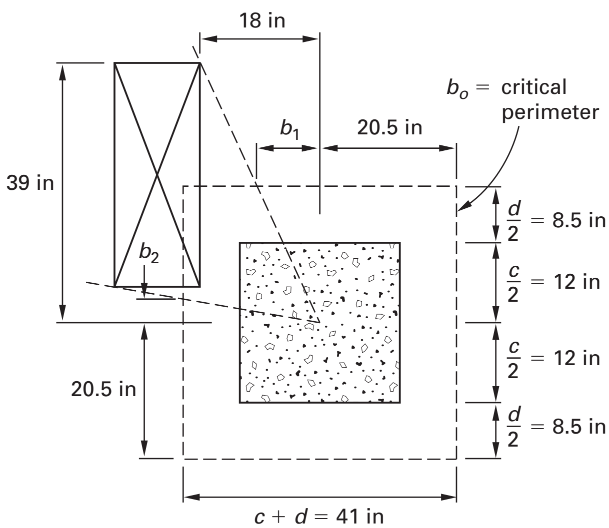
The required punching shear strength of the solid head region, ϕV_n, based on ACI 318 criteria, is most nearly

(A) 300 kip
(B) 380 kip
(C) 500 kip
(D) 560 kip

Design Criteria

- service level dead load = 90 lbf/ft^2 (averaged)
- service live load = 100 lbf/ft^2 (not reducible)
- use an average effective depth equal to the difference between the overall depth and 1.5 in

Solution



The critical perimeter is located at $d/2$ from the face of the column, except where the perimeter intersects the opening. For these two sides, ACI 318 requires that the critical perimeter stop where it intersects the lines drawn from the center of the column to the tangents at the edge of opening. Similar triangles give the required lengths. For example, to find the distance b_1,

$$\frac{b_1}{\frac{c}{2}+\frac{d}{2}} = \frac{18 \text{ in}}{39 \text{ in}}$$

$$\begin{aligned} b_1 &= \frac{18 \text{ in}}{39 \text{ in}}\left(\frac{c}{2}+\frac{d}{2}\right) \\ &= \left(\frac{18 \text{ in}}{39 \text{ in}}\right)\left(\frac{24 \text{ in}}{2}+\frac{17 \text{ in}}{2}\right) \\ &= 9.5 \text{ in} \end{aligned}$$

b_2 is similarly found by using similar triangles.

$$\frac{b_2}{\frac{c}{2}+\frac{d}{2}} = \frac{3 \text{ in}}{12 \text{ in} + 24 \text{ in}}$$

$$\begin{aligned} b_2 &= \left(\frac{3 \text{ in}}{12 \text{ in} + 24 \text{ in}}\right)\left(\frac{c}{2}+\frac{d}{2}\right) \\ &= \left(\frac{3 \text{ in}}{36 \text{ in}}\right)\left(\frac{24 \text{ in}}{2}+\frac{17 \text{ in}}{2}\right) \\ &= 1.71 \text{ in} \end{aligned}$$

The critical perimeter for punching shear is

$$\begin{aligned} b_o &= b_1 + b_2 + 3c + 3d \\ &= 9.5 \text{ in} + 1.71 \text{ in} + (3)(24 \text{ in}) + (3)(17 \text{ in}) \\ &= 134.2 \text{ in} \end{aligned}$$

From ACI Sec. 11.11.2.1, calculate the nominal punching shear strength.

$$\beta_c = \frac{\text{longer side length}}{\text{shorter side length}} = \frac{24 \text{ in}}{24 \text{ in}} = 1.0$$

$$V_c \le \begin{cases} \left(2+\frac{4}{\beta_c}\right)\lambda\sqrt{f'_c}b_o d \\ \quad = \left(2+\frac{4}{1}\right)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}(134.2 \text{ in})(17 \text{ in}) \\ \quad = 750{,}000 \text{ lbf} \\ \left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f'_c}b_o d \\ \quad = \left(\frac{(30)(17 \text{ in})}{134.2 \text{ in}}+2\right)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}} \\ \quad\quad \times (134.2 \text{ in})(17 \text{ in}) \\ \quad = 725{,}000 \text{ lbf} \\ 4\lambda\sqrt{f'_c}b_o d \\ \quad = (4)(1)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}(134.2 \text{ in})(17 \text{ in}) \\ \quad = 500{,}000 \text{ lbf} \quad \text{[controls]} \end{cases}$$

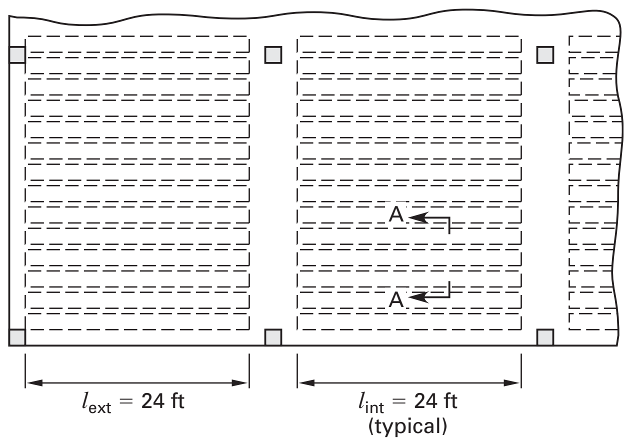
The design punching shear strength is

$$\begin{aligned} \phi V_n &= \phi V_c \\ &= (0.75)(500{,}000 \text{ lbf})\left(\frac{1 \text{ kip}}{1000 \text{ lbf}}\right) \\ &= 375 \text{ kip} \quad (380 \text{ kip}) \end{aligned}$$

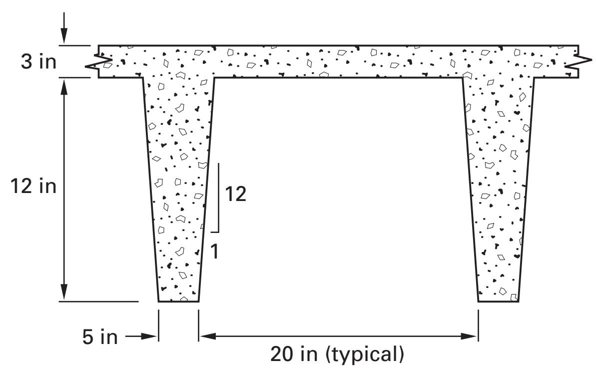
The answer is (B).

Practice Problem 27

A partial framing plan for a one-way ribbed slab system is shown.



partial floor plan



section A-A
(not to scale)

The factored shear force in a typical joist at the most critical location is most nearly

(A) 6200 lbf
(B) 7200 lbf
(C) 7800 lbf
(D) 8400 lbf

Design Criteria

- normal weight concrete (150 lbf/ft^3)
- standard module joists spaced 25 in on centers
- allow 1 in for cover and depth to main reinforcement
- superimposed dead load of 20 lbf/ft^2
- live load of 100 lbf/ft^2 (nonreducible)
- use approximate analysis method of ACI Sec. 8.3

Solution

The joists taper 1 to 12, so their tops are 2 in wider than their bottoms. Thus, their average width is (5 in + 7 in)/2 = 6 in. The dead load on a typical ribbed slab element is

$$\begin{aligned} w_d &= (b_{\text{ave}}h_j + h_s B)\,w_c + w_{d,\text{sup}}B \\ &= \left(\frac{(6\text{ in})(12\text{ in}) + (3\text{ in})(25\text{ in})}{144\ \frac{\text{in}^2}{\text{ft}^2}}\right)\left(150\ \frac{\text{lbf}}{\text{ft}^3}\right) \\ &\quad + \left(20\ \frac{\text{lbf}}{\text{ft}^2}\right)\left(\frac{25\text{ in}}{12\ \frac{\text{in}}{\text{ft}}}\right) \\ &= 195\text{ lbf/ft} \end{aligned}$$

The service live load on a typical element is

$$\begin{aligned} w_l &= w_{l,\text{sup}}B = \left(100\ \frac{\text{lbf}}{\text{ft}^2}\right)\left(\frac{25\text{ in}}{12\ \frac{\text{in}}{\text{ft}}}\right) \\ &= 208\text{ lbf/ft} \end{aligned}$$

The factored load is

$$w_u \geq \begin{cases} 1.4w_d = (1.4)\left(195\ \frac{\text{lbf}}{\text{ft}}\right) \\ \qquad = 273\text{ lbf/ft} \\ 1.2w_d + 1.6w_l \\ \qquad = (1.2)\left(195\ \frac{\text{lbf}}{\text{ft}}\right) + (1.6)\left(208\ \frac{\text{lbf}}{\text{ft}}\right) \\ \qquad = 567\text{ lbf/ft} \quad \text{[controls]} \end{cases}$$

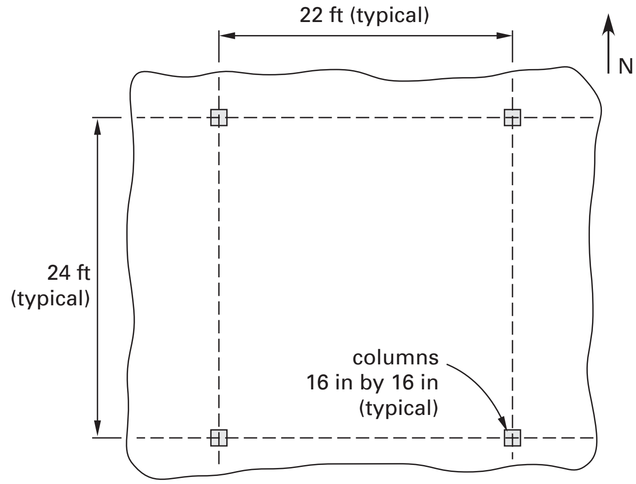
The critical shear occurs at d-distance from the face of the first interior support.

$$\begin{aligned} d &= h - 1\text{ in} = (15\text{ in} - 1\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right) \\ &= 1.17\text{ ft} \\ V_u &= \frac{1.15w_u l_{\text{ext}}}{2} - w_u d \\ &= \frac{(1.15)\left(567\ \frac{\text{lbf}}{\text{ft}}\right)(24\text{ ft})}{2} - \left(567\ \frac{\text{lbf}}{\text{ft}}\right)(1.17\text{ ft}) \\ &= 7161\text{ lbf} \quad (7200\text{ lbf}) \end{aligned}$$

The answer is (B).

Practice Problem 28

The partial plan of a two-way flat plate slab is shown. The typical interior bay shown extends for at least one additional bay in all directions.



partial floor framing plan

The required area of flexural steel in the column strip in the north-south direction is most nearly

(A) 0.14 in^2/ft
(B) 0.17 in^2/ft
(C) 0.58 in^2/ft
(D) 0.72 in^2/ft

Design Criteria

- $f'_c = 3.5$ ksi
- $f_y = 60$ ksi
- service dead load = 120 lbf/ft^2 (includes slab weight)
- service live load = 100 lbf/ft^2 (nonreducible)
- effective slab depth = 6.5 in
- use direct design method of ACI 318

Solution

The factored design load is

$$w_u \geq \begin{cases} 1.4w_d = (1.4)\left(120\ \frac{\text{lbf}}{\text{ft}^2}\right) \\ \quad = 168\ \text{lbf/ft}^2 \\ 1.2w_d + 1.6w_l \\ \quad = (1.2)\left(120\ \frac{\text{lbf}}{\text{ft}^2}\right) + (1.6)\left(100\ \frac{\text{lbf}}{\text{ft}^2}\right) \\ \quad = 304\ \text{lbf/ft}^2 \quad \text{[controls]} \end{cases}$$

The clear span in the north-south direction is

$$l_n = l_1 - h = 24\ \text{ft} - (16\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) = 22.67\ \text{ft}$$

The statical moment is

$$M_o = \frac{w_u l_2 l_n^2}{8} = \frac{\left(304\ \frac{\text{lbf}}{\text{ft}^2}\right)(22\ \text{ft})(22.67\ \text{ft})^2}{8} = 429{,}600\ \text{ft-lbf}$$

The width of column strip is

$$B \leq \begin{cases} 0.5l_1 = (0.5)(24\ \text{ft}) = 12\ \text{ft} \\ 0.5l_2 = (0.5)(22\ \text{ft}) = 11\ \text{ft} \quad \text{[controls]} \end{cases}$$

The direct design method assigns the percentage of static moment in the negative regions of an interior panel as 65% and the percentage assigned to the column strip as 75% of this value. Thus, the bending moment in the north-south direction of the column strip, on a per-foot-of-width basis, is

$$m_{cs}^- = \frac{(0.65)(0.75)M_o}{B} = \frac{(0.65)(0.75)(429{,}600\ \text{lbf-ft})}{11\ \text{ft}} = 19{,}040\ \text{ft-lbf/ft}$$

On a per-foot basis, with b equal to 12 in, the required area of flexural steel is

$$m_{cs}^- = \phi M_n = \phi\rho b d^2 f_y\left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right)$$

$$\left(19{,}040\ \frac{\text{ft-lbf}}{\text{ft}}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right) = 0.9\rho(12\ \text{in})(6.5\ \text{in})^2\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right) \times \left(1 - 0.59\rho\left(\frac{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}{3500\ \frac{\text{lbf}}{\text{in}^2}}\right)\right)$$

$$\rho = 0.0092$$

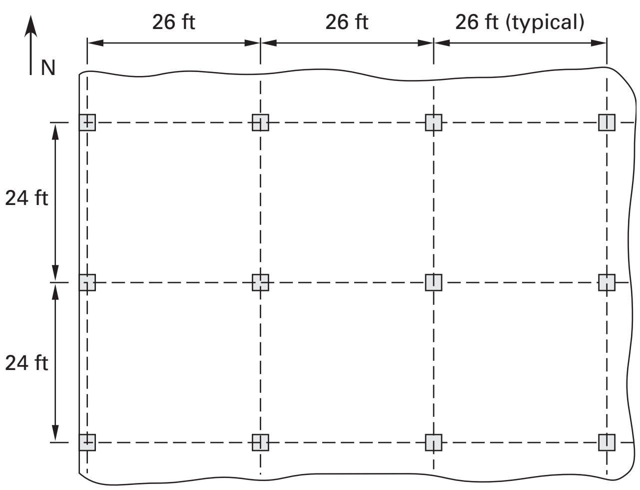
The required steel area is

$$A_s \geq \begin{cases} \rho bd = (0.0092)\left(12\ \frac{\text{in}}{\text{ft}}\right)(6.5\ \text{in}) \\ \quad = 0.72\ \text{in}^2/\text{ft} \quad [\text{controls}] \\ 0.0018bh = (0.0018)\left(12\ \frac{\text{in}}{\text{ft}}\right)(8\ \text{in}) \\ \quad = 0.17\ \text{in}^2/\text{ft} \end{cases}$$

The answer is (D).

Practice Problem 29

The two-way flat plate shown consists of rectangular panels spanning 24 ft in the north-south direction and 26 ft in the east-west direction. Columns are 16 in square and there are no spandrel beams on the perimeter.



partial floor framing plan

The minimum slab thickness to satisfy ACI serviceability requirements is most nearly

(A) 5 in
(B) 6 in
(C) 8 in
(D) 10 in

Design Criteria

- normal weight concrete, $f'_c = 3.5$ ksi
- grade 60 reinforcement, $f_y = 60$ ksi

Solution

Because all panels measure 24 ft by 26 ft and there are no spandrel beams, the larger clear span in the exterior panels controls the minimum slab thickness. The clear span is

$$l_n = l_1 - c_1 = 26\ \text{ft} - (16\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) = 24.67\ \text{ft}$$

For grade 60 reinforcement, ACI Table 9.5(c) gives the thickness for slabs without drop panels or edge beams as

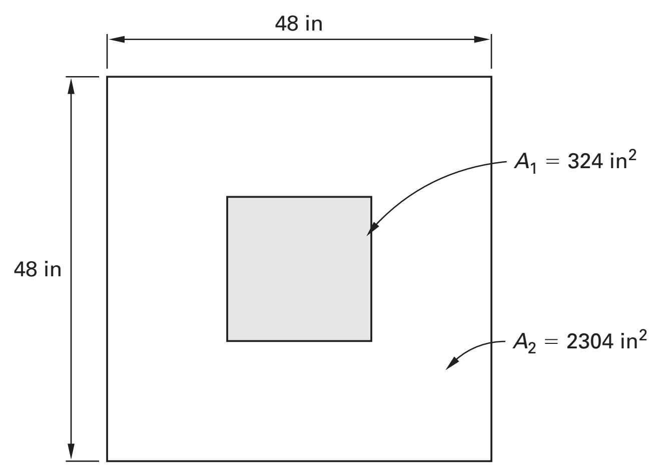
$$h \geq \frac{l_n}{30} = \frac{(24.67\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{30} = 9.9\ \text{in} \quad (10\ \text{in})$$

This exceeds the lower limit of 5 in required by ACI Sec. 9.5.3.2. Therefore, use a thickness of 10 in.

The answer is (D).

Practice Problem 30

An 18 in by 18 in column with a compressive strength of 5 ksi is reinforced with six no. 10 grade 60 rebars concentrically loaded in compression with a factored force of 980 kip. The column bears on a pedestal of normal weight, 4 ksi concrete having an effective depth of 36 in and plan dimensions as shown.



plan

The minimum area of grade 60 dowels that is required from the pedestal into the column is most nearly

(A) 1.6 in^2
(B) 2.2 in^2
(C) 4.4 in^2
(D) 6.2 in^2

Solution

From ACI Sec. 10.14.1, calculate the bearing strength of the pedestal.

$$\Omega \leq \begin{cases} \sqrt{\dfrac{A_2}{A_1}} = \sqrt{\dfrac{2304 \text{ in}^2}{324 \text{ in}^2}} = 2.7 \\ 2.0 \quad \text{[controls]} \end{cases}$$

$$\begin{aligned} \phi P_{n,\text{ped}} &= \phi\Omega(0.85 f'_{c,\text{ped}} A_1) \\ &= (0.65)(2.0)(0.85)\left(4 \ \frac{\text{kip}}{\text{in}^2}\right)(324 \text{ in}^2) \\ &= 1432 \text{ kip} \end{aligned}$$

Bearing strength of the column at the base is

$$\begin{aligned} \phi P_{n,\text{col}} &= \phi(0.85 f'_{c,\text{col}} A_1) \\ &= (0.65)(0.85)\left(5 \ \frac{\text{kip}}{\text{in}^2}\right)(324 \text{ in}^2) \\ &= 895 \text{ kip} \\ &< P_u = 980 \text{ kip} \end{aligned}$$

Because the bearing strength is smaller than the factored axial force in the column, dowels are required.

$$\begin{aligned} A_{\text{st}} &\geq \frac{P_u - \phi P_n}{\phi f_y} \\ &\geq \frac{980 \text{ kip} - 895 \text{ kip}}{(0.65)\left(60 \ \dfrac{\text{kip}}{\text{in}^2}\right)} \\ &\geq 2.18 \text{ in}^2 \quad (2.2 \text{ in}^2) \end{aligned}$$

ACI Sec. 15.8.2.1 requires a minimum area of

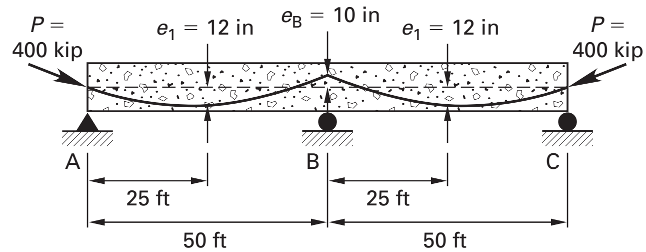
$$\begin{aligned} A_{s,\min} &= 0.005 A_g = (0.005)(324 \text{ in}^2) \\ &= 1.62 \text{ in}^2 \end{aligned}$$

The strength requirement controls, so A_{st} equals 2.18 in^2.

The answer is (B).

Practice Problem 31

A two-span post-tensioned concrete beam has an effective prestress of 400 kip in the idealized profile shown. The beam's modulus of elasticity is 3600 ksi and the moment of inertia about the axis of bending is 32,000 in^4.

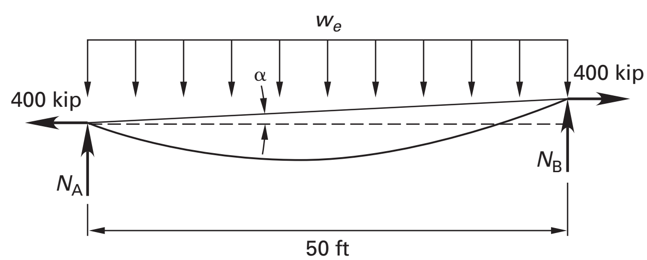


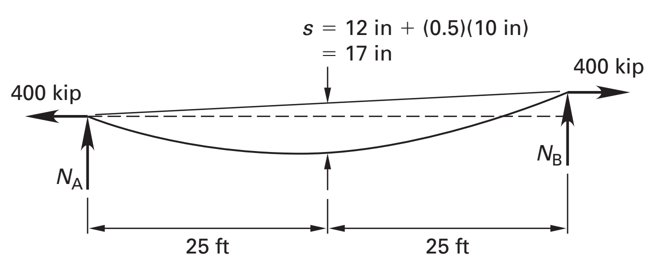
The secondary reaction that develops at support B due to prestressing is most nearly equal to

(A) 0 kip
(B) 5 kip
(C) 10 kip
(D) 20 kip

Solution

Draw a free-body diagram of the tendon from A to B.





The equivalent prestress forces are

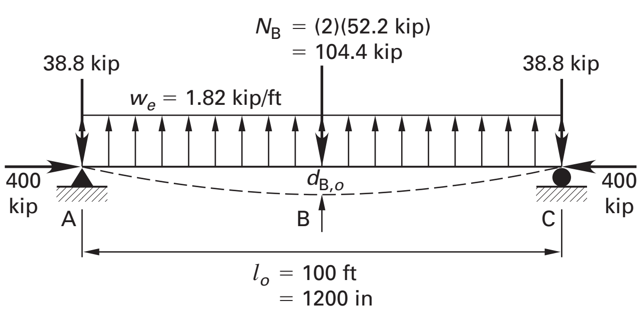
$$\begin{aligned} s &= e_1 + 0.5 e_{\text{B}} \\ &= \big(12 \text{ in} + (0.5)(10 \text{ in})\big)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \\ &= 1.42 \text{ ft} \end{aligned}$$

$$w_e = \frac{8Ps}{L^2} = \frac{(8)(400 \text{ kip})(1.42 \text{ ft})}{(50 \text{ ft})^2} = 1.82 \text{ kip/ft}$$

$$N_\text{A} = \frac{w_e L}{2} - \frac{Pe_\text{B}}{L} = \frac{\left(1.82 \ \frac{\text{kip}}{\text{ft}}\right)(50 \text{ ft})}{2} - \frac{(400 \text{ kip})(10 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)}{50 \text{ ft}} = 38.8 \text{ kip}$$

$$N_\text{B} = \frac{w_e L}{2} + \frac{Pe_\text{B}}{L} = \frac{\left(1.82 \ \frac{\text{kip}}{\text{ft}}\right)(50 \text{ ft})}{2} + \frac{(400 \text{ kip})(10 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)}{50 \text{ ft}} = 52.2 \text{ kips}$$

The structure is statically indeterminate to one degree. Choose the reaction at B as the unknown force and release it.



$$d_{\text{B},o} = \frac{5w_e l_o^4}{384EI} - \frac{N_B l_o^3}{48EI} = \frac{(5)\left(1.82 \ \frac{\text{kip}}{\text{ft}}\right)(100 \text{ ft})^4\left(12 \ \frac{\text{in}}{\text{ft}}\right)^3}{(384)\left(3600 \ \frac{\text{kip}}{\text{in}^2}\right)(32{,}000 \text{ in}^4)} - \frac{(104.4 \text{ kip})(100 \text{ ft})^3\left(12 \ \frac{\text{in}}{\text{ft}}\right)^3}{(48)\left(3600 \ \frac{\text{kip}}{\text{in}^2}\right)(32{,}000 \text{ in}^4)} = 2.92 \text{ in} \quad [\text{upward}]$$

The flexibility coefficient is obtained by applying a unit force upward at the released point and computing the deflection caused by that force.

$$f_\text{BB} = \frac{l_o^3}{48EI} = \frac{(100 \text{ ft})^3\left(12 \ \frac{\text{in}}{\text{ft}}\right)^3}{(48)\left(3600 \ \frac{\text{kip}}{\text{in}^2}\right)(32{,}000 \text{ in}^4)} = 0.3125 \text{ in/kip} \quad [\text{upward}]$$

For consistent displacement at B,

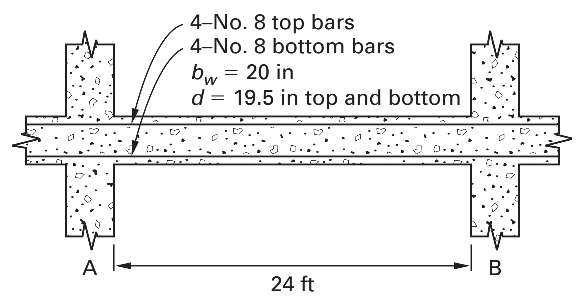
$$R_\text{B} f_\text{BB} + d_{\text{B},o} = 0 \text{ in}$$

$$R_\text{B} = \frac{-d_{\text{B},o}}{f_\text{BB}} = \frac{-2.92 \text{ in}}{0.3125 \ \frac{\text{in}}{\text{kip}}} = 9.34 \text{ kip} \quad [\text{downward}]$$

The answer is (C).

Practice Problem 32

The rectangular reinforced concrete beam shown is part of a special moment frame in a region of high seismic risk. The beam is subjected to service dead load of 2.5 kip/ft and live load due to office occupancy equal to 1.8 kip/ft. Treat the beam as singly reinforced for purpose of computing nominal moment strength and assume axial force is zero.



elevation

The design shear in the vicinity of the support is most nearly

(A) 25 kip
(B) 38 kip
(C) 64 kip
(D) 77 kip

Design Criteria

- $f'_c = 5$ ksi
- $f_y = 60$ ksi

Solution

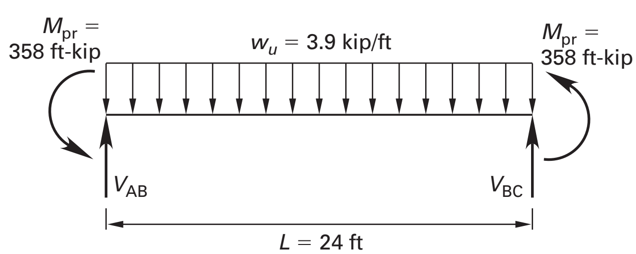
From ACI Sec. 21.5.4, the probable moment strengths of the member are

$$\begin{aligned}a &= \frac{A_{\text{st}}(1.25f_y)}{0.85f_c'b_w}\\ &= \frac{(4)(0.79\ \text{in}^2)(1.25)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(5\ \frac{\text{kip}}{\text{in}^2}\right)(20\ \text{in})}\\ &= 2.79\ \text{in}\end{aligned}$$

$$\begin{aligned}M_{\text{pr}} &= A_{\text{st}}\,(1.25f_y)\left(d-\frac{a}{2}\right)\\ &= (4)(0.79\ \text{in}^2)(1.25)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)\\ &\quad\times\left(19.5\ \text{in}-\frac{2.79\ \text{in}}{2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)\\ &= 358\ \text{ft-kip}\end{aligned}$$

The gravity load acting in combination with earthquake load is

$$\begin{aligned}w_u &= 1.2w_d + 0.5w_l\\ &= (1.2)\left(2.5\ \frac{\text{kip}}{\text{ft}}\right) + (0.5)\left(1.8\ \frac{\text{kip}}{\text{ft}}\right)\\ &= 3.9\ \text{kip/ft}\end{aligned}$$



Maximum shear occurs at the left end when hinges form in the pattern shown.

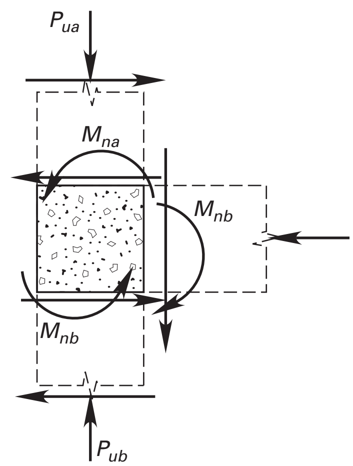
$$\begin{aligned}V_{\text{AB}} &= \frac{w_uL}{2} + \frac{2M_{\text{pr}}}{L}\\ &= \frac{\left(3.9\ \frac{\text{kip}}{\text{ft}}\right)(24\ \text{ft})}{2} + \frac{(2)(358\ \text{ft-kip})}{24\ \text{ft}}\\ &= 76.6\ \text{kip}\quad(77\ \text{kip})\end{aligned}$$

The answer is (D).

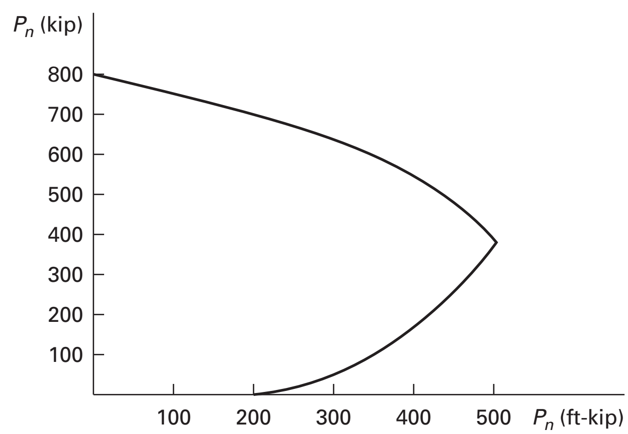
Practice Problem 33

The rectangular reinforced concrete column shown is part of a special moment frame in a region of high seismic risk. The columns above and below the joint have the same cross section containing properly developed and confined concrete whose interaction diagram is shown. Under factored gravity and seismic loads, the design axial compression forces on the columns are

- case 1: $P_{ua} = 260$ kip, $P_{ub} = 320$ kip
- case 2: $P_{ua} = 55$ kip, $P_{ub} = 100$ kip



elevation – exterior joint



column interaction diagram

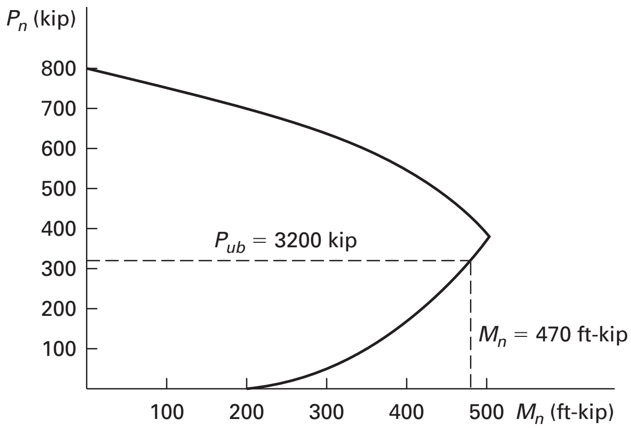
The maximum nominal moment strength of the beam permitted by ACI to alleviate plastic hinges forming first in the columns for the loading cases is most nearly

(A) 440 ft-kip
(B) 540 ft-kip
(C) 640 ft-kip
(D) 740 ft-kip

Solution

For the largest axial compression force acting on the column, the nominal moment strength is controlled by tension yielding in the column. Therefore, the critical case, which gives the smallest moment capacity, is based on load case 2. For these loads,

1) when P_{ua} equals 55 kip, M_{nca} is equal to 300 ft-kip
2) when P_{ub} equals 100 kip, M_{ncb} is equal to 350 ft-kip



column interaction diagram

The limiting nominal moment strength for the beam must satisfy ACI Eq. 21-1.

$$\sum M_{nc} \geq \tfrac{6}{5} \sum M_{nb}$$

$$M_{nca} + M_{ncb} \geq 1.2 M_{nb}$$

$$M_{nb} \leq \frac{M_{nca} + M_{ncb}}{1.2}$$

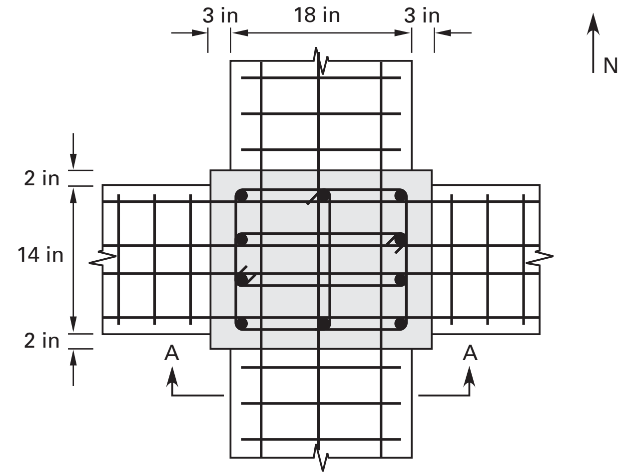
$$\leq \frac{300 \text{ ft-kip} + 350 \text{ ft-kip}}{1.2}$$

$$\leq 542 \text{ ft-kip} \quad (540 \text{ ft-kip})$$

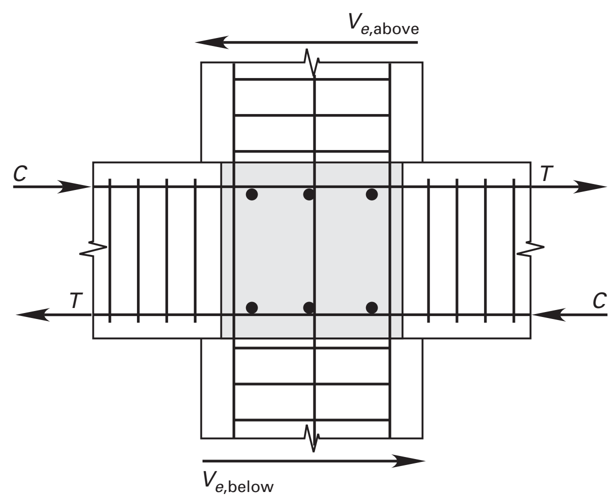
The answer is (B).

Practice Problem 34

The column shown supports a 14 in wide by 18 in deep girder reinforced with four no. 8 bars top and bottom in one direction, and 18 in wide by 18 in deep girders in the perpendicular direction, as shown.



plan



section A-A through joint
(bending moments and axial forces omitted for clarity)

Concrete is normal weight with f'_c equal to 5 ksi, and steel has a specified yield strength of 60 ksi. For ground motion in the east-west direction, the design earthquake shear in the column is 20 kip. The ratio of the design strength of the joint, ϕV_n, to the design shear in the confined joint, V_u, is most nearly equal to

(A) 0.75
(B) 1.00
(C) 1.30
(D) 1.50

Solution

Because beams confine the joint on four faces, hoops spaced at one-half the required spacing above and below the joint may be used. For the direction considered, the joint depth, h, is 24 in and the effective width is given by ACI Sec. 21.7.4 (see Fig. R21.7.4) as

$$b_e \leq \begin{cases} b + h = 14 \text{ in} + 24 \text{ in} = 38 \text{ in} \\ b + 2x = 14 \text{ in} + (2)(2 \text{ in}) \\ \quad = 18 \text{ in} \quad \text{[controls]} \end{cases}$$

$$\begin{aligned} \phi V_n &= \phi 20\sqrt{f'_c}A_j \\ &= (0.85)(20)\sqrt{5000 \ \frac{\text{lbf}}{\text{in}^2}}(18 \text{ in})(24 \text{ in})\left(\frac{1 \text{ kip}}{1000 \text{ lbf}}\right) \\ &= 519 \text{ kip} \end{aligned}$$

Note that ACI 9.3.4(c) permits $\phi = 0.85$ for this case. Including the shear in the column, the shear in the joint is

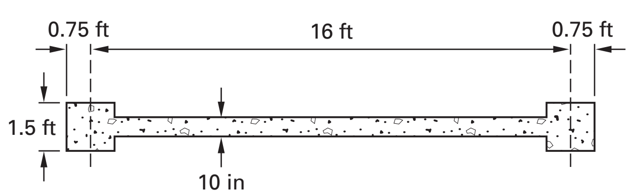
$$\begin{aligned} V_u &= (2)(1.25f_y)A_s - V_{ea} \\ &= (2)(1.25)\left(60 \ \frac{\text{kip}}{\text{in}^2}\right)(4)(0.79 \text{ in}^2) - 20 \text{ kip} \\ &= 454 \text{ kip} < \phi V_n \\ \frac{\phi V_n}{V_u} &= \frac{458 \text{ kip}}{454 \text{ kip}} = 1.01 \quad (1.00) \end{aligned}$$

Therefore, the joint strength is adequate.

The answer is (B).

Practice Problem 35

The reinforced concrete shear wall shown in cross section is part of the lateral force resisting system in a high-rise building located in a region of high seismic risk. Concrete is normal weight, 4 ksi; reinforcing steel has a specified yield stress of 60 ksi. The overall wall height is 135 ft and the controlling factored load case at the base $(0.9D + 1.0E)$ subjects the wall to a seismic shear, V_u, of 575 kip.



section at base

(not to scale)

The area of vertical steel required in the web per foot of length is most nearly

(A) 0.2 in^2/ft
(B) 0.3 in^2/ft
(C) 0.4 in^2/ft
(D) 0.5 in^2/ft

Solution

ACI Sec. 21.9.4 controls the shear strength. Calculate the height-to-length ratio of the wall.

$$V_u \leq \phi V_n$$

$$\frac{h_w}{l_w} = \frac{135 \text{ ft}}{17.5 \text{ ft}} = 7.7$$

This ratio is larger than 2.0, so α_c is equal to 2.0.

$$\begin{aligned} A_{cv} &= l_w t_w = (17.5 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)(10 \text{ in}) \\ &= 2100 \text{ in}^2 \\ \phi V_n &= V_u = \phi A_{cv}\left(\alpha_c\sqrt{f'_c} + \rho_t f_y\right) \\ \rho_t &= \frac{\dfrac{V_u}{\phi A_{cv}} - \alpha_c\sqrt{f'_c}}{f_y} \\ &= \frac{\dfrac{575{,}000 \text{ lbf}}{(0.75)(2100 \text{ in}^2)} - 2.0\sqrt{4000 \ \dfrac{\text{lbf}}{\text{in}^2}}}{60{,}000 \ \dfrac{\text{lbf}}{\text{in}}^2} \\ &= 0.004 \end{aligned}$$

The reinforcing percentage required to resist the shear is greater than the minimum specified by ACI Sec. 21.9.2.

$$\rho_t \geq \rho_{t,\min} = 0.0025$$

Therefore,

$$\begin{aligned} A_t &= \rho_t b t_w \\ &= (0.004)\left(12 \ \frac{\text{in}}{\text{ft}}\right)(10 \text{ in}) \\ &= 0.48 \text{ in}^2/\text{ft} \quad (0.5 \text{ in}^2/\text{ft}) \end{aligned}$$

The answer is (D).

SE Depth Exam Problems

Problems 36 and 37 are essay (design) problems similar to those on the depth sections of the SE exam.

Practice Problem 36

A preliminary partial plan and a typical cross section are shown for a two-story reinforced concrete building.

Design Criteria

- code is current ACI 318
- $f'_c = 3$ ksi, normal weight concrete
- ASTM A615 grade 60 reinforcement
- design live load = 100 lbf/ft^2 (nonreducible)
- ceiling, mechanical, and partitions = 30 lbf/ft^2
- columns = 18 in square

In addition to these general criteria, the following criteria apply to the second floor framing system.

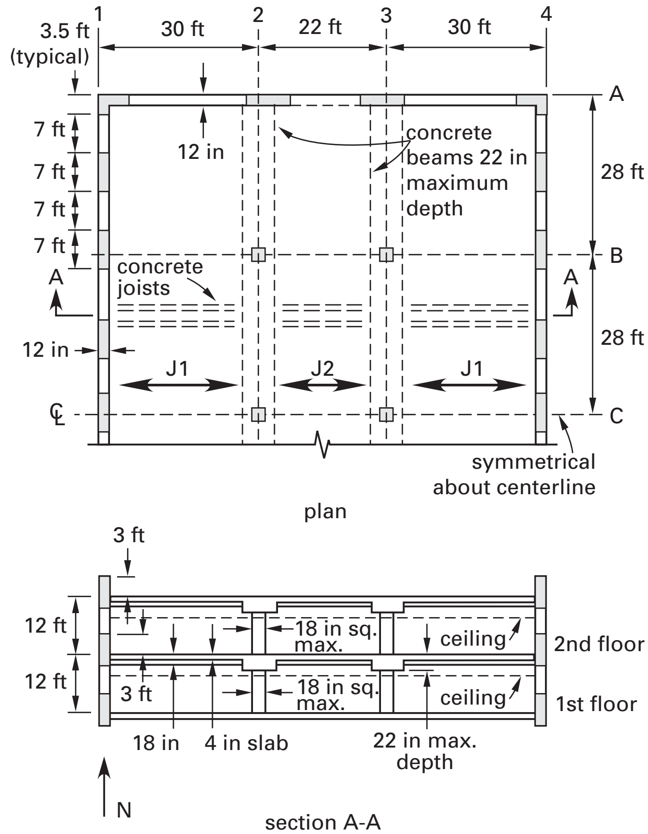
- 4 in thick concrete slab
- 20 in wide by 14 in deep joist pans
- joists are oriented to span east-west as indicated.
- beams on lines 2 and 3 are limited to a depth of 22 in (18 in + 4 in slab) and a width of 30 in.
- take advantage of moment redistribution and assume a maximum redistribution of 10%.

Required

A. Considering only the vertical loads, prepare a design for the second floor slab and for the second floor joists. Assume a pin-centered support at exterior walls.

B. Design the second floor beams on lines 2 and 3.

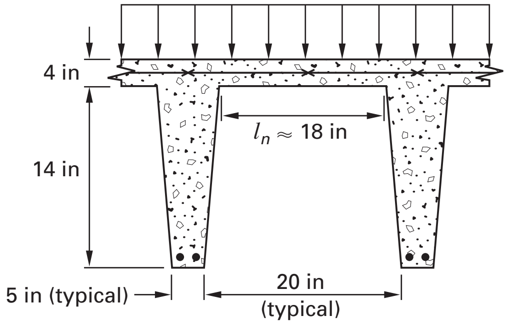
C. Draw appropriate sketches to show a complete design and reinforcement for the joints and the beams. Use scale 3/4 in = 1 ft or larger. Lengths may be cut to fit the paper. Show placing of all bars including splices, clearances, and cut-off points.



Solution

A. Design the 4 in thick one-way slab over the joist on a per-linear-foot-of-width basis. Use the strength design method (ACI 318), with $f'_c = 3$ ksi, $f_y = 60$ ksi, and normal weight (150 lbf/ft^3) concrete. Therefore, the 4 in slab weight is 50 lbf/ft^2, partitions plus ceiling impose an additional 30 lbf/ft^2 dead load, and the live load is 100 lbf/ft^2 which is nonreducible on the slab. Thus,

$$\begin{aligned} w_u &= 1.2w_d + 1.6w_l \\ &= (1.2)\left(50\ \frac{\text{lbf}}{\text{ft}^2} + 30\ \frac{\text{lbf}}{\text{ft}^2}\right)(1\text{ ft}) \\ &\quad + (1.6)\left(100\ \frac{\text{lbf}}{\text{ft}^2}\right)(1\text{ ft}) \\ &= 256\ \text{lbf/ft} \end{aligned}$$



section through typical joists

Reinforce the slab using welded wire fabric. Take $d^- = d^+ = 2$ in; joists taper such that clear span, l_n, is approximately 18 in. Using the moment coefficients of ACI Sec. 8.3,

$$M_u = \frac{w_u l_n^2}{12} = \frac{\left(256\ \frac{\text{lbf}}{\text{ft}}\right)(1.5\ \text{ft})^2}{12}$$
$$= 48\ \text{ft-lbf}$$

$$\phi M_n = M_u = \phi\rho b d^2 f_y \left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right)$$

$$(0.048\ \text{ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right) = 0.9\rho(12\ \text{in})(2\ \text{in})^2\left(60\ \frac{\text{kip}}{\text{in}^2}\right) \times \left(1 - 0.59\rho\left(\frac{60\ \frac{\text{kip}}{\text{in}^2}}{3\ \frac{\text{kip}}{\text{in}^2}}\right)\right)$$

$$\rho = 0.00022$$

$$A_s = \rho b d = (0.00022)(2.0\ \text{in})\left(12\ \frac{\text{in}}{\text{ft}}\right) = 0.0053\ \text{in}^2/\text{ft}$$

ACI Secs. 7.12 and 8.13.6.2 give the minimum steel for temperature and shrinkage transverse to the joist as

$$A_{s,\min} = 0.0018bh = (0.0018)(4\ \text{in})\left(12\ \frac{\text{in}}{\text{ft}}\right) = 0.0864\ \text{in}^2/\text{ft} \quad \text{[controls]}$$

Because the steel in the longitudinal direction is limited by joist reinforcement, only spacer wires are needed in that direction. Use 4 in×12 in W2.9×W0.9 welded wire fabric for the 4 in slab reinforcement.

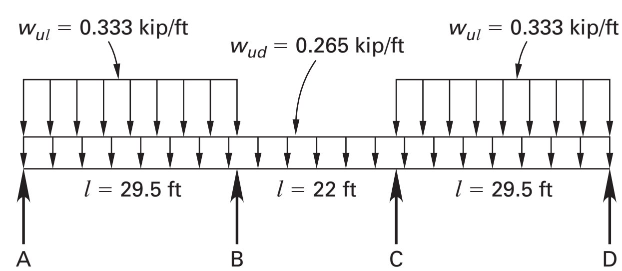
For the joists, the adjacent clear spans differ by more than 20%; therefore, the approximate coefficients of ACI Sec. 8.3 do not apply. Use moment distribution to evaluate moments and shears in the three-span joists. Joists are standard module 20 in plus 5 in soffit; therefore, joists are at 2.08 ft on centers. The weight of a typical joist and slab is

$$\begin{aligned} w_j &= w_c A_j \\ &= w_c\left(Bh_s + b_{\text{ave}}h_j\right) \\ &= \left(150\ \frac{\text{lbf}}{\text{ft}^3}\right) \times \left((4\ \text{in})(25\ \text{in}) + \left(\frac{5\ \text{in} + \left(5\ \text{in} + \frac{(2)(14\ \text{in})}{12}\right)}{2}\right) \times (14\ \text{in})\right) \times \left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) \\ &= 194\ \text{lbf/ft} \end{aligned}$$

$$\begin{aligned} w_{ud} &= 1.2w_d \\ &= (1.2)\left(194\ \frac{\text{lbf}}{\text{ft}} + \left(13\ \frac{\text{lbf}}{\text{ft}^2}\right)(2.08\ \text{ft})\right) \\ &= 265\ \text{lbf/ft} \end{aligned}$$

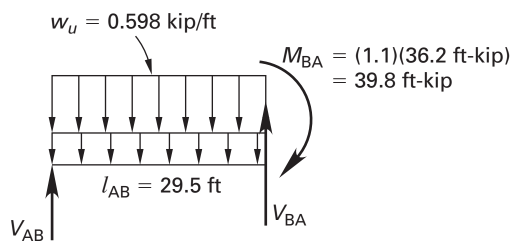
$$\begin{aligned} w_{ul} &= 1.6w_l \\ &= (1.6)\left(100\ \frac{\text{lbf}}{\text{ft}^2}\right)(2.08\ \text{ft}) \\ &= 333\ \text{lbf/ft} \end{aligned}$$

For maximum positive moments in exterior spans, load all spans with factored dead plus live on the exterior spans.



distribution factor	DF_{AB} —	DF_{BA} 0.36	DF_{BC} 0.64	DF_{CB} 0.64	DF_{CD} 0.36	DF_{DC} —
fixed-end moment	0	65.1	−10.7	10.7	−65.1	0
distribution B	0	$\underline{-19.6}$	$\underline{-34.8}$	−17.4	—	—
distribution C	—	—	23.0	$\underline{46.0}$	$\underline{25.8}$	0
distribution B	0	$\underline{-8.3}$	$\underline{-14.7}$	−7.4	—	—
distribution C	—	—	2.4	$\underline{4.7}$	$\underline{2.7}$	0
distribution B	0	$\underline{-0.9}$	$\underline{-1.5}$	−0.8	—	—
distribution C	—	—	0.3	$\underline{0.5}$	$\underline{0.3}$	0
distribution B	0	$\underline{-0.1}$	$\underline{-0.2}$	—	—	—
$\sum$	0	36.2	−36.2	36.3	−36.3	0

Underlined distributed moments are those in which equilibrium is satisfied. ACI Sec. 8.4 permits a 10% increase in the negative moment with corresponding decrease in positive moments.



$$\sum M_B = \frac{w_u l_{AB}^2}{2} - V_{AB} l_{AB} - M_{AB} = 0 \text{ in-kips}$$

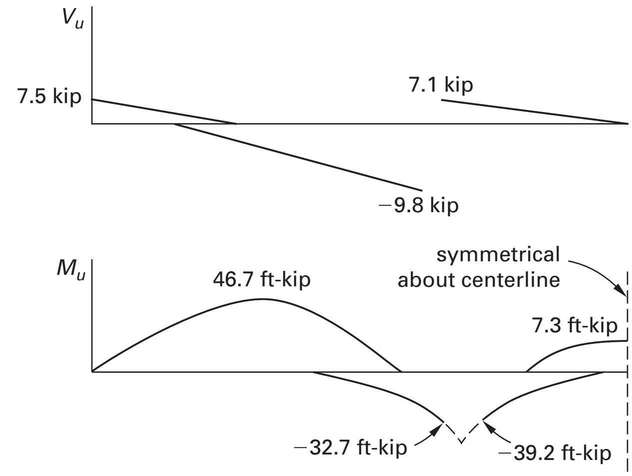
$$V_{AB} = \frac{\frac{w_u l_{AB}^2}{2} - M_{AB}}{l_{AB}} = \frac{\frac{\left(0.598 \, \frac{\text{kip}}{\text{ft}}\right)(29.5 \text{ ft})^2}{2} - 39.8 \text{ ft-kip}}{29.5 \text{ ft}} = 7.47 \text{ kip}$$

$$V = 7.47 \text{ kip} - \left(0.598 \, \frac{\text{kip}}{\text{ft}}\right) x = 0 \text{ kip}$$

$$x = 12.5 \text{ ft}$$

$$M_u^+ = 0.5 V_{AB} x = (0.5)(7.49 \text{ kip})(12.5 \text{ ft}) = 46.8 \text{ ft-kip}$$

Similarly, analysis for factored dead plus live load on two adjacent spans, with redistribution of moments and reduction to support face, gives the shear and moment envelopes for the joists (symmetrical about the middle of the interior span).



Design for shear using ACI Sec. 8.13.8 with an effective depth 1 in less than the overall depth.

$$\begin{aligned} d &= h_s + h_j - 1 \text{ in} \\ &= 4 \text{ in} + 14 \text{ in} - 1 \text{ in} \\ &= 17 \text{ in} \\ V_{u,\max} &= V_{u,BA} - w_u d \\ &= 9.8 \text{ kip} - \left(0.598 \, \frac{\text{kip}}{\text{ft}}\right)(17 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \\ &= 9.0 \text{ kip} \end{aligned}$$

Thus, the design shear exceeds the strength of the unreinforced web. Use tapered end forms to increase the soffit width from the standard 5 in to 9 in at the critical shear location. This gives proportional increase in shear strength, which is adequate. (Alternatively, stirrups could be provided, but stirrups are difficult to place in narrow web width of joists.)

For flexural steel in joists,

$$\begin{aligned} A_{s,\min} &= \rho_{\min} b_w d = \left(\frac{200}{f_y}\right) b_w d \\ &= \left(\frac{200 \, \frac{\text{lbf}}{\text{in}^2}}{60{,}000 \, \frac{\text{lbf}}{\text{in}^2}}\right)(5 \text{ in})(17 \text{ in}) \\ &= 0.28 \text{ in}^2 \end{aligned}$$

For positive moment in exterior spans, the compression flange is 25 in wide.

$$\phi M_n = M_u = \phi\rho b d^2 f_y \left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right)$$

$$\begin{aligned}(46.8\text{ ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right) &= 0.9\rho(25\text{ in})(17\text{ in})^2\left(60\ \frac{\text{kip}}{\text{in}^2}\right)\\ &\quad\times\left(1 - 0.59\rho\left(\frac{60\ \frac{\text{kip}}{\text{in}^2}}{3\ \frac{\text{kip}}{\text{in}^2}}\right)\right)\\ \rho &= 0.00146\\ A_s = \rho b d &= (0.00146)(25\text{ in})(17.0\text{ in})\\ &= 0.62\text{ in}^2\end{aligned}$$

Choose two no. 5 bars for the bottom steel.

$$A_{s,\text{prov}} = (2)(0.31\text{ in}^2) = 0.62\text{ in}^2$$

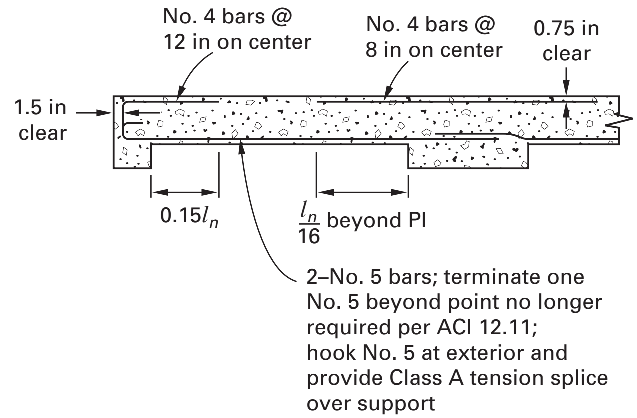
For the negative steel over the first interior supports, the compression flange is on the bottom. If tapered forms are used, the width is 9 in at the critical location.

$$\begin{aligned}(39.2\text{ ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right) &= 0.9\rho(9\text{ in})(17\text{ in})^2\left(60\ \frac{\text{kip}}{\text{in}^2}\right)\\ &\quad\times\left(1 - 0.59\rho\left(\frac{60\ \frac{\text{kip}}{\text{in}^2}}{3\ \frac{\text{kip}}{\text{in}^2}}\right)\right)\\ \rho &= 0.00349\\ A_s = \rho b d &= (0.00349)(9\text{ in})(17.0\text{ in})\\ &= 0.53\text{ in}^2\end{aligned}$$

Choose three no. 4 bars spread over the 25 in width for each joist. For the interior span, the small factored moment (7.3 ft-kip) requires minimum steel. Extend the top steel in the exterior spans, as required by ACI Sec. 12.10, for an embedment length of at least

$$\frac{l_n}{16} = \frac{(29.5\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{16} = 22\text{ in}$$

ACI Sec. 7.13 requires that at least one bottom bar terminate at the exterior with a standard hook and splice over support with a Class A tension splice.



elevation of joists

B. Design girders on lines 2 and 3. Treat the girders as rectangular beams 30 in wide by 22 in overall depth with an effective depth, d, of 19.5 in (22 in − 2.5 in). Clear spans are

$$\begin{aligned}l_{\text{ext}} &= 28\text{ ft} - 0.75\text{ ft} - 0.5\text{ ft}\\ &= 26.75\text{ ft}\\ l_{\text{int}} &= 28\text{ ft} - 1.5\text{ ft}\\ &= 26.5\text{ ft}\end{aligned}$$

These are approximately equal, and therefore the approximate coefficients of ACI Sec. 8.3 apply. The factored load on a girder is the sum of reactions from the adjacent spans plus added weight of solid girder.

$$\begin{aligned}w_g &= w_c A_g = \left(0.15\ \frac{\text{kip}}{\text{ft}^3}\right)(2.5\text{ ft})(1.83\text{ ft})\\ &= 0.69\text{ kip/ft}\\ w_u &= 1.2w_g + \frac{w_{uj}(l_{j,\text{ext}} + l_{j,\text{int}})}{2}\\ &= (1.2)\left(0.69\ \frac{\text{kip}}{\text{ft}}\right)\\ &\quad + \left(\frac{\left(\frac{0.598\ \frac{\text{kip}}{\text{ft}}}{2.08\text{ ft}}\right)(29.5\text{ ft} + 22\text{ ft})}{2}\right)\\ &= 8.23\text{ kip/ft}\end{aligned}$$

Minimum flexural steel is

$$\begin{aligned}A_{s,\text{min}} &= \left(\frac{200}{f_y}\right)b_w d\\ &= \left(\frac{200\ \frac{\text{lbf}}{\text{in}^2}}{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right)(30\text{ in})(19.5\text{ in})\\ &= 1.95\text{ in}^2\end{aligned}$$

Provide minimum steel at exterior top for crack control. For the positive moment in the exterior span,

$$M_u^+ = \frac{w_u l_n^2}{11} = \frac{\left(8.24\ \frac{\text{kip}}{\text{ft}}\right)(26.75\ \text{ft})^2}{11}$$
$$= 536\ \text{ft-kip}$$
$$\phi M_n = M_u = \phi\rho b d^2 f_y \left(1 - 0.59\rho\left(\frac{f_y}{f_c'}\right)\right)$$
$$(536\ \text{ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right)$$
$$= 0.9\rho(30\ \text{in})(19.5\ \text{in})^2\left(60\ \frac{\text{kip}}{\text{in}^2}\right)$$
$$\times\left(1 - 0.59\rho\left(\frac{60\ \frac{\text{kip}}{\text{in}^2}}{3\ \frac{\text{kip}}{\text{in}^2}}\right)\right)$$
$$\rho = 0.0122$$
$$A_s = \rho b d = (0.0122)(30\ \text{in})(19.5\ \text{in})$$
$$= 7.14\ \text{in}^2$$

Use eight no. 9 bars for bottom steel in the exterior span.

Over the first interior support,

$$l_n = \frac{l_{\text{ext}} + l_{\text{int}}}{2} = \frac{26.75\ \text{ft} + 26.5\ \text{ft}}{2}$$
$$= 26.63\ \text{ft}$$
$$M_u^- = \frac{w_u l_n^2}{10} = \frac{\left(8.24\ \frac{\text{kip}}{\text{ft}}\right)(26.63\ \text{ft})^2}{10}$$
$$= 584\ \text{ft-kip}$$
$$(584\ \text{ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right)$$
$$= 0.9\rho(30\ \text{in})(19.5\ \text{in})^2\left(60\ \frac{\text{kip}}{\text{in}^2}\right)$$
$$\times\left(1 - 0.59\rho\left(\frac{60\ \frac{\text{kip}}{\text{in}^2}}{3\ \frac{\text{kip}}{\text{in}^2}}\right)\right)$$
$$\rho = 0.0135$$
$$A_s = \rho b d = (0.0135)(30\ \text{in})(19.5\ \text{in})$$
$$= 7.90\ \text{in}^2$$

Use eight no. 9 bars over the first interior support.

For the positive moment in the interior span,

$$M_u^+ = \frac{w_u l_n^2}{16} = \frac{\left(8.24\ \frac{\text{kip}}{\text{ft}}\right)(26.5\ \text{ft})^2}{16}$$
$$= 362\ \text{ft-kip}$$
$$\phi M_n = M_u = \phi\rho b d^2 f_y \left(1 - 0.59\rho\left(\frac{f_y}{f_c'}\right)\right)$$
$$(362\ \text{ft-kip})\left(12\ \frac{\text{in}}{\text{ft}}\right)$$
$$= 0.9\rho(30\ \text{in})(19.5\ \text{in})^2\left(60\ \frac{\text{kip}}{\text{in}^2}\right)$$
$$\times\left(1 - 0.59\rho\left(\frac{60\ \frac{\text{kip}}{\text{in}^2}}{3\ \frac{\text{kip}}{\text{in}^2}}\right)\right)$$
$$\rho = 0.0078$$
$$A_s = \rho b d = (0.0078)(30\ \text{in})(19.5\ \text{in})$$
$$= 4.56\ \text{in}^2 \quad \text{[say, two no. 9 and two no. 10]}$$

Use two no. 9 and two no. 10 bars for the bottom steel in the interior span.

For shear design in the exterior span, critical condition occurs at d-distance from first interior support.

$$V_u = \frac{1.15 w_u l_n}{2} - w_u d$$
$$= \frac{(1.15)\left(8.24\ \frac{\text{kip}}{\text{ft}}\right)(26.75\ \text{ft})}{2}$$
$$- \left(8.24\ \frac{\text{kip}}{\text{ft}}\right)(19.5\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$
$$= 113.4\ \text{kip}$$
$$\phi V_c = \phi 2\lambda\sqrt{f_c'}b_w d$$
$$= (0.75)(2)(1)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right)$$
$$\times (30\ \text{in})(19.5\ \text{in})$$
$$= 48\ \text{kip}$$

Stirrups are required. Try no. 4 U-stirrups, $A_v = 0.40\ \text{in}^2$. The spacing at the critical location is

$$
\begin{aligned}
V_s &= \frac{V_u - \phi V_c}{\phi} \\
&= \frac{113.4\ \text{kip} - 48.0\ \text{kip}}{0.75} \\
&= 87.2\ \text{kip}
\end{aligned}
$$

$$
s \le \begin{cases}
\dfrac{d}{2} = \dfrac{19.5\ \text{in}}{2} = 9.8\ \text{in} \\[2ex]
\dfrac{A_v f_y}{50 b_w} = \dfrac{(0.40\ \text{in}^2)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{\left(50\ \dfrac{\text{lbf}}{\text{in}^2}\right)(30\ \text{in})} = 16\ \text{in} \\[2ex]
\dfrac{A_v f_y d}{V_s} = \dfrac{(0.40\ \text{in}^2)\left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)(19.5\ \text{in})}{87.2\ \text{kip}} \\
\quad = 5.3\ \text{in} \quad [\text{controls}]
\end{cases}
$$

Use no. 4 U-stirrups at 5 in on centers at the critical location. For the interior span,

$$
\begin{aligned}
V_u &= \frac{w_u l_n}{2} - w_u d \\
&= \frac{\left(8.24\ \dfrac{\text{kip}}{\text{ft}}\right)(26.5\ \text{ft})}{2} - \left(8.24\ \frac{\text{kip}}{\text{ft}}\right)(19.5\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\
&= 95.8\ \text{kip}
\end{aligned}
$$

The spacing at the critical location is

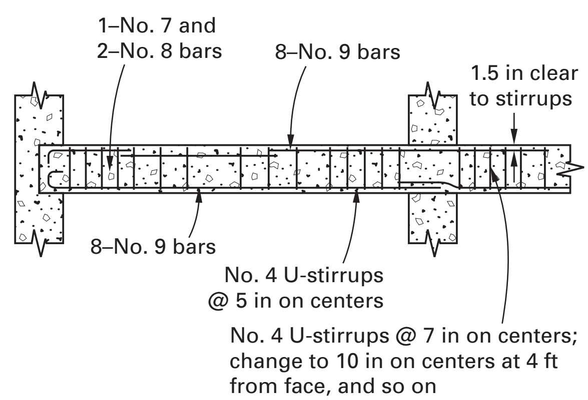
$$
\begin{aligned}
V_s &= \frac{V_u - \phi V_c}{\phi} \\
&= \frac{95.8\ \text{kip} - 48.0\ \text{kip}}{0.75} \\
&= 63.7\ \text{kip}
\end{aligned}
$$

$$
s \le \begin{cases}
\dfrac{d}{2} = \dfrac{19.5\ \text{in}}{2} = 9.8\ \text{in} \\[2ex]
\dfrac{A_v f_y}{50 b_w} = \dfrac{(0.40\ \text{in}^2)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{\left(50\ \dfrac{\text{lbf}}{\text{in}^2}\right)(30\ \text{in})} = 16\ \text{in} \\[2ex]
\dfrac{A_v f_y d}{V_s} = \dfrac{(0.40\ \text{in}^2)\left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)(19.5\ \text{in})}{63.7\ \text{kip}} \\
\quad = 7.4\ \text{in} \quad [\text{controls}]
\end{cases}
$$

Use no. 4 U-stirrups at 7 in on centers at the critical location. Stirrup spacing may increase at locations toward midspan. For example, at a distance of 4 ft from face of interior support,

$$
\begin{aligned}
V_u &= \frac{w_u l_n}{2} - 4 w_u d \\
&= \frac{\left(8.24\ \dfrac{\text{kip}}{\text{ft}}\right)(26.5\ \text{ft})}{2} - (4\ \text{ft})\left(8.24\ \frac{\text{kip}}{\text{ft}}\right) \\
&= 76.2\ \text{kip} \\
V_s &= \frac{V_u - \phi V_c}{\phi} \\
&= \frac{76.2\ \text{kip} - 48.0\ \text{kip}}{0.75} \\
&= 37.6\ \text{kip}
\end{aligned}
$$

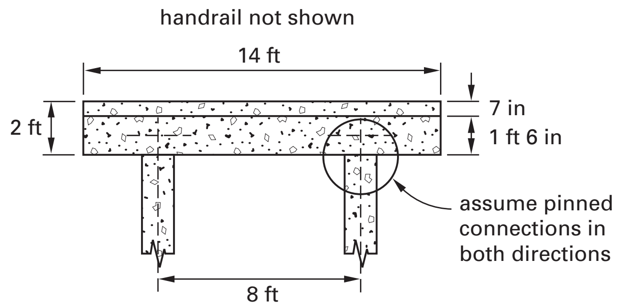
$$
s \le \begin{cases}
\dfrac{d}{2} = \dfrac{19.5\ \text{in}}{2} = 9.8\ \text{in} \quad [\text{controls}] \\[2ex]
\dfrac{A_v f_y}{50 b_w} = \dfrac{(0.40\ \text{in}^2)\left(60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{\left(50\ \dfrac{\text{lbf}}{\text{in}^2}\right)(30\ \text{in})} = 16\ \text{in} \\[2ex]
\dfrac{A_v f_y d}{V_s} = \dfrac{(0.40\ \text{in}^2)\left(60\ \dfrac{\text{kip}}{\text{in}^2}\right)(19.5\ \text{in})}{37.6\ \text{kip}} \\
\quad = 12.4\ \text{in}
\end{cases}
$$



partial elevation of girders at line 1 and 2

Practice Problem 37

A cast-in-place reinforced concrete pedestrian bridge is to be constructed between two existing buildings as shown. The bridge deck is supported by cantilevered columns. The deck slab is continuous over all supports.



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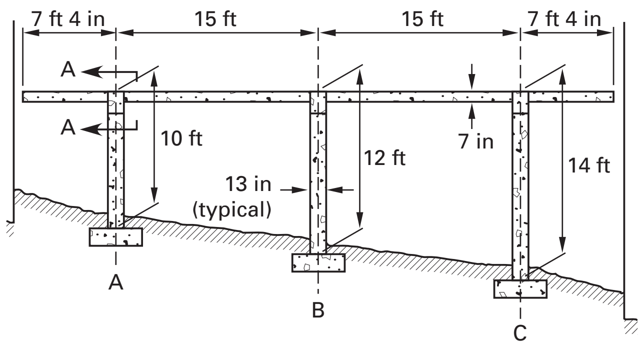


Design Criteria

- $f'_c = 3$ ksi
- ASTM A615 grade 40 reinforcing steel
- $f_y = 40$ ksi
- live load = 100 lbf/ft^2
- seismic performance category D
- neglect vertical acceleration in factored load combinations

Required

A. Design all reinforcing steel for the pedestrian bridge deck slab for full dead load plus live load on all spans. Provide 1½ in clear to top reinforcement and 1 in clear to bottom reinforcement.

B. Draw a detail showing all slab reinforcement.

C. Determine the deflection of the cantilevered slab for dead load on the entire deck considering long-term effects.

D. Design the reinforcing steel for columns using 13 in square columns. Do not combine live load with dead load under lateral loading. Consider lateral loading perpendicular to the bridge. Use $V = 0.2W$ (which includes the reliability factor) for lateral loads.

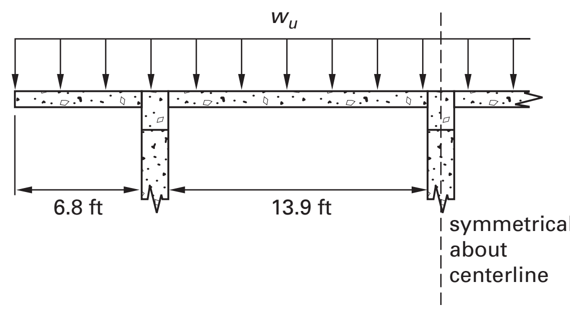
E. Detail column reinforcement.

F. Determine the maximum column deflection under lateral load. Neglect footing rotation.

Solution

A. Design the slab reinforcement. The clear spans are

$$l_{\text{int}} = l_1 - h_c = 15\text{ ft} - (13\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)$$
$$= 13.9\text{ ft}$$
$$a = l_0 - \frac{h_c}{2} = 7.33\text{ ft} - \left(\frac{13\text{ in}}{2}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)$$
$$= 6.8\text{ ft}$$



For 7 in normal weight concrete slab and a 100 lbf/ft^2 nonreducible live load, the factored gravity load is

$$w_u = 1.2w_d + 1.6w_l = 1.2(w_c h_s) + 1.6w_l$$
$$= (1.2)\left(0.150\ \frac{\text{kip}}{\text{ft}^3}\right)(7\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)$$
$$+ (1.6)\left(0.1\ \frac{\text{kip}}{\text{ft}^2}\right)$$
$$= 0.265\text{ kip/ft}^2$$

Assuming no. 4 main steel, the effective depths in the negative regions, d^-, and positive regions, d^+, are

$$d^- = h_s - \text{cover} - \frac{d_b}{2}$$
$$= 7\text{ in} - 1.5\text{ in} - \frac{0.5\text{ in}}{2}$$
$$= 5.25\text{ in}$$
$$d^+ = h_s - \text{cover} - \frac{d_b}{2}$$
$$= 7\text{ in} - 1.0\text{ in} - \frac{0.5\text{ in}}{2}$$
$$= 5.75\text{ in}$$

Check shear (design on a unit width basis, $b_w = 1$ ft = 12 in).

$$V_u \geq \begin{cases} \dfrac{1.15 w_u l_{\text{int}}}{2} - w_u d^- \\ \quad = \dfrac{(1.15)\left(0.265\ \dfrac{\text{kip}}{\text{ft}^2}\right)(13.9\ \text{ft})}{2} \\ \qquad - \left(0.265\ \dfrac{\text{kip}}{\text{ft}^2}\right)(5.25\ \text{in})\left(\dfrac{1\ \text{ft}}{12\ \text{in}}\right) \\ \quad = 2.00\ \text{kip/ft} \quad [\text{controls}] \\ w_u a - w_u d^- \\ \quad = \left(0.265\ \dfrac{\text{kip}}{\text{ft}^2}\right)(6.8\ \text{ft}) \\ \qquad - \left(0.265\ \dfrac{\text{kip}}{\text{ft}^2}\right)(5.25\ \text{in})\left(\dfrac{1\ \text{ft}}{12\ \text{in}}\right) \\ \quad = 1.69\ \text{kip/ft} \end{cases}$$

$$\begin{aligned} \phi V_c &= \phi 2\lambda\sqrt{f'_c}\, b_w d \\ &= (0.75)(2)(1)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\left(12\ \frac{\text{in}}{\text{ft}}\right) \\ &\quad \times (5.25\ \text{in})\left(\frac{1\ \text{kip}}{1000\ \text{lbf}}\right) \\ &= 5.2\ \text{kip/ft} \\ \phi V_c &> V_u \end{aligned}$$

Therefore, shear is no problem.

Minimum steel is controlled by temperature and shrinkage.

$$\begin{aligned} A_{s,\min} &= 0.002 b h_s \\ &= (0.002)\left(12\ \frac{\text{in}}{\text{ft}}\right)(7\ \text{in}) \\ &= 0.17\ \text{in}^2/\text{ft} \end{aligned}$$

This could be, say, no. 4 bars at 14 in on centers. For flexural steel, use the coefficients of ACI Sec. 8.3.

$$\begin{aligned} M^-_{u,1} &= \frac{w_u a^2}{2} \\ &= \frac{\left(0.265\ \dfrac{\text{kip}}{\text{ft}^2}\right)(6.8\ \text{ft})^2}{2} \\ &= 6.13\ \text{ft-kip/ft} \end{aligned}$$

$$\begin{aligned} \phi M_n &= M^-_{u,1} = \phi\rho f_y b d^2\left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right) \\ \left(6.13\ \frac{\text{ft-kip}}{\text{ft}}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right) &= 0.9\rho\left(40\ \frac{\text{kip}}{\text{in}^2}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)(5.25\ \text{in})^2 \\ &\quad \times \left(1 - 0.59\rho\left(\frac{40\ \dfrac{\text{kip}}{\text{in}^2}}{3\ \dfrac{\text{kip}}{\text{in}^2}}\right)\right) \\ \rho &= 0.0065 \\ A^-_{s,1} &= \rho b d = (0.0065)\left(12\ \frac{\text{in}}{\text{ft}}\right)(5.25\ \text{in}) \\ &= 0.41\ \text{in}^2/\text{ft} \end{aligned}$$

This could be no. 5 bars at 9 in on centers. For horizontal cantilevers in seismic design category D, ASCE 7 (Sec. 9.5.2.6.4.3) requires a net upward force 0.2 times the dead load to account for vertical ground acceleration. By comparison with the values above, the resulting moment requires minimum steel (no. 4 bars at 14 in) on the bottom, which must fully develop at the exterior column faces. For other locations,

$$\begin{aligned} M^+_{u,\text{int}} &= \frac{w_u l_n^2}{14} \\ &= \frac{\left(0.265\ \dfrac{\text{kip}}{\text{ft}^2}\right)(13.9\ \text{ft})^2}{14} \\ &= 3.66\ \text{ft-kip/ft} \end{aligned}$$

$$\begin{aligned} \phi M_n &= M^+_{u,\text{int}} = \phi\rho f_y b d^2\left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right) \\ \left(3.66\ \frac{\text{ft-kip}}{\text{ft}}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right) &= 0.9\rho\left(40\ \frac{\text{kip}}{\text{in}^2}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)(5.25\ \text{in})^2 \\ &\quad \times \left(1 - 0.59\rho\left(\frac{40\ \dfrac{\text{kip}}{\text{in}^2}}{3\ \dfrac{\text{kip}}{\text{in}^2}}\right)\right) \\ \rho &= 0.0038 \\ A^+_{s,\text{int}} &= \rho b d = (0.0038)\left(12\ \frac{\text{in}}{\text{ft}}\right)(5.25\ \text{in}) \\ &= 0.24\ \text{in}^2/\text{ft} \end{aligned}$$

Use no. 4 bars at 10.5 in on centers.

$$M^-_{u,\mathrm{int}} = \frac{w_u l_n^2}{9} = \frac{\left(0.265\ \frac{\mathrm{kip}}{\mathrm{ft}^2}\right)(13.9\ \mathrm{ft})^2}{9} = 5.69\ \text{ft-kip/ft}$$

$$\phi M_n = M^-_{u,\mathrm{int}} = \phi\rho f_y b d^2\left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right)$$

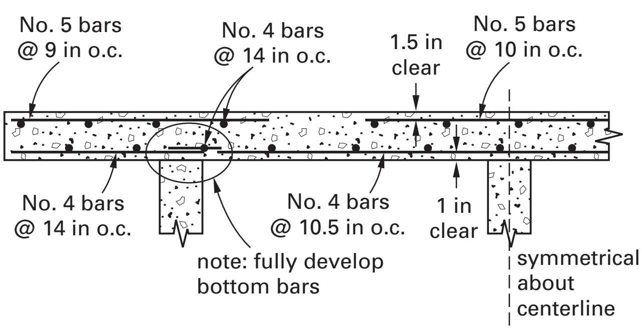
$$\left(5.69\ \frac{\text{ft-kip}}{\mathrm{ft}}\right)\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right) = 0.9\rho\left(40\ \frac{\mathrm{kip}}{\mathrm{in}^2}\right)\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)(5.25\ \mathrm{in})^2 \times\left(1 - 0.59\rho\left(\frac{40\ \frac{\mathrm{kip}}{\mathrm{in}^2}}{3\ \frac{\mathrm{kip}}{\mathrm{in}^2}}\right)\right)$$

$$\rho = 0.0060$$

$$A^-_{s,\mathrm{int}} = \rho b d = (0.0060)\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)(5.25\ \mathrm{in}) = 0.38\ \mathrm{in}^2/\mathrm{ft}$$

Use no. 5 bars at 10 in on centers.

B. Detail all slab reinforcement.



section

(not to scale)

C. Compute the immediate and long-term deflection of the cantilever. Use the effective moment of inertia of ACI Sec. 9.5 (be conservative and neglect the compression steel).

$$E_c = 57{,}000\sqrt{f'_c} = 57{,}000\sqrt{3000\ \frac{\mathrm{lbf}}{\mathrm{in}^2}}\left(\frac{1\ \mathrm{kip}}{1000\ \mathrm{lbf}}\right) = 3120\ \mathrm{ksi}$$

$$n = \frac{E_s}{E_c} = \frac{29{,}000\ \frac{\mathrm{kip}}{\mathrm{in}^2}}{3120\ \frac{\mathrm{kip}}{\mathrm{in}^2}} = 9.29$$

$$\rho n = \left(\frac{A_s}{bd}\right)n = \left(\frac{0.41\ \mathrm{in}^2}{(12\ \mathrm{in})(5.25\ \mathrm{in})}\right)(9.29) = 0.060$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n = \sqrt{(0.060)^2 + (2)(0.060)} - 0.060 = 0.29$$

$$I_{\mathrm{cr}} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2 = \frac{(12\ \mathrm{in})\big((0.29)(5.25\ \mathrm{in})\big)^3}{3} + (9.29)(0.41\ \mathrm{in}^2)\big(5.25\ \mathrm{in} - (0.29)(5.25\ \mathrm{in})\big)^2 = 67.0\ \mathrm{in}^4$$

$$I_g = \frac{bh^3}{12} = \frac{(12\ \mathrm{in})(7\ \mathrm{in})^3}{12} = 343\ \mathrm{in}^4$$

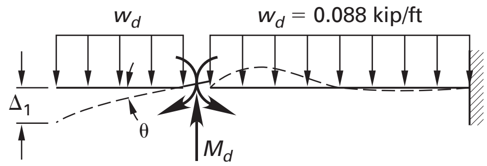
$$M_{\mathrm{cr}} = \frac{7.5\sqrt{f'_c}I_g}{y_t} = \left(\frac{7.5\sqrt{3000\ \frac{\mathrm{lbf}}{\mathrm{in}^2}}(343\ \mathrm{in}^4)}{3.5\ \mathrm{in}}\right)\times\left(\frac{1\ \mathrm{kip}}{1000\ \mathrm{lbf}}\right)\left(\frac{1\ \mathrm{ft}}{12\ \mathrm{in}}\right) = 3.35\ \text{ft-kip}$$

The extent of cracking is a function of the maximum service load moment.

$$M_a = \frac{(w_d + w_l)a^2}{2} = \frac{\left(0.088\ \frac{\mathrm{kip}}{\mathrm{ft}} + 0.1\frac{\mathrm{kip}}{\mathrm{ft}}\right)(7.33\ \mathrm{ft})^2}{2} = 5.05\ \text{ft-kip}$$

$$I_e = \left(\frac{M_{\mathrm{cr}}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{\mathrm{cr}}}{M_a}\right)^3\right)I_{\mathrm{cr}} = \left(\frac{3.35\ \text{ft-kip}}{5.05\ \text{ft-kip}}\right)^3(343\ \mathrm{in}^4) + \left(1 - \left(\frac{3.35\ \text{ft-kip}}{5.05\ \text{ft-kip}}\right)^3\right)(67.0\ \mathrm{in}^4) = 148\ \mathrm{in}^4$$

$$E_c I_e = \left(3120\ \frac{\mathrm{kip}}{\mathrm{in}^2}\right)(148\ \mathrm{in}^4) = 461{,}760\ \text{kip-in}^2$$



The dead load deflection at the tip is found by superposition of the deflection for a uniformly loaded cantilever and the rotation, θ, at the support.

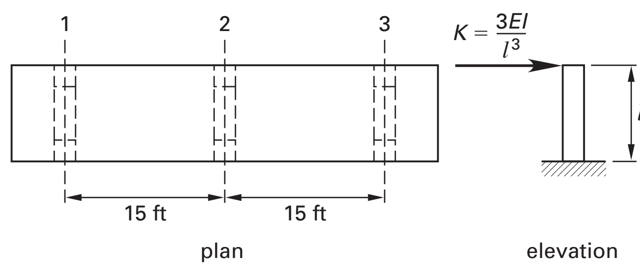
$$
\begin{aligned}
M_d &= \frac{w_d a^2}{2} = \frac{\left(0.088\ \frac{\text{kip}}{\text{ft}}\right)(7.33\ \text{ft})^2}{2} \\
&= 2.36\ \text{ft-kip} \\
\theta &= \frac{w_d L^3}{48 E_c I_e} - \frac{M_d L}{4 E_c I_e} \\
&= \frac{\left(0.088\ \frac{\text{kip}}{\text{ft}}\right)(15\ \text{ft})^3 \left(144\ \frac{\text{in}^2}{\text{ft}^2}\right)}{(48)(461{,}760\ \text{in}^2\text{-kip})} \\
&\quad - \frac{(2.36\ \text{ft-kip})(15\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{(4)(461{,}760\ \text{in}^2\text{-kip})} \\
&= 0.0017\ \text{rad} \quad [\text{counterclockwise}] \\
\Delta_i &= \frac{w_d a^4}{48 E_c I_e} + \theta a \\
&= \frac{\left(0.088\ \frac{\text{kip}}{\text{ft}}\right)(7.33\ \text{ft})^4 \left(12\ \frac{\text{in}}{\text{ft}}\right)^3}{(48)(461{,}760\ \text{in}^2\text{-kip})} \\
&\quad + (0.0017\ \text{rad})(7.33\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right) \\
&= 0.17\ \text{in} \quad [\text{downward}]
\end{aligned}
$$

Long-term deflection (considering the benefits of the no. 4 bars at 14 in in the compression region) is

$$
\begin{aligned}
\rho' &= \frac{A'_s}{bd} = \frac{0.17\ \text{in}^2}{(12\ \text{in})(5.25\ \text{in})} \\
&= 0.0027 \\
\lambda &= \frac{\xi}{1 + 50\rho'} = \frac{2}{1 + (50)(0.0027)} \\
&= 1.76 \\
\Delta_{\text{lt}} &= \Delta_i + \lambda \Delta_i = 0.17\ \text{in} + (1.76)(0.17\ \text{in}) \\
&= 0.47\ \text{in} \quad [\text{downward}]
\end{aligned}
$$

D. Design the reinforcing steel. The effective weight for seismic loading consists of the 7 in slab plus one-half the column weight.

$$
\begin{aligned}
W &= w_c \left(h_s B L + 2h_c^2 \left(\frac{l_1 + l_2 + l_3}{2} \right) \right) \\
&= \left(0.15\ \frac{\text{kip}}{\text{ft}^3}\right) \left(\begin{array}{l} \left(\dfrac{7\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} \right)(14\ \text{ft})(44.67\ \text{ft}) \\ + (2)\left(\dfrac{13\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} \right)^2 \\ \times \left(\dfrac{10\ \text{ft} + 12\ \text{ft} + 14\ \text{ft}}{2} \right) \end{array} \right) \\
&= 61\ \text{kip}
\end{aligned}
$$



Assuming that bending deformation occurs only in the columns and that EI is the same in every column, the relative rigidities of the column bents is proportional to the unsupported length cubed.

$$
\begin{aligned}
K &= \frac{3EI}{L^3} \\
l_1 &= 14\ \text{ft} - 1\ \text{ft} = 13\ \text{ft} \\
K_1 &= \frac{3EI}{(13\ \text{ft})^3} = K \\
l_2 &= 12\ \text{ft} - 1\ \text{ft} = 11\ \text{ft} \\
K_2 &= \frac{3EI}{(11\ \text{ft})^3} = 1.6K \\
l_3 &= 10\ \text{ft} - 1\ \text{ft} = 9\ \text{ft} \\
K_3 &= \frac{3EI}{(9\ \text{ft})^3} = 3.0K \\
\sum K_i &= K + 1.6K + 3.0K = 5.6K \\
\overline{x} &= \frac{\sum K_i x_i}{\sum K_i} = \frac{1.6K(15\ \text{ft}) + K(30\ \text{ft})}{5.6K} \\
&= 9.6\ \text{ft} \quad [\text{from shortest columns}]
\end{aligned}
$$

Due to the difference in column lengths, the center of mass is slightly off center, but negligibly so. Assume the center of mass is over the middle columns and apply the minimum eccentricity provision of ASCE 7 (that is, $e = \pm 0.05L$). The base shear is

$$\begin{aligned}
V &= 0.2W = (0.2)(61\text{ kip}) \\
&= 12.2\text{ kip} \\
M_t &= Ve = V(15\text{ ft} - \overline{x} + 0.05L) \\
&= (12.2\text{ kip})\begin{pmatrix} 15\text{ ft} - 9.6\text{ ft} \\ \quad + (0.05)(44.67\text{ ft}) \end{pmatrix} \\
&= 93\text{ ft-kip} \\
\sum K_i x_i^2 &= 3.0K(9.6\text{ ft})^2 + 1.6K(5.4\text{ ft})^2 \\
&\quad + 1.0K(20.4\text{ ft})^2 \\
&= (739\text{ ft}^2)K \\
V_i &= \frac{K_i V}{\sum K_i} + \frac{M_t K_i x_i}{\sum K_i x_i^2} \\
M_i &= \frac{V_i l_i}{2} \\
V_1 &= \frac{3.0K(12.2\text{ kip})}{5.6K} \\
&\quad + \frac{(93\text{ ft-kip})(3.0K)(-9.6\text{ ft})}{(739\text{ ft}^2)K} \\
&= 2.9\text{ kip} \\
V_2 &= \frac{1.6K(12.2\text{ kip})}{5.6K} \\
&\quad + \frac{(93\text{ ft-kip})(1.6K)(5.4\text{ ft})}{(739\text{ ft}^2)K} \\
&= 4.6\text{ kip} \\
V_3 &= \frac{K(12.2\text{ kip})}{5.6K} + \frac{(93\text{ ft-kip})K(20.4\text{ ft})}{(739\text{ ft}^2)K} \\
&= 4.7\text{ kip}
\end{aligned}$$

Check for torsional irregularity.

$$\begin{aligned}
\delta_1 &= \frac{V_1 l_1^3}{3EI} = \frac{(2.9\text{ kip})(9\text{ ft})^3}{3EI} = \delta \\
\delta_2 &= \frac{V_2 l_2^3}{3EI} = \frac{(4.6\text{ kip})(11\text{ ft})^3}{3EI} = 2.9\delta \\
\delta_3 &= \frac{V_3 l_3^3}{3EI} = \frac{(4.7\text{ kip})(13\text{ ft})^3}{3EI} = 4.9\delta \\
\delta_{\text{ave}} &= \frac{\delta + 2.9\delta + 4.9\delta}{3} = 2.9\delta \\
\delta_{\text{max}} &= 4.9\delta > 1.2\delta_{\text{ave}} = 3.5\delta
\end{aligned}$$

Therefore, torsional irregularity requires moment amplification per ASCE 7 Eq. 12.8-14.

$$\begin{aligned}
A_x &= \left(\frac{\delta_{\text{max}}}{1.2\delta_{\text{ave}}}\right)^2 = \left(\frac{4.9\delta}{(1.2)(2.9\delta)}\right)^2 \\
&= 2.0 < 3.0
\end{aligned}$$

Critical bending moment is in the longest column. Recompute using the amplified torsion moment.

$$\begin{aligned}
V_i &= \frac{K_i V}{\sum K_i} + \frac{A_x M_t K_i x_i}{\sum K_i x_i^2} \\
V_3 &= \frac{K(12.2\text{ kip})}{5.6K} + \frac{(2.0)(93\text{ ft-kip})K(20.4\text{ ft})}{(739\text{ ft}^2)K} \\
&= 7.3\text{ kip} \\
M_3 &= \left(\frac{V_3}{2}\right) l_3 = \frac{(7.3\text{ kip})(13\text{ ft})}{2} \\
&= 47.5\text{ ft-kip}
\end{aligned}$$

Per the problem statement, ignore axial load effects on the column. Place reinforcement in opposite faces to resist reversible moments, but conservatively ignore the contribution of steel on the compression side to flexural strength. The given $0.2W$ seismic load includes the reliability factor. Take

$$\begin{aligned}
d &= h - 2.5\text{ in} = 13\text{ in} - 2.5\text{ in} \\
&= 10.5\text{ in}
\end{aligned}$$

$$\phi M_n = M_u = \phi\rho f_y b d^2 \left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right)$$

$$\begin{aligned}
\left(47.5\ \frac{\text{ft-kip}}{\text{ft}}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right) \\
&= 0.9\rho\left(40\ \frac{\text{kip}}{\text{in}^2}\right)(13\text{ in})(10.5\text{ in})^2 \\
&\quad \times \left(1 - 0.59\rho\left(\frac{40\ \frac{\text{kip}}{\text{in}^2}}{3\ \frac{\text{kip}}{\text{in}^2}}\right)\right) \\
\rho &= 0.0123 \\
A_s &= \rho b d = (0.0123)(13\text{ in})(10.5\text{ in}) \\
&= 1.68\text{ in}^2
\end{aligned}$$

This could be two no. 8 bars on each side. Check ductility.

$$\begin{aligned}
c &= \frac{A_s f_y}{\beta_1 0.85 f'_c b} \\
&= \frac{(2)(0.79\text{ in}^2)\left(40\ \frac{\text{kip}}{\text{in}^2}\right)}{(0.85)(0.85)\left(3\ \frac{\text{kip}}{\text{in}^2}\right)(13\text{ in})} \\
&= 2.24\text{ in} \\
&< 0.375d
\end{aligned}$$

Therefore, failure is controlled by tension, as assumed ($\phi = 0.9$).

E. The assumption of fixed-pinned condition is conservative for flexure; however, the shear design should be based on formation of the probable moments at both ends. Shear will be critical in the shortest column. For the two no. 8 bars (per ACI Sec. 21.5.4.1),

$$a = \frac{1.25 f_y A_s}{0.85 f'_c b} = \frac{(1.25)\left(40\ \frac{\text{kip}}{\text{in}^2}\right)(2)(0.79\ \text{in}^2)}{(0.85)\left(3\ \frac{\text{kip}}{\text{in}^2}\right)(13\ \text{in})}$$
$$= 2.38\ \text{in}$$

$$M_{\text{pr}} = 1.25 f_y A_s \left(d - \frac{a}{2}\right)$$
$$= (1.25)\left(40\ \frac{\text{kip}}{\text{in}^2}\right)(2)(0.79\ \text{in}^2) \times \left(10.5\ \text{in} - \frac{2.38\ \text{in}}{2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$
$$= 61.3\ \text{ft-kip}$$

$$V_u = \frac{2M_{\text{pr}}}{l_1} = \frac{(2)(61.3\ \text{ft-kip})}{9\ \text{ft}}$$
$$= 13.6\ \text{kip}$$

Provide transverse reinforcing per ACI Sec. 21.6.4.

$$A_{sh} \geq \begin{cases} 0.3\left(\frac{s b_c f'_c}{f_{yt}}\right)\left(\frac{A_g}{A_{ch}} - 1\right) \\ \quad = (0.3)\left(\frac{s b_c\left(3\ \frac{\text{kip}}{\text{in}^2}\right)}{40\ \frac{\text{kip}}{\text{in}^2}}\right) \times \left(\frac{(13\ \text{in})(13\ \text{in})}{(10.5\ \text{in})(10.5\ \text{in})} - 1\right) \\ \quad = 0.012 s b_c \quad \text{[controls]} \\ 0.09\left(\frac{s b_c f'_c}{f_{yt}}\right) \\ \quad = (0.09)\left(\frac{s b_c\left(3 \frac{\text{kip}}{\text{in}^2}\right)}{40\ \frac{\text{kip}}{\text{in}^2}}\right) \\ \quad = 0.0068 s b_c \end{cases}$$

For the column, b_c equals 10.5 in. Try two legs no. 4 hoops to confine the concrete.

$$s \leq \frac{A_{sh}}{0.012 b_c} = \frac{(2)(0.2\ \text{in}^2)}{(0.012)(10.5\ \text{in})}$$
$$= 3.2\ \text{in}$$

Other spacing limits of ACI Sec. 21.6.4 require

$$h_x = \frac{h}{2} - \text{cover} - d_h + \frac{d_{bl}}{2}$$
$$= 6.5\ \text{in} - 1.5\ \text{in} - 0.5\ \text{in} + \frac{1.0\ \text{in}}{2}$$
$$= 5.0\ \text{in}$$

$$s_o = 4\ \text{in} + \left(\frac{14\ \text{in} - h_x}{3}\right)$$
$$= 4\ \text{in} + \left(\frac{14\ \text{in} - 5.0\ \text{in}}{3}\right)$$
$$= 7.0\ \text{in}$$

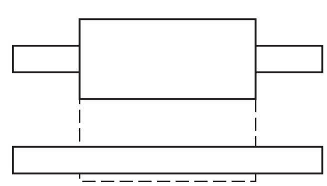
$$s \leq \begin{cases} 0.25 \times \text{least member dimension} \\ \quad = (0.25)(13\ \text{in}) \\ \quad = 3.25\ \text{in} \\ 6d_b \quad \text{[of longitudinal steel]} \\ \quad = (6)(1.0\ \text{in}) \\ \quad = 6.0\ \text{in} \\ s_o = 7.0\ \text{in} \end{cases}$$

The spacing of 3.2 in computed for the strong axis confinement controls. Check the shear strength furnished by the hoops (V_c is zero).

$$\phi V_s = \frac{\phi A_v f_y d}{s}$$
$$= \frac{(0.75)(0.40\ \text{in}^2)\left(40\ \frac{\text{kip}}{\text{in}^2}\right)(10.5\ \text{in})}{3.0\ \text{in}}$$
$$= 42\ \text{kip}$$
$$> V_u$$

Shear is OK.

F. The maximum deflection occurs in the two 13 ft columns resisting the amplified design shear of 7.3 kip. For the doubly reinforced cross section,



$$\frac{b(kd)^2}{2} + (n-1)A'_s(kd - d')$$
$$= nA_s$$

$$\frac{(13\ \text{in})(kd)^2}{2} + (9.29 - 1)(1.58\ \text{in}^2)(kd - 2.5\ \text{in})$$
$$= (9.29)(1.58\ \text{in}^2)(10.5\ \text{in} - kd)$$

$$kd = 3.54\ \text{in}$$

$$
\begin{aligned}
I_{\text{cr}} &= \frac{b(kd)^3}{3} + nA_s(d-kd)^2 + (n-1)A_s'(kd-d')^2 \\
&= \frac{(13 \text{ in})(3.54 \text{ in})^3}{3} \\
&\quad + (9.29)(1.58 \text{ in}^2)(10.5 \text{ in} - 3.54 \text{ in})^2 \\
&\quad + (9.29-1)(1.58 \text{ in}^2)(3.54 \text{ in} - 2.5 \text{ in})^2 \\
&= 917 \text{ in}^4 \\
I_g &= \frac{bh^3}{12} = \frac{(13 \text{ in})(13 \text{ in})^3}{12} \\
&= 2380 \text{ in}^4 \\
M_a &= M_3 = 47.5 \text{ ft-kip} \\
M_{\text{cr}} &= \frac{7.5\sqrt{f_c'}I_g}{y_t} \\
&= \frac{7.5\sqrt{3000 \ \frac{\text{lbf}}{\text{in}^2}}(2380 \text{ in}^4)}{6.5 \text{ in}}\left(\frac{1 \text{ kip}}{1000 \text{ lbf}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \\
&= 12.5 \text{ ft-kip}
\end{aligned}
$$

The extent of cracking is a function of the maximum service load.

$$
\begin{aligned}
I_e &= \left(\frac{M_{\text{cr}}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{\text{cr}}}{M_a}\right)^3\right) I_{\text{cr}} \\
&= \left(\frac{12.5 \text{ ft-kip}}{47.5 \text{ ft-kip}}\right)^3 (2380 \text{ in}^4) \\
&\quad + \left(1 - \left(\frac{12.5 \text{ ft-kip}}{47.5 \text{ ft-kip}}\right)^3\right)(917 \text{ in}^4) \\
&= 944 \text{ in}^4 \\
\delta_{xe} &= \left(\frac{V_3}{2}\right)\left(\frac{l_3^3}{3E_cI_e}\right) \\
&= \left(\frac{7.3 \text{ kip}}{2}\right)\left(\frac{(13 \text{ ft})^3\left(12 \ \frac{\text{in}}{\text{ft}}\right)^3}{(3)\left(3122 \ \frac{\text{kip}}{\text{in}^2}\right)(944 \text{ in}^4)}\right) \\
&= 1.57 \text{ in}
\end{aligned}
$$

The deflection δ_{xe} is the elastic deflection computed using the design seismic force. To account for inelastic action, this force was scaled by the system response factor, R. Applying ASCE 7 Eq. 12.8-15 gives a more realistic estimate.

$$
\begin{aligned}
\delta_x &= \frac{C_d\delta_{xe}}{I_e} = \frac{(1.5)(1.57 \text{ in})}{1.0} \\
&= 2.4 \text{ in}
\end{aligned}
$$

C_d is the deflection amplification factor taken from Table 9.5.2.2 treating the structure as an inverted pendulum, special reinforced concrete moment frame.

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Concrete Design

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Second Edition

An In-Depth Review of Concrete Design Methods and Standards

Concrete Design for the Civil PE and Structural SE Exams presents the concrete design and analysis methods most needed by civil and structural engineering students. The book's 12 chapters provide a concise but thorough review of concrete theory, code application, design principles, and structural analysis. The 51 example problems demonstrate how to apply concepts, codes, and equations, and over 40 figures and tables provide essential support material. A complete nomenclature list defines the industry-standard variables and symbols used in each chapter.

This book includes code references to familiarize you with exam-adopted codes, such as ASCE7 and ACI 318. It also includes 35 multiple-choice problems and 2 scenario-based design problems to enhance your problem-solving skills. Each problem's complete solution lets you check your solving approach. On exam day, you can use this book's thorough index to quickly locate important codes and concepts.

Topics Covered

- Columns and Compression Members
- Continuous One-Way Systems
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- Development of Reinforcement
- Flexural Design of Reinforced Concrete Beams
- Materials
- Prestressed Concrete
- Seismic Design of Reinforced Concrete Members
- Serviceability of Reinforced Concrete Beams
- Shear Design of Reinforced Concrete
- Two-Way Slab Systems

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